Relaxation Frequency and Relaxation Time Estimation for Phantom Modeling by Proposed Fitting Linear Models

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***Abstract*— This study aims to investigate the effect of quantity of sugar to the relaxation time needed for a mixture to accomplish steady state. Fitting linear models were proposed for fitting and extrapolating the incomplete permittivity graph due to the lacking of Vector Network Analyzer (VNA) machine unable to operate at frequency higher than 20GHz. The proposed models gave an adequate internal consistency of relaxation time calculation for the four mixtures. Result shows that the quantity of element sugar is the main reason of affecting the permittivity and relaxation time of the mixture. This findings enable contribute to related field researchers due to the effect of sugar that may influence the relaxation time for a mixture model.**

Keywords-dielectric; relative permittivity; relaxation frequency ; relaxation time; extrapolation.

I. INTRODUCTION

Every material has a unique set of electrical characteristics which dependent on its dielectric properties. The dielectric properties of material at radio and microwave frequencies range gain a great interest in research field. The information provides another alternative in material properties' study.

The dielectric properties consist of a complex number which consisting of a real permittivity ($\mathcal{E}_r'$ or e') and an imaginary permittivity ($\mathcal{E}_r''$or e'') [1]. The dielectric constant ($\mathcal{E}_r$ or κ) is an indication of the polarizability of the molecules and their ability to store electric energy. The dielectric loss factor ($\mathcal{E}_r''$ or e'') is related to the energy absorption and dissipation of electromagnetic energy from the field [2].

Based on these dielectric properties, relaxation time can be calculated to measure the mobility of the exits molecules. The relaxation time provides important information regarding the steady stage and relaxation frequency of the material.

II. LITERATURE REVIEW

In this study, dielectric properties measurement has been done using HP 8720B Vector Network Analyzer (VNA) with HP 85070B open-ended coaxial high temperature sensor probe.

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A. Relaxation Time

Relaxation time, τ describes the mobility of the molecules which occurs inside a material. Liquid and solid materials have molecules that are in a condensed state condition with limited freedom for moving when an electric field is applied. Constant collisions cause internal friction and hence that particular molecules turn slowly and exponentially approach the final state of orientation polarization with relaxation time constant, τ.

When the field is switched off, the sequence is reversed and random distribution is restored with the same time constant. The relaxation frequency, f_c is inversely related to relaxation time [3]:

$$\tau = \frac{1}{\omega_c} = \frac{1}{2\pi f_c} \quad (1)$$

B. Debye Relaxation

Debye relaxation used to model material under test (MUT). Debye consists of a single relaxation time and represented in a semi-log graph as in Figure 1. Debye relaxation calculation is based on Agilent application note [4].

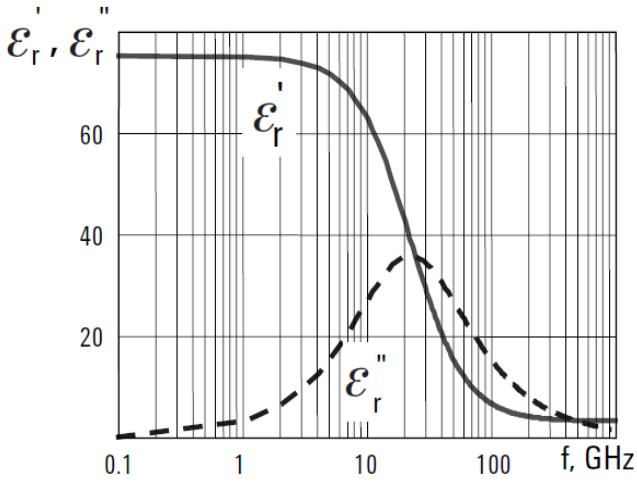


Figure 1. Debye relaxation.

C. Cole-cole diagram

A lot of the dielectric equations were modified from the ideal Debye theory, such as Cole-Cole (CC) and Davidson-Cole (DC) [3]. Cole-cole diagram was formed by plotting the imaginary permittivity (ε_r'') at the y-axis and real permittivity (ε_r') at the x-axis. Material consists of a single relaxation frequency, f_c will appears as a semicircle with its centre lying on the x-axis. Material with multiple relaxation frequencies, f_c is appearing as a semicircle (symmetric distribution) or an arc (non-symmetrical distribution) with its centre lying below the x-axis. The peak of the imaginary permittivity (ε_r'') is equal to the radius as shown in Figure 2 [4].

