

A New Sign Distance-Based Ranking Method for Fuzzy Numbers

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Abstract. In this paper, a new sign distance-based ranking method for fuzzy numbers is proposed. It is a synthesis of geometric centroid and sign distance. The use of centroid and sign distance in fuzzy ranking is not new. Most existing methods (e.g., distance-based method [9]) adopt the Euclidean distance from the origin to the centroid of a fuzzy number. In this paper, a fuzzy number is treated as a polygon, in which a new geometric centroid for the fuzzy number is proposed. Since a fuzzy number can be represented in different shapes with different spreads, a new dispersion coefficient pertaining to a fuzzy number is formulated. The dispersion coefficient is used to fine-tune the geometric centroid, and subsequently sign distance from the origin to the tuned geometric centroid is considered. As discussed in [5-9], an ideal fuzzy ranking method needs to satisfy seven reasonable fuzzy ordering properties. As a result, the capability of the proposed method in fulfilling these properties is analyzed and discussed. Positive experimental results are obtained.

Keywords: Geometric centroid, sign distance, fuzzy numbers, reasonable ordering properties, dispersion coefficient.

1 Introduction

Fuzzy ranking attempts to order a set of fuzzy numbers. The importance of fuzzy ranking in many application domains has been highlighted in the literature, e.g., decision-making [1], data analysis [2], risk assessment [3] and artificial intelligence [4]. Indeed, many fuzzy ranking methods have also been developed [1][5][6]. These methods can be categorized into three categories [5-6]; i.e., (1) transforming a set of fuzzy numbers into crisp numbers and subsequently ranking the crisp numbers; (2) mapping a set of fuzzy numbers to crisp numbers based on a pre-defined reference set(s) for comparison; (3) ranking through pairwise comparison of fuzzy numbers.

In this paper, our focus is on the first category, which is straightforward as compared with the other two categories. Nevertheless, the challenge is to develop a method that satisfies a set of reasonable ordering properties [5-9], as detailed in Section 2.2. Recently, a number of methods, e.g., deviation degree-based [8][10], distance-based [9][11], centroid-based [12], and area-based [13], have been proposed. However, many of these methods [8][10-13] do not have analysis pertaining to fulfillment

of the reasonable ordering properties [5-9]. Besides that, fuzzy ranking is complicated when fuzzy numbers are presented as different shapes with different spreads, e.g., as illustrated in Fig. 1 [8].

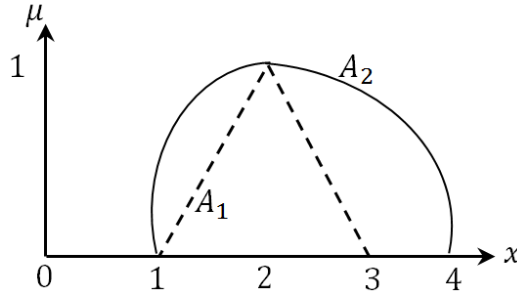


Fig. 1. Triangular (i.e., A_1) and generalized (i.e., A_2) fuzzy numbers [8]

In this paper, a new fuzzy ranking method that deals with fuzzy numbers (Definition 2), which is a synthesis of geometric centroid and sign distance, is proposed. The proposed method is capable of handling more complex fuzzy numbers i.e., generalized fuzzy numbers, as illustrated in Fig. 1. The method can be summarized into a few steps, as follows: (1) Fuzzy numbers are firstly converted into normalized fuzzy numbers (i.e., a set of fuzzy numbers available in space $R^* \in [-1,1]$). (2) A discretization technique [14], i.e., slicing the support of each normalized fuzzy number into N slices vertically at even intervals, is employed. The intersections between the normalized fuzzy number and the N vertical slices are called intersection points or discretized points. (3) Using the discretized points, geometric centroid of the normalized fuzzy number is then computed using Bourke's method [15]. To deal with more complex fuzzy numbers, as depicted in Fig. 1, a new dispersion coefficient of the normalized fuzzy numbers is proposed. The dispersion coefficient attempts to fine-tune the geometric centroid. (4) Finally, sign distance between the origin and the tuned geometric centroid is measured. In this paper, the results obtained from the sign distances are used to order a set of fuzzy numbers. The fulfilment of the seven reasonable ordering properties as stated in [5-9] is further examined. To evaluate the proposed method, benchmark examples from [8] are used.

