EFFECT OF DEPOSITION THICKNESS ON CRITICAL SHEAR STRESS FOR INCIPIENT MOTION OF SEDIMENTS

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The current understanding on how sediment deposition thickness could affect the incipient motion of sediment is still lacking in the literature. Few existing literature suggested that the current equation for incipient motion in rigid boundary channel with limited sediment depth become less accurate as the sediment bed thickness increased. The current study highlights the effect of sediment deposits thickness on critical shear stress of sediment in rigid rectangular channel. Results of experimental work on incipient motion were used to establish the effect of sediment deposits thickness on critical shear stress. Findings from the analysis shows that sediment deposits thickness has effect on the critical shear stress at low sediment bed thickness and the effect will reduce with the increased in sediment bed thickness. A new equation for predicting critical shear stress was proposed by incorporating the sediment deposits thickness and appears to be more consistent and was not much affected by the sediment deposits thickness as compares to the existing rigid boundary equations. The new equation is an attempt toward unifying the equations for both rigid boundary and loose boundary conditions.

Keywords: critical shear stress, incipient motion, loose boundary, rigid boundary, sediment

Introduction

Incipient motion is defined as the critical condition that is just adequate to initiate sediment motion (Dey and Papanicolaou, 2008). Though the literature on incipient motion of sediment is enormous, the majority of the literature were for loose boundary channel while only few researches were for rigid boundary channel (Novak and Nalluri, 1984, Mohammadi, 2005). For the purpose of design, the Shields diagram (Shields, 1936) was conventionally used to predict incipient motion of granular particles especially for loose boundary channel with unlimited supply of sediment such as alluvial channel (Vongvisessomjai et al., 2010). Despite the very different boundary conditions in sewers/drains (Ashley et al., 2004) which are of rigid boundary with limitation in terms of sediment depth and source of new sediment for transport; the Shields diagram has been applied in a number of studies on sewers and drains (Verbanck et al., 1994, Almedeij, 2012). The Shields diagram was developed using a relationship based on the balance between particle weight and boundary shear stress as shown in Equation (1):

$$\theta_c = \frac{\tau_c}{gd(\rho_s - \rho)} = f\left(\frac{u_* d}{\nu}\right) = f(\mathrm{Re}_*) \qquad (1)$$

where Θ_c is the dimensionless Shields stress; τ_c is the critical shear stress [N/m^2]; g is the gravitational acceleration [m/s^2]; ρ_s is the sediment density [kg/m^3]; ρ is the fluid density [kg/m^3]; d is the grain size (normally $d=d_{50}$ for uniform sediment) [m]; $u_*=(\tau_c/\rho)^{1/2}$ is the shear velocity [m/s]; v is the kinematic viscosity of fluid [m^2/s] and Re_* is the dimensionless grain Reynolds number. From 29 sources of data, a single curve representing the mean threshold values for $0.01<Re_*<10^5$ (Paphitis, 2001) is as described in Equation (2):

$$\theta_c = \frac{0.188}{1+\mathrm{Re}_*} + 0.0475\left(1 - 0.669 e^{-0.015\,\mathrm{Re}_*}\right) \qquad (2)$$

Novak and Nalluri (1975) derived the following critical shear stress expression for rectangular channels with rigid smooth bed:

$$\frac{\tau_c}{\rho g d(S_s - 1)} = 0.065\,\mathrm{Re}_*^{-0.52} \qquad (3)$$

where S_s is the specific gravity of sediment.

A universal equation for smooth and rough rigid beds was developed by El-Zaemey (1991) incorporating the effects of channel shape by the parameter y_0/B as follows:

$$\frac{\tau_c}{\rho g d(S_s - 1)} = 5.37(\mathrm{Re}_*)^{-0.44}(\lambda_0)^{1.00}\left(\frac{y_0}{B}\right)^{0.51} \qquad (4)$$

where λ_0 is the bed friction factor, B is the channel bed width [m] and y_0 is the normal flow depth in the channel [m] (El-Zaemey, 1991). The bed friction factor λ_0 can be calculated using the Darcy-Weisbach's equation (El-Zaemey, 1991, Salem, 2013):

$$\lambda_0 = \frac{8gRS}{V^2} \qquad (5)$$

where R is the hydraulic radius; S is the channel slope and V is the flow velocity [m/s].