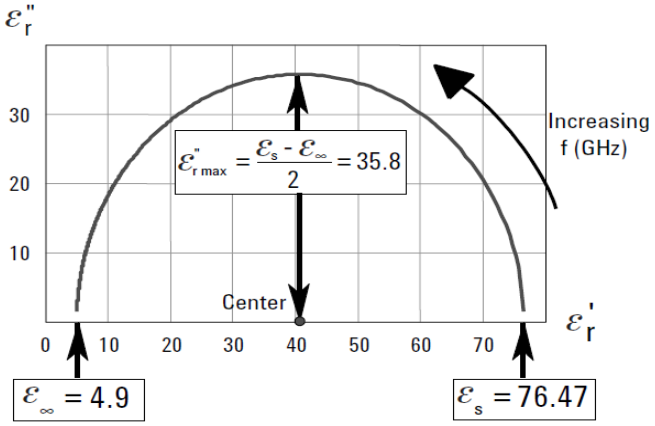


Figure 2. Cole-cole diagram.

The measured loss of material can actually be expressed as below [4]:

$$\varepsilon_r'' = \varepsilon_{rd}'' + \frac{\sigma}{\omega\varepsilon_0} \quad (2)$$

III. METHODOLOGIES

A. Phantom Probing

The open-ended coaxial probe is a cut off section of transmission line. The material is measured by immersing the probe into a liquid or touching it to the flat surface of a solid (or powder) material.

Before start measuring, calibration at the tip of the probe must be performed. It is important to allow enough time for the cable which connects the probe to the network analyzer to stabilize and should ensure that the cable is not flexed between calibration and measurement.

For solid materials, an air gap between the probe and sample can be a significant source of error unless the sample surface is machined to be at least as flat as the probe face. For liquid samples, air bubbles on the tip of the probe may act as the same way of error. Besides, the sample must also be thick enough to appear "infinite" to the probe for transmission and receiving during measurement as shown in Figure 3 [4].

The requirements for measurement using High Temperature Probe are including diameter, thickness and granule size (for granular materials) of MUT. In order to maintain the warranted performance of VNA, diameter of MUT should be more than 20mm in length as tabulated in Table 1 while thicknesses of MUT are based on below [5].

$$\text{Thickness, } h > \frac{20}{(\sqrt{|\varepsilon_r|})} \text{ mm} \quad (3)$$

TABLE I. MINIMUM THICKNESS FOR MUT FROM 1-6 GHZ.

MUT	Relative permittivity, ε_r	Thickness (mm)
Filtered Water	78.33-74.50	2.25-2.31
Piped Water	78.23- 74.38	2.26-2.32
Water + 5g Sugar	76.72- 71.64	2.28-2.36
Water + 10g Sugar	75.39- 69.03	2.30-2.41
Water + 15g Sugar	74.05- 66.54	2.32-2.45

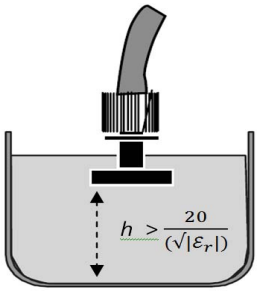


Figure 3. Thickness (in mm unit) requirements for material under test.

B. Phantom Element

All these samples are mixture of simple and low cost elements as follow:

- Sample 1 = water
- Sample 2 = water + 5g sugar
- Sample 3 = water + 10g sugar
- Sample 4 = water + 15g sugar

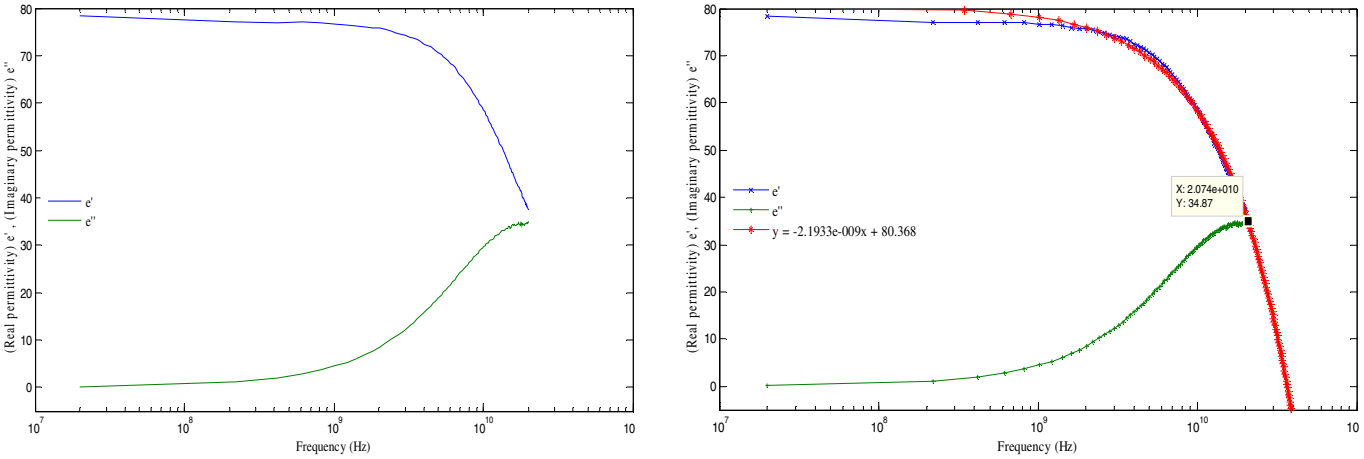
The samples were contained inside a small beaker for permittivity probing.