The rest of the paper is organized as follows. In Section 2, some background studies are presented. In Section 3, the proposed ranking method is explained. In Section 4, an experimental study and the associated results are presented. Finally, concluding remarks are given in Section 5.

2 Background

2.1 Definitions

The following mathematical notations are widely used. R denotes a set of real numbers, μ represents a fuzzy number, and $\mu(x)$ for its membership function, $\forall x \in R$. Definition 1, as follows, is considered.

Definition 1 [16]: A fuzzy number, A is a fuzzy set such that $\mu : R \rightarrow [0,1]$ which satisfies the following properties:

- A is represented as (a_1, a_2, a_3, a_4)
- μ is upper semi-continuous
- a_1, a_2, a_3 and a_4 are real numbers, in such $a_1 \leq a_2 \leq a_3 \leq a_4$. μ is strictly increasing for interval $[a_1, a_2]$ and strictly decreasing for interval $[a_3, a_4]$.
- $\mu(x) = 1$, for $a_2 \leq x \leq a_3$,
- $\mu(x) = 0$ for x not in the interval of $[a_1, a_4]$,
- Support of A , i.e., $sup(A) = \{x \in R | \mu(x) > 0\}$.
- μ is expressed as follows:

$$\mu(x) = \begin{cases} \mu^L(x), & a_1 \leq x \leq a_2, \\ 1, & a_2 \leq x \leq a_3, \\ \mu^R(x), & a_3 \leq x \leq a_4, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

where $\mu^L: [a_1, a_2] \rightarrow [0,1]$ and $\mu^R: [a_3, a_4] \rightarrow [0,1]$ are left and right membership functions of fuzzy number u . If $\mu^L(x)$ and $\mu^R(x)$ are linear functions and $a_2 < a_3$, then A is a trapezoidal fuzzy number. If $\mu^L(x)$ and $\mu^R(x)$ are linear functions, $a_2 = a_3$, then A is a special case of a trapezoidal fuzzy number i.e., a triangular fuzzy number.

Definition 2 [17]: Normalized fuzzy numbers (i.e., $A_i^*, i = 1,2,3, \dots, m$) are a set of fuzzy numbers in space $R^* \in [-1,1]$. Consider a set of fuzzy numbers A_i where $i = 1,2,3, \dots, m$ in space $R \in (-\infty, \infty)$. A_i^* is obtained with Eq. (2).

$$A_i^* = \left(\frac{a_{i1}}{k}, \frac{a_{i2}}{k}, \frac{a_{i3}}{k}, \frac{a_{i4}}{k}\right) = (a_{i1}^*, a_{i2}^*, a_{i3}^*, a_{i4}^*) \tag{2}$$

where $k = \max(|a_{ij}|, 1)$, $|a_{ij}|$ is the absolute value of a_{ij} , $1 \leq i \leq m$ and $1 \leq j \leq 4$. As an example, $A_1^* = (a_{11}^*, a_{12}^*, a_{13}^*, a_{14}^*)$ is illustrated in Fig.2, and centroids of A_1^* is written as $cX_{A_1^*}, cY_{A_1^*}$.

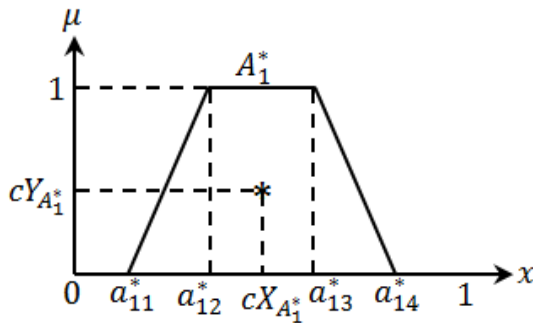


Fig. 2. The membership function of a normalized trapezoidal fuzzy number

2.2 Seven Reasonable Ordering Properties

Consider three fuzzy numbers, i.e., A_1 , A_2 , and A_3 in space R . An ideal fuzzy ranking method should satisfy seven reasonable ordering properties, as follows. The first six reasonable ordering properties (i.e., **P1-P6**) are introduced in [5-6], while **P7** is introduced in recent literatures [7-9].