Existing literatures (Novak and Nalluri, 1975, Salem, 2013) on rigid boundary channel have shown that the sediment particles are eroded at lower shear stress and critical velocity than that predicted by Shields diagram for alluvial channels. This means that the critical values for incipient motion for rigid boundary channels are substantially lower for any particle size than for loose boundary channels (Novak and Nalluri, 1975). Verification on existing critical velocity equations for rigid boundary channel has shown that these equations become less accurate as the thickness of sediment deposits increased (Bong et al., 2013). Hence there are evidences that the use of Shields diagram for rigid boundary channels will produce significant errors.

This paper aims to understand the effect of sediment deposits thickness on incipient motion which is still lacking in the literature. Data were obtained from experimental work on incipient motion in a rigid rectangular flume with varying sediment deposits thickness. New critical shear stress equation was proposed by incorporating the effect of sediment deposits thickness in the equation.

Dimensionless Analysis

Flow involving fluid and sediment such as the study of sediment transport and incipient motion are two-phase phenomenon and can be described by three components, i) fluid; ii) non-cohesive granular medium; and iii) flow (Yalin, 1977). The fluid is defined by its density ρ [kg/m^3]; the non-cohesive granular medium is defined by its density ρ_s [kg/m^3] and particle size d [m]; and the flow is defined by the hydraulic radius R [m] and gravity acceleration g [m^2/s] (Bong et al., 2013). The term for gravity acceleration g can be replaced with $\gamma_s=g(\rho_s-\rho)$ where γ_s is the specific weight for the sediment γ_s [N/m^3]. By choosing d, ρ and u_* as the repeating variables, the dimensionless variables of the two-phase phenomenon are as follows:

$$\frac{\tau_c}{\rho g d(S_s-1)}, \frac{u_* d}{\nu}, \frac{R}{d}, \frac{\rho_s}{\rho} \qquad (6)$$

The term ρ_s/ρ can be excluded since the density of fluid and sediment were not varied during the experiment and would be a constant. To incorporate sediment deposition thickness, the dimensionless terms of t_s/d and t_s/y_0; where t_s is the sediment bed thickness [m], d is the sediment particle size [m] and y_0 is the normal flow depth [m] can be included in the analysis. Hence, the dimensionless terms of the incipient motion function for critical shear stress are given by:

$$\frac{\tau_c}{\rho g d(S_s-1)} = f\left(\frac{u_* d}{\nu}, \frac{R}{d}, \frac{t_s}{d}, \frac{t_s}{y_0}\right) \qquad (7)$$

Methodology

The incipient motion experiment was conducted for six sediment deposition thickness namely; one layer ($t_s = d_{50}$), 5 mm, 10 mm, 24 mm, 48 mm and 100 mm. The schematic diagram for the experimental setup is as shown in Figure 1 while Table 1 shows the experimental parameters. The definition of incipient motion for the current study was of general movement (Kramer, 1935). During the experiment, water level and discharge were slightly increased by controlling the pump that supplies water into the flume until incipient motion was observed. Electronic flow meter was used to determine the velocity and discharge values during the experiment. More information on the experimental procedure for the present study could be referred to Bong (2013) and Bong et al. (2013). The observed critical shear stress was calculated using the following equation:

$$\tau_c = \rho g R S \tag{8}$$

where τ_c is the critical shear stress [N/m^2]; ρ is the density of fluid [kg/m^3]; g is the acceleration of gravity [m^2/s]; R is the hydraulic radius of flow and S is the flume slope.