C. Instrument Revision

The experiment took place in a laboratory site and temperature was controlled under 25±1ºC. Due to the lacking of this VNA model, the frequency range for MUT is restricted within 0.02-20GHz. Thus, a broader frequency range of data was needed due to the relaxation time, τ of every MUT is located at frequency higher than 20GHz, which is outside the measuring ability of the instrument.

Thereupon, four models were proposed respectively for four different mixtures. As in Figure 4(a) of ε_r' and ε_r'' was limited by the frequency range, therefore an equation was proposed to extrapolate both of the lines for meeting up at an intersection point (Figure 4(b)).

The intersection point is the relaxation frequency of MUT or in other words relaxation frequency and relaxation time was obtained when $\varepsilon_r' = \varepsilon_r'' = \varepsilon_{rd}''$.



(a) (b)

Figure 4. Proposed interpolation model (b) for an incomplete graph (a).

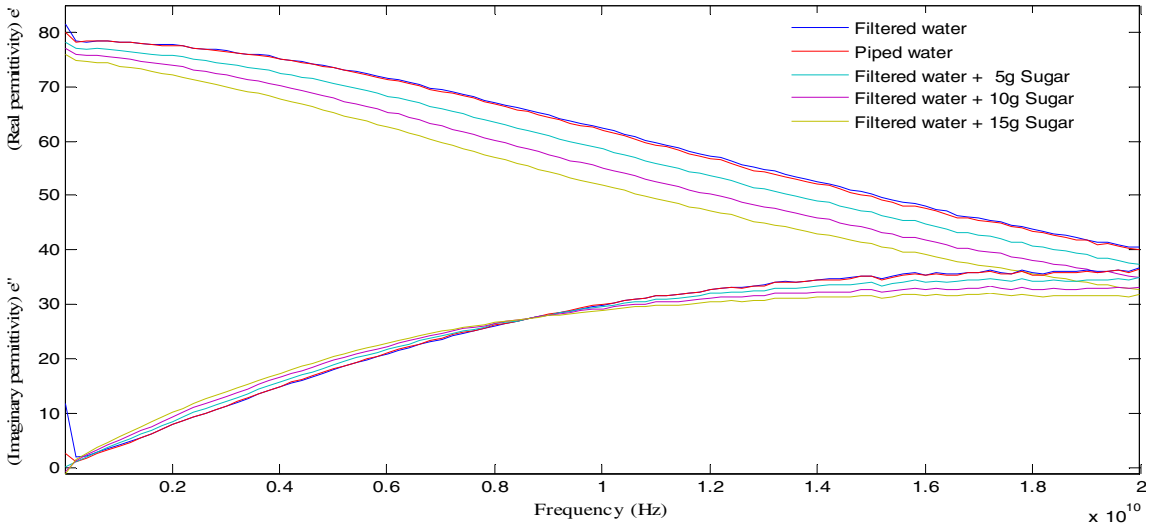
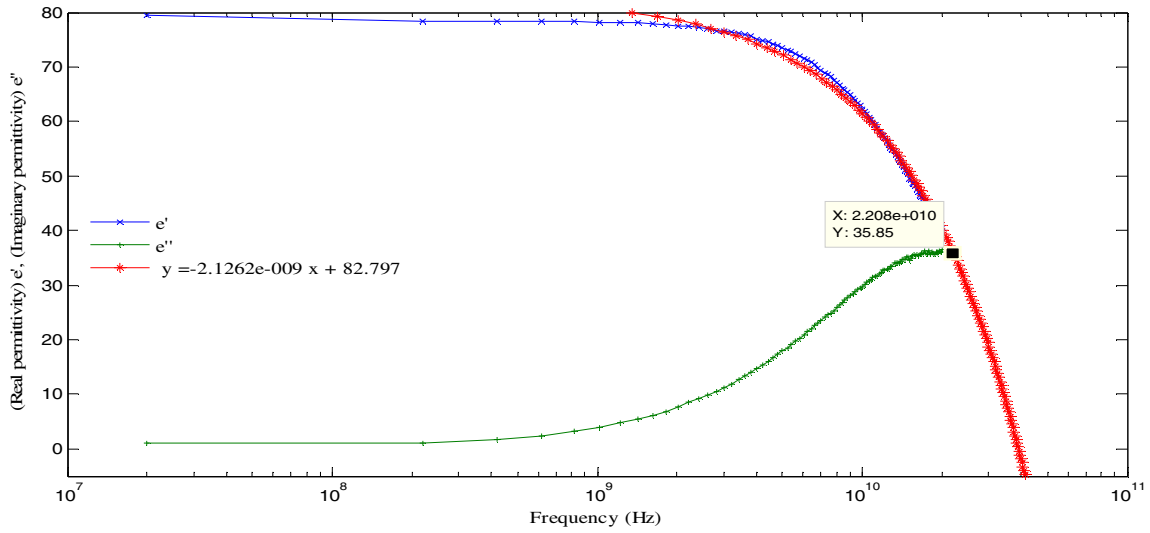
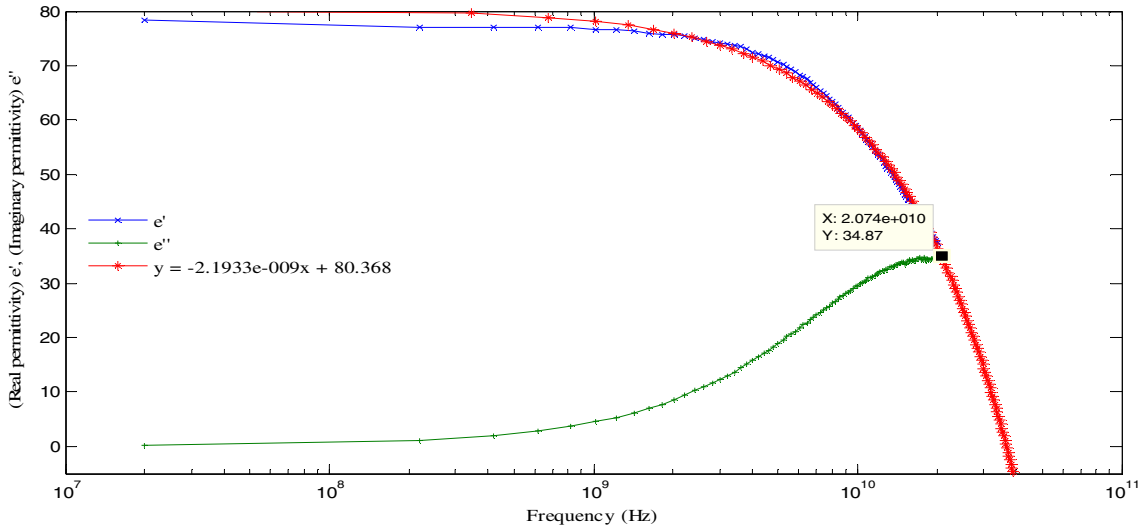


Figure 5. Graphical plot of real and imaginary permittivity (ε_r', ε_r'') of sample 1-4



(a)