P1: If $A_1 \succcurlyeq A_2$ and $A_2 \succcurlyeq A_1$, then $A_1 \sim A_2$.

P2: If $A_1 \succcurlyeq A_2$ and $A_2 \succcurlyeq A_3$, then $A_1 \succcurlyeq A_3$.

P3: If $A_1 \cap A_2 = \emptyset$ and A_1 is on the right of A_2 , then $A_1 \succcurlyeq A_2$.

P4: The order of A_1 and A_2 is not affected by other fuzzy numbers under comparison.

P5: If $A_1 \succcurlyeq A_2$, then $A_1 + A_3 \succcurlyeq A_2 + A_3$.

P6: If $A_1 \succcurlyeq A_2$, then $A_1 A_3 \succcurlyeq A_2 A_3$.

P7: If $A_1 \succcurlyeq A_2 \succcurlyeq A_3$ then $-A_1 \preccurlyeq -A_2 \preccurlyeq -A_3$.

This paper focuses on fulfillment of these seven reasonable ordering properties for trapezoidal fuzzy numbers. Note that the addition and multiplication operators in **P5** and **P6** are based on fuzzy arithmetic operators as follows:

Fuzzy Addition operator [11]:

$$\begin{aligned} A_1 \oplus A_2 &= (a_{11}, a_{12}, a_{13}, a_{14}) \oplus (a_{21}, a_{22}, a_{23}, a_{24}) \\ &= (a_{11} + a_{21}, a_{12} + a_{22}, a_{13} + a_{23}, a_{14} + a_{24}) \end{aligned} \quad (3)$$

Fuzzy Multiplication [11]:

$$\begin{aligned} A_1 \otimes A_2 &= (a_{11}, a_{12}, a_{13}, a_{14}) \otimes (a_{21}, a_{22}, a_{23}, a_{24}) \\ &= (a_{11} \times a_{21}, a_{12} \times a_{22}, a_{13} \times a_{23}, a_{14} \times a_{24}) \end{aligned} \quad (4)$$

Fuzzy Image [7]-[9]:

Fuzzy image of $A_1 = (a_{11}, a_{12}, a_{13}, a_{14})$ is $-A_1 = (-a_{14}, -a_{13}, -a_{12}, -a_{11})$.

3 The Proposed Methodology

Consider m fuzzy numbers, A_i , i.e., A_1, A_2, \dots, A_m , in space $R \in [-\infty, \infty]$, which have to be ranked. The proposed method is summarized into five steps as follows:

Step 1: Transform each fuzzy number $A_i = (a_{i1}, a_{i2}, a_{i3}, a_{i4})$ into a normalized fuzzy number $A_i^* = (a_{i1}^*, a_{i2}^*, a_{i3}^*, a_{i4}^*)$ using Eq. (2).

Step 2: Discretize the support of A_i^* into N points, i.e., x_{ij}^* , $j = 1, 2, 3, \dots, N$, and obtain $\mu_{A_i^*}(x_{ij}^*)$, where $x_{ij}^* \in R^*$. The discretized points are expressed in a sequence of $x_{i1}^*, x_{i2}^*, \dots, x_{iN}^*$, where x_{i1}^* and x_{iN}^* are the left- and right-end points of A_i^* , i.e., a_{i1}^* and a_{i4}^* , respectively. The discretized points in the horizontal component, i.e., x_{ij}^* , are computed using Eq. (5). On the other hand, the discretized points in the vertical component, i.e., $\mu_{A_i^*}(x_{ij}^*)$, are computed using Eq. (1). The discretized points for A_i^* are expressed in Eqs. (6) and (7) as follows.