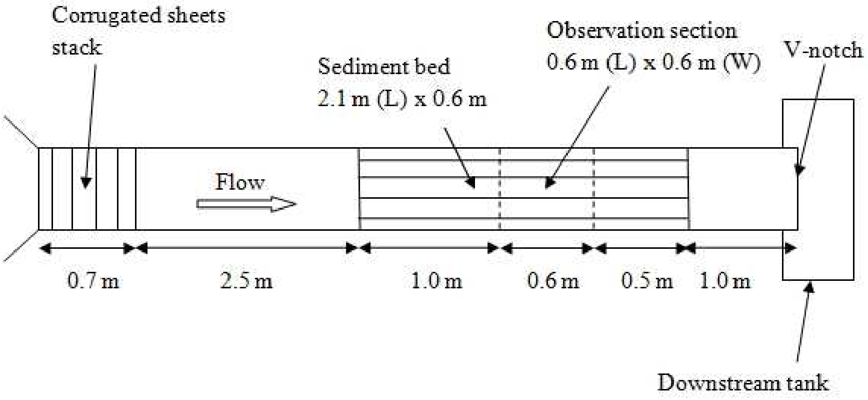


Figure 1: Schematic diagram experimental set up (not to scale) (Bong, 2013)

Multiple linear regressions were performed using the data from the current study for the dimensionless terms in Equation (7). The best regression model from the combination of dimensionless groups in Equation (7) was selected based on four criteria: (a) coefficient of determination R^2; (b) adjusted coefficient of determination R^2_{adj}; (c) mean square error *MSE*; and (d) Mallow's C_p statistics. Performance test on the new equation was done by calculating the discrepancy ratio using Equation (9). The acceptable range of discrepancy ratio is between 0.5 and 2.0 which is normally used for the study on sediment transport (Yang, 1996).

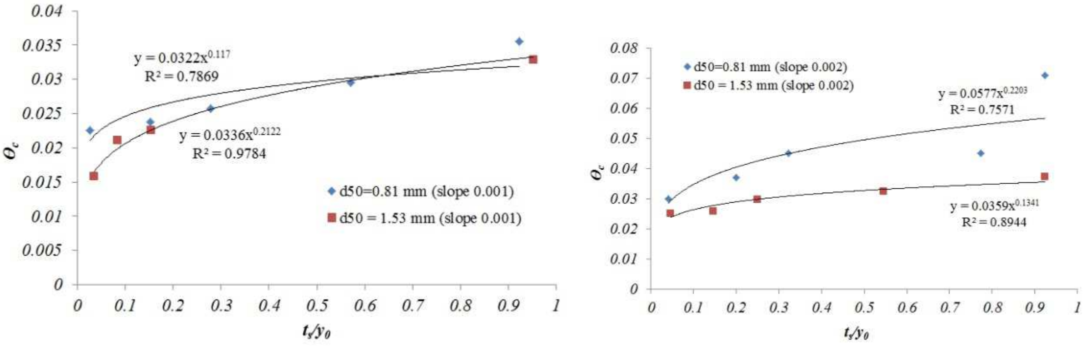
$$\text{Discrepancy ratio} = \frac{\tau_c \text{ predicted (m/s)}}{\tau_c \text{ observed (m/s)}} \tag{9}$$

Table 1: Range of experimental parameters for the current study

Parameter	Current study
Flume width B (m)	0.6
Critical velocity V_c (m/s)	0.216 – 0.400
Normal flow depth y_0 (m)	0.02 – 0.11
Flume slope S_0	0.001 and 0.002
Sediment median size d_{50} (mm)	0.81 and 1.53
Sediment specific gravity S_s	2.54 and 2.55
Sediment bed thickness t_s (mm)	0.81 – 100