$$x_{ij}^* = x_{i1}^* + (j - 1) \left(\frac{x_{iN}^* - x_{i1}^*}{N-1} \right), \quad (5)$$

$$x^* = \begin{bmatrix} x_{11}^* & \cdots & x_{1N}^* \\ \vdots & \ddots & \vdots \\ x_{m1}^* & \cdots & x_{mN}^* \end{bmatrix}, \quad (6)$$

$$\mu_{A_i^*}(x^*) = \begin{bmatrix} \mu_{A_i^*}(x_{11}^*) & \cdots & \mu_{A_i^*}(x_{1N}^*) \\ \vdots & \ddots & \vdots \\ \mu_{A_i^*}(x_{m1}^*) & \cdots & \mu_{A_i^*}(x_{mN}^*) \end{bmatrix}, \quad (7)$$

where $i = 1, 2, \dots, m$, $j = 1, 2, 3, \dots, N$.

Step 3: Compute the centroid of A_i^* (i.e., $cX_{A_i^*}$, $cY_{A_i^*}$). In this paper, the geometric centroid [15] is adopted, whereby $cX_{A_i^*}$ and $cY_{A_i^*}$ are obtained with Eqs. (8) and (9), as follows.

$$cX_{A_i^*} = \frac{1}{6L_{A_i^*}} \sum_{j=1}^N (x_{ij}^* + x_{i(j+1)}^*) (x_{ij}^* \mu_{A_i^*}(x_{i(j+1)}^*) + x_{i(j+1)}^* \mu_{A_i^*}(x_{ij}^*)), \quad (8)$$

$$cY_{A_i^*} = \begin{cases} \frac{1}{6L_{A_i^*}} \sum_{j=1}^N [(\mu_{A_i^*}(x_{ij}^*) + \mu_{A_i^*}(x_{i(j+1)}^*)) (x_{ij}^* \mu_{A_i^*}(x_{i(j+1)}^*) - x_{i(j+1)}^* \mu_{A_i^*}(x_{ij}^*))], & \text{if } x_{i1}^* \neq x_{iN}^* \\ \frac{\mu_{A_i^*}(x_i^*)}{2}, & \text{if } x_{i1}^* = x_{iN}^* \end{cases}, \quad (9)$$

where $L_{A_i^*} = \frac{1}{2} \sum_{j=1}^N (x_{ij}^* \mu_{A_i^*}(x_{i(j+1)}^*) - x_{i(j+1)}^* \mu_{A_i^*}(x_{ij}^*))$, $i = 1, 2, 3, \dots, m$, $j = 1, 2, 3, \dots, N$.

Step 4: Refine the centroid (i.e., $cX_{A_i^*}$, $cY_{A_i^*}$) using Eqs. (10)-(13) to obtain the new centroid, i.e., $cX_{A_i^*}^*$, $cY_{A_i^*}^*$.

$$\bar{X}_{A_i^*} = \frac{1}{N} \sum_{j=1}^N x_{ij}^*, \quad (10)$$

$$\delta_{A_i^*} = \sqrt{\frac{1}{N-1} \sum_{j=1}^N (x_{ij}^* - \bar{X}_{A_i^*})^2}, \quad (11)$$

$$cX_{A_i^*}^* = cX_{A_i^*}, \quad (12)$$

$$cY_{A_i^*}^* = \frac{H_i}{2} - \delta_{A_i^*} cY_{A_i^*}, \quad (13)$$

where $\delta_{A_i^*}$ denotes a dispersion coefficient of A_i^* , and $H_i = \max_{j=1,2,\dots,N} \{\mu_{A_i^*}(x_{ij}^*)\}$, $i = 1, 2, 3, \dots, m$.

Step 5: Compute the ordering index, $I_{A_i^*}$, using Eqs. (14)-(15) as follows.

$$\varphi_{A_i^*} = \begin{cases} 1, & \text{sign}(cX_{A_i^*}^*) \geq 0 \\ -1, & \text{sign}(cX_{A_i^*}^*) < 0 \end{cases} \quad (14)$$

$$I(A_i^*) = \varphi_{A_i^*} \sqrt{cX_{A_i^*}^{*2} + cY_{A_i^*}^{*2}}, \quad (15)$$

where $\varphi_{A_i^*}$ indicates the position of the fuzzy numbers. $I(A_i^*)$ is the distance between point $(cX_{A_i^*}, cY_{A_i^*})$ and the origin $(0,0)$. A larger ordering index (i.e., $I(A_i^*)$) indicate a higher the ranking order.

4 Experimental Study

In this section, benchmark examples from [8] are used to evaluate the proposed method for ranking fuzzy numbers. An analysis of P1-P7 [5-9] is further reported.

4.1 Ranking Generalized Fuzzy Numbers

An example from [8], as depicted in Fig.1, is considered. Fuzzy membership functions for A_1 and A_2 , are as follows.

$$\mu_{A_1}(x) = \begin{cases} x - 1, & 1 \leq x \leq 2, \\ 3 - x, & 2 \leq x \leq 3, \\ 0, & \text{otherwise,} \end{cases} \tag{16}$$

$$\mu_{A_2}(x) = \begin{cases} [1 - (x - 2)^2]^{1/2}, & 1 \leq x \leq 2, \\ [1 - \frac{1}{4}(x - 2)^2]^{1/2}, & 2 \leq x \leq 4, \\ 0, & \text{otherwise.} \end{cases} \tag{17}$$

The fuzzy numbers, A_1 and A_2 in Fig. 1, are transformed into normalized fuzzy numbers (i.e., A_1^*, A_2^* respectively), as illustrated in Fig.3.

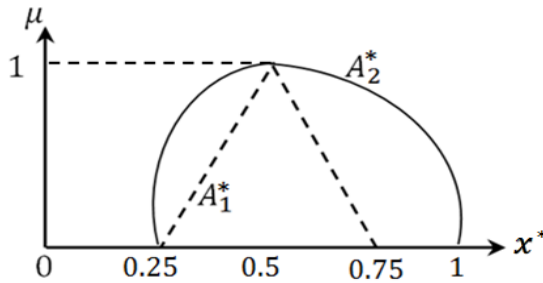


Fig. 3. Normalized triangular (i.e., A_1^*) and generalized (i.e., A_2^*) fuzzy numbers [8]

With the proposed method, $I(A_1^*) = 0.669$ and $I(A_2^*) = 0.723$ i.e., $I(A_1^*) < I(A_2^*)$; therefore $A_1^* \preceq A_2^*$ and $A_1 \preceq A_2$. The ranking results are in agreement with those in [8].

4.2 Analysis of the Seven Reasonable Ordering Properties

Three fuzzy numbers A_1, A_2 , and A_3 , are considered, where A_1^*, A_2^* , and A_3^* are normalized fuzzy numbers. An ideal fuzzy ranking method should comply with the seven reasonable ordering properties [5-9] as follows:

- P1:** $I(A_1^*) \geq I(A_2^*)$ occurs if $A_1 \succcurlyeq A_2$ and $I(A_2^*) \geq I(A_1^*)$ occurs if $A_2 \succcurlyeq A_1$. Therefore, $I(A_1^*) = I(A_2^*)$ is always true, if $A_1 \sim A_2$.
- P2:** $I(A_1^*) \geq I(A_2^*)$ occurs if $A_1 \succcurlyeq A_2$ and $I(A_2^*) \geq I(A_3^*)$ occurs if $A_2 \succcurlyeq A_3$. Therefore, $I(A_1^*) \geq I(A_3^*)$ is always true, for $A_1 \succcurlyeq A_2 \succcurlyeq A_3$.
- P3:** If $A_1 \cap A_2 = \emptyset$, and if A_1 is on the right side of A_2 , then $I(A_1^*) > I(A_2^*)$ is always true.
- P4:** $I(A_1^*)$ and $I(A_2^*)$ are computed separately to order A_1 and A_2 . Therefore, the ranking outcome is not affected by other fuzzy numbers under comparison.
- P5:** If $A_1 \succcurlyeq A_2$, then $I(A_1^*) \geq I(A_2^*)$. An addition of A_3 to A_1 and A_2 changes the centroids of A_1 and A_2 correspondingly. Therefore, $I(A_1 + A_3^*) \geq I(A_2 + A_3^*)$, if $A_1 + A_3 \succcurlyeq A_2 + A_3$.
- P6:** If $A_1 \succcurlyeq A_2$, $I(A_1^*) \geq I(A_2^*)$. Similar to P5, a multiplication of A_3 to A_1 and A_2 changes the centroids of A_1 and A_2 correspondingly. Therefore, $I(A_1 A_3^*) \geq I(A_2 A_3^*)$, if $A_1 A_3 \succcurlyeq A_2 A_3$.
- P7:** If $A_1 \succcurlyeq A_2 \succcurlyeq A_3$, then $I(A_1^*) \geq I(A_2^*) \geq I(A_3^*)$. Consider $\varphi_{A_1^*} = \varphi_{A_2^*} = \varphi_{A_3^*} = 1$ for A_1, A_2 and A_3 . For $-A_1, -A_2$ and $-A_3$, $\varphi_{-A_1^*} = \varphi_{-A_2^*} = \varphi_{-A_3^*} = -1$. Therefore, $-I(-A_1^*) \leq -I(-A_2^*) \leq -I(-A_3^*)$, for $-A_1 \preccurlyeq -A_2 \preccurlyeq -A_3$.

The proof for **P1-P4** is straightforward. Analysis of **P5, P6** and **P7** are more complicated, and are further illustrated with an example in [8]. The details are presented in the following section.

4.3 An Empirical Study

Consider an example in [8], $A_1 = (2,4,4,6)$; $A_2 = (1,5,5,6)$, and $A_3 = (3,5,5,6)$ as shown in Fig. 4(a). $A_1 + A_3$, $A_2 + A_3$, $A_1 A_3$, and $A_2 A_3$ (as depicted in Fig. 4(b) and Fig. 4(c)) are computed using fuzzy addition and fuzzy multiplication as summarized in Eqs. (3) and (4). Fuzzy images of A_1, A_2 , and A_3 are presented in Fig. 4(d). Fuzzy numbers in Fig. 4 are normalized and presented in Fig. 5.

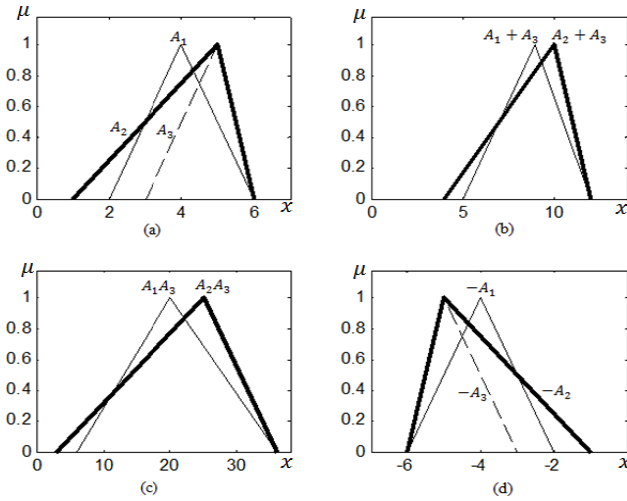


Fig. 4. (a) Three fuzzy numbers, A_1 , A_2 , and A_3 , (b) fuzzy addition, $A_1 + A_3$ and $A_2 + A_3$, (c) fuzzy multiplication, A_1A_3 and A_2A_3 , and (d) fuzzy images $-A_1, -A_2$ and $-A_3$