Results and Discussion

To determine the effect of sediment deposition thickness on the critical shear stress for incipient motion, Θ_c was plotted against t_s/y_0 as shown in Figure 2. It was observed that Θ_c increases at diminishing rate as t_s/y_0 increases for both the flume slopes used in the current study. This shows that sediment deposits thickness have effect on the incipient motion of the sediment particle at low deposits thickness and the effect will slowly diminished with the increased in thickness of sediment deposits. It was also observed from both the graphs in Figure 2 that the relationship between Θ_c and t_s/y_0 is best fitted with power relationships (R^2 value close to unity). The effect could be due to the increase of friction that existed between the sediment particles with the increase in sediment deposits thickness. At thicker sediment deposits, the increase of friction between the sediment particles will slowly become negligible with further increase of the deposits thickness. Due to this effect, the existing equations for incipient motion in rigid boundary channel which are for limited sediment deposits thickness become less accurate as the deposits thickness increase since the equations did not incorporate the sediment bed thickness.



(a) (b)

Figure 2: Θ_c against t_s/y_0 for flume slope of: (a) 0.001; and (b) 0.002

Figure 3 shows the data from the current study when plotted on the Shields Diagram. The single curve on the Shields Diagram was obtained using Equation (2) developed by Paphitis (2001). Comparing the Θ_c calculated from the data of the current study using Equation (1) for both the flume slopes with the single curve, Θ_c values from the current study generally tend to be below the single curve for low sediment deposits thickness ($t_s=d_{50}$, 5 mm and 10 mm). As the sediment deposits thickness increase, the Θ_c values from the data of the current study tend to become closer to the single curve. This confirms the observation by Novak and Nalluri (1975) where the sediment particles for rigid boundary channels (limited sediment depth) were eroded at lower shear stress than that predicted by Shields diagram for alluvial channels.

From Figure 2 and Figure 3, it was obvious that sediment deposits thickness has effect on incipient motion of rigid boundary channel. This effect can be included in the critical shear stress equation by incorporating the dimensionless terms t_s/d and t_s/y_0 as shown in Equation (7). Table 2 shows the results of Pearson correlation analysis between Θ_c with t_s/d and t_s/y_0. Results from the Pearson correlation analysis showed that t_s/d has weak correlation with Θ_c (having correlation value less than 0.7) while t_s/y_0 has strong correlation with Θ_c (having correlation value more than 0.7).

Multiple linear regressions were performed and the results were as shown in Table 3. Table 3 shows the three best models among the regression models that incorporate the dimensionless terms t_s/d and t_s/y_0 for the current study. Equation (12) was chosen for further analysis in terms of performance test since it has R^2 and R^2_{adj} values closest to unity among the best models in the current study and having the smallest MSE value with C_p value close to the number of terms in the model.