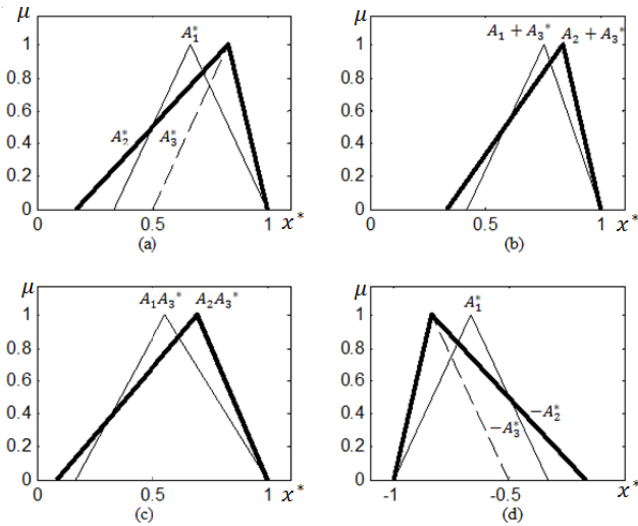


Fig. 5. Normalized fuzzy numbers of fuzzy numbers in Fig.4

The objective of this section is to analyze **P5**, **P6**, and **P7** empirically. In this paper, $N = 11$ is considered. The experimental results are summarized in Table 1. Columns " $I(A_1^*)$ ", " $I(A_2^*)$ ", and " $I(A_3^*)$ " show the ordering indexes for A_1 , A_2 , and A_3 , respectively. Columns " $I(A_1 + A_3^*)$ " and " $I(A_2 + A_3^*)$ " show the ordering indexes for $A_1 + A_3$ and $A_2 + A_3$, respectively. Columns " $I(A_1A_3^*)$ " and " $I(A_2A_3^*)$ " show the ordering indexes for A_1A_3 and A_2A_3 , respectively. Columns " $I(-A_1^*)$ ", " $I(-A_2^*)$ " and " $I(-A_3^*)$ " show the ordering indexes for $-A_1$, $-A_2$, and $-A_3$, respectively.

Table 1. Ranking results of the proposed method

$I(A_1^*)$	$I(A_2^*)$	$I(A_3^*)$	$I(A_1 + A_3^*)$	$I(A_2 + A_3^*)$	$I(A_1A_3^*)$	$I(A_2A_3^*)$	$I(-A_1^*)$	$I(-A_2^*)$	$I(-A_3^*)$
0.791	0.782	0.872	0.827	0.822	0.687	0.684	-0.791	-0.782	-0.872
$A_2 \leq A_1 \leq A_3$			$A_2 + A_3 \leq A_1 + A_3$		$A_2A_3 \leq A_1A_3$		$-A_3 \leq -A_1 \leq -A_2$		

P5, **P6** and **P7** can be observed from Table 1. With the proposed method, $I(A_1^*) = 0.791$, $I(A_2^*) = 0.782$, $I(A_3^*) = 0.872$; as such $A_2 \leq A_1 \leq A_3$; $I(A_1 + A_3^*) = 0.827$, and $I(A_2 + A_3^*) = 0.822$; therefore $I(A_1 + A_3^*) > I(A_2 + A_3^*)$ is satisfied. On the other hand, $I(A_1A_3^*) = 0.687$ and $I(A_2A_3^*) = 0.684$; therefore $I(A_1A_3^*) > I(A_2A_3^*)$ is satisfied. Lastly, $I(-A_1^*) = -0.791$, $I(-A_2^*) = -0.782$ and $I(-A_3^*) = -0.872$; therefore $-A_3 \leq -A_1 \leq -A_2$. In short, **P5**, **P6** and **P7** are satisfied.

5 Concluding Remarks

In this paper, a new fuzzy ranking method is proposed. It constitutes a solution for ranking fuzzy numbers. The proposed method has been empirically analyzed using benchmark examples. The seven reasonable ordering properties i.e., **P1-P7** [5-9], have been satisfied empirically. The rationale and implications of the proposed method have also been analyzed and discussed.

For future work, the proposed method can be extended to measure similarity between fuzzy numbers. Application of the proposed method to decision making [17] will also be investigated.

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