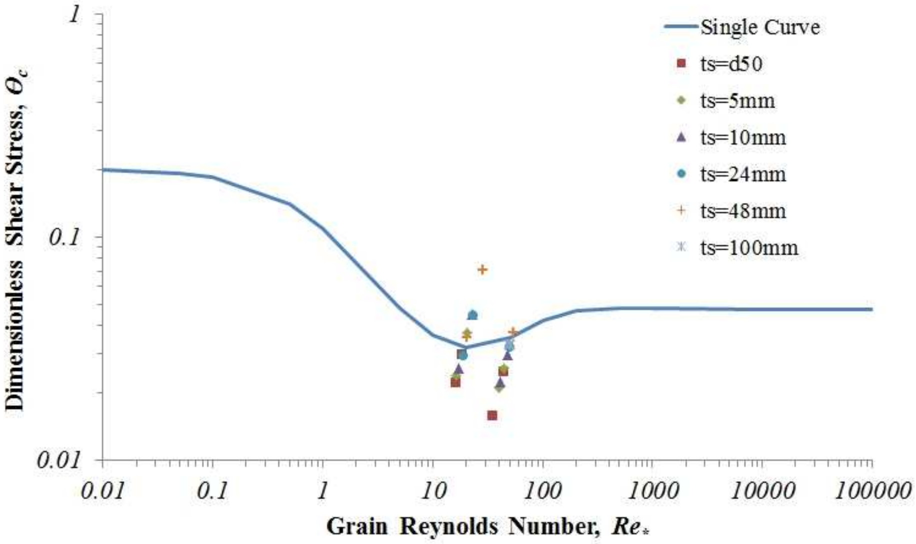


Figure 3: Data for the current study as plotted on the Shields Diagram

Table 2: Pearson correlation analysis for Θ_c with t_s/d and t_s/y_0

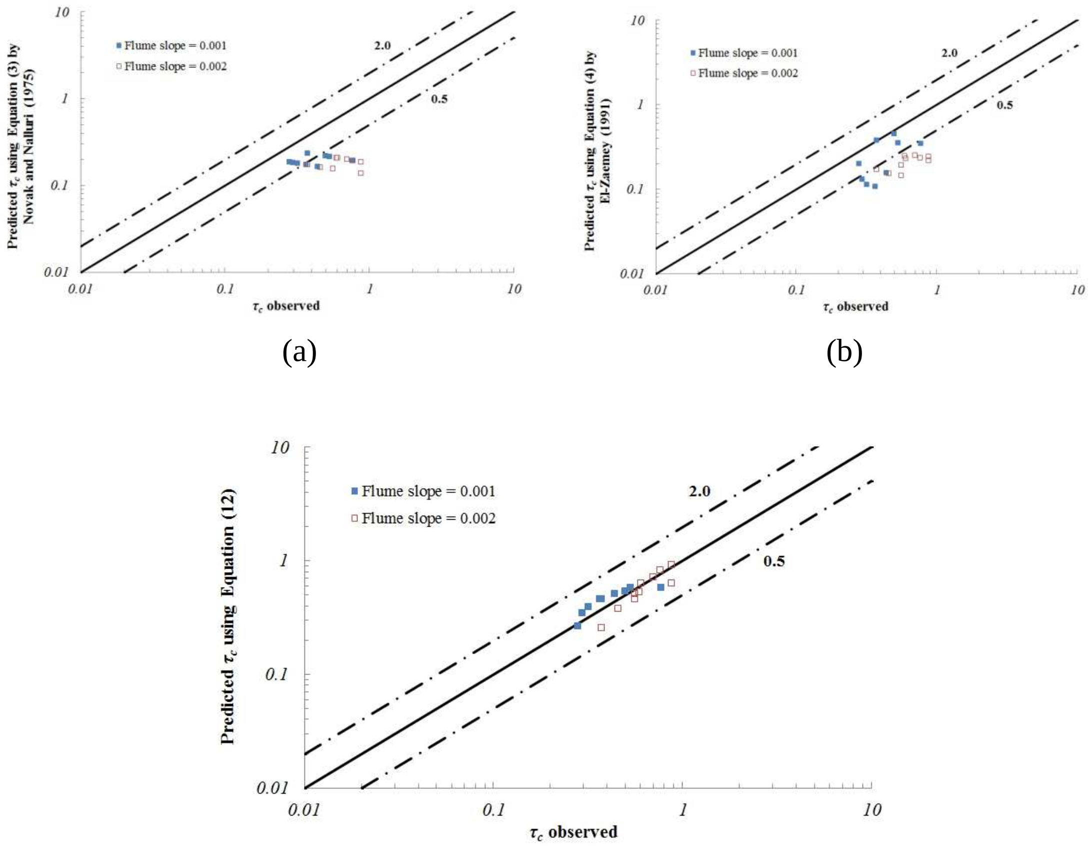
Dimensionless term	Θ_c
t_s/d	0.689 (p-value = 0.001)
t_s/y_0	0.714 (p-value = 0.001)

Table 3: Results from multiple linear regressions analysis

Model	R^2	R^2_{adj}	MSE	C_p	Eq.
$\frac{\tau_c}{\rho g d(S_s-1)}=0.190\left(\frac{u_* d}{\nu}\right)^{-0.205}\left(\frac{t_s}{d}\right)^{-0.215}\left(\frac{t_s}{y_0}\right)^{0.462}$	0.57	0.48	0.011	3.99	(10)
$\frac{\tau_c}{\rho g d(S_s-1)}=0.035\left(\frac{R}{d}\right)^{2.72}\left(\frac{t_s}{d}\right)^{-2.57}\left(\frac{t_s}{y_0}\right)^{2.79}$	0.63	0.55	0.001	4.00	(11)
$\frac{\tau_c}{\rho g d(S_s-1)}=0.00029\left(\frac{u_* d}{\nu}\right)^{0.636}\left(\frac{R}{d}\right)^{8.33}\left(\frac{t_s}{d}\right)^{-7.27}\left(\frac{t_s}{y_0}\right)^{7.40}$	0.69	0.60	0.009	3.00	(12)

Performance test was done by calculating the discrepancy ratio between τ_c predicted by Equation (12) with the τ_c observed from the experiment in the current study. Discrepancy ratio values were calculated using Equation (9) and the number of values within the acceptable range of 0.5 to 2.0 was noted. As comparison, performance test were also done using existing rigid boundary equations by Novak and Nalluri (1975) and El-Zaemey (1991). Figure 4(a), Figure 4(b) and Figure 4(c) show the comparison between observed and predicted critical shear stress using the data from the current study for equations by Novak and Nalluri (1975), El-Zaemey (1991) and Equation (12) respectively. Results from Figure 4 for the performance test are as summarized in Table 4. From Table 4, it was observed that Equation (12) performed better than the existing equations by Novak and Nalluri (1975) and El-Zaemey (1991) by having all the predicted values within the acceptable range.

Table 5 shows the results of performance test in terms of various sediment deposits thickness. With the discrepancy ratio acceptable range values of between 0.5 and 2.0, it was observed that equation by Novak and Nalluri (1975) only predicts the critical shear stress satisfactory for a single layer thickness of the sediment deposit. Equation by El-Zaemey (1991) predicts satisfactory the critical shear stress for a single layer and 5 mm of sediment deposits thickness. The newly developed Equation (12) from the current study predicts the critical shear stress reasonably well for all the sediment deposits thickness used in the current study and appears to be consistent and not much affected by the sediment deposits thickness. Hence, Equation (12) developed from the current study brought improvement in terms of the prediction power for critical shear stress values as compared to the existing equations for rigid boundary channel in the literature.



(a) (b)

(c)

Figure 4: Comparison between observed and predicted critical shear stress for equations by (a) Novak and Nalluri (1975); (b) El-Zaemey (1991); and (c) Equation (12)

Table 4: Comparison of results from performance test

Equation	Values within acceptable discrepancy ratio (0.5 – 2.0)	
	Number of data	Percentage of data (%)
Novak and Nalluri (1975)	4	21.1
El-Zaemey (1991)	4	21.1
Equation (12)	19	100.0

Table 5: Comparison of results from performance test

Equation	Discrepancy ratio values for various sediment deposits thickness t_s						Average
	d_{50}	5 mm	10 mm	24 mm	48 mm	100 mm	
Novak and Nalluri (1975)	0.543	0.450	0.392	0.344	0.253	0.258	0.373
El-Zaemey (1991)	0.666	0.532	0.444	0.295	0.302	0.467	0.450
Equation (12)	0.971	1.058	1.071	1.115	1.001	0.772	0.998

Conclusions

The current study aims to understand the effect of sediment deposits thickness on the critical shear stress for incipient motion. Experimental work on incipient motion was conducted in a rectangular flume by varying the thickness of sediment deposits. Results show that sediment deposits thickness has effect on the incipient motion of the sediment particle at low sediment deposits thickness and the effect will slowly diminished with the increased in thickness of sediment deposits. Existing critical shear stress equations for rigid boundary channel were proven to be not accurate as the sediment deposits thickness increased. A new equation was proposed in the current study to predict the critical shear stress during incipient motion by incorporating the sediment deposits thickness. This new equation appears to be more consistent and was not much affected by the sediment deposits thickness as compares to the existing rigid boundary equations.

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