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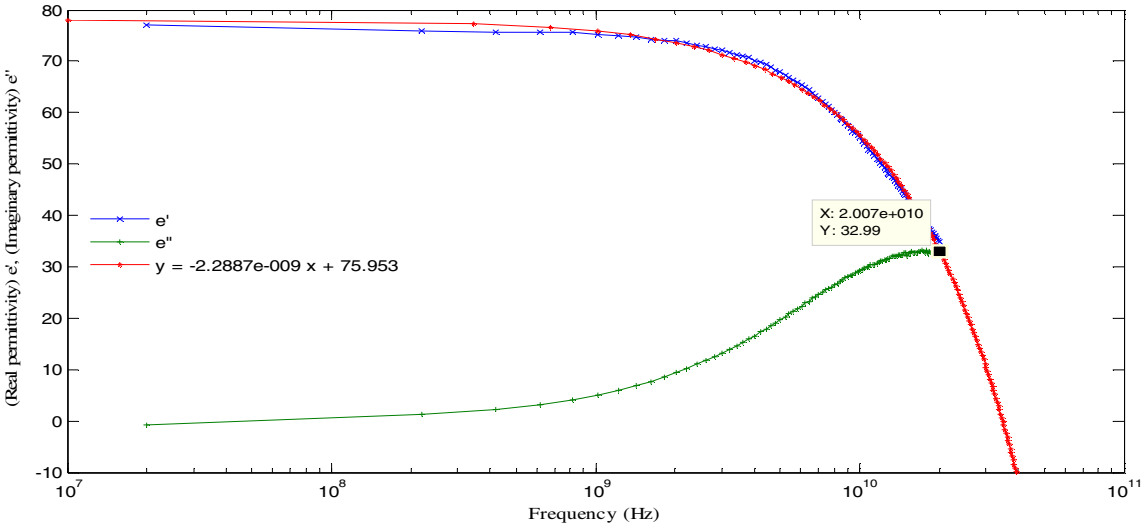
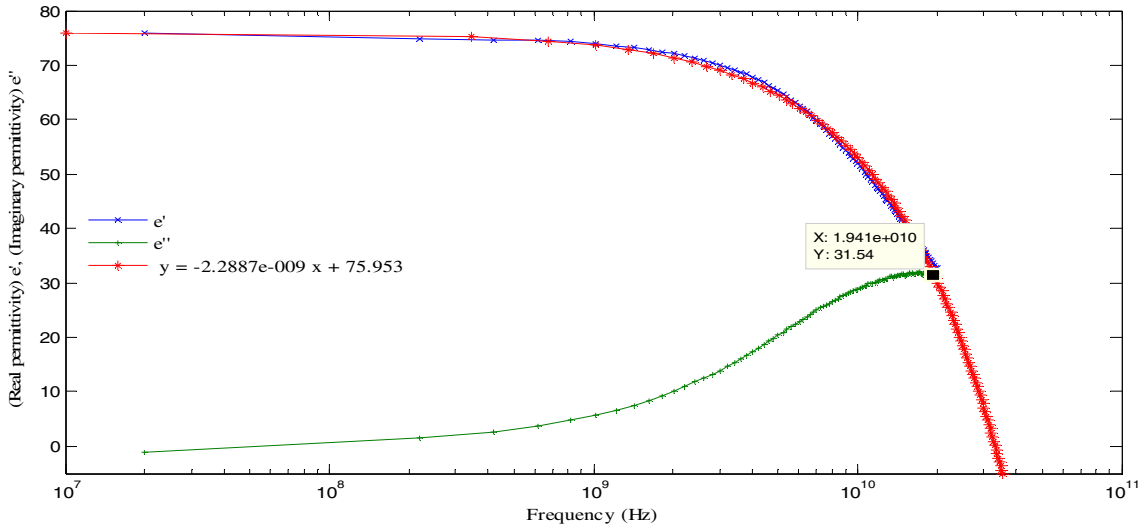


(c)



(d)

Figure 6. Extrapolation plot of fitting linear models for sample 1-4.

TABLE II. FITTING PROPOSED MODEL FOR EXTRAPOLATION FOR CALCULATION OF RELAXATION FREQUENCY AND TIME

Mixture	Proposed Model	Permittivity $\varepsilon_r' / \varepsilon_r''$	Relaxation Frequency, f (GHz)	Relaxation Time, τ (ps)
(a) Water	$\varepsilon_r' / \varepsilon_r'' = -2.1262f + 82.797$	35.85	22.08	7.208
(b) Water + 5g Sugar	$\varepsilon_r' / \varepsilon_r'' = -2.1933f + 80.368$	34.87	20.74	7.674
(c) Water + 10g Sugar	$\varepsilon_r' / \varepsilon_r'' = -2.2887f + 75.953$	32.99	20.07	7.930
(d) Water + 15g Sugar	$\varepsilon_r' / \varepsilon_r'' = -2.2887f + 75.953$	31.54	19.41	8.204

IV. RESULT

The study is still in a preliminary stage and yet we had conducted this pilot test as a fundamental testing and analysis on the collected data. The main goal of this study is to investigate and discover the most suitable mixture for building a human brain phantom with the closest relative permittivity to the real human brain. Besides, the relaxation time for every MUT needs to be considered for investigating the time consumed by MUT to reach a steady state.

From Figure 5, as the water is adding with increasingly amount of sugar, the trend of ε_r' slope will getting lower while for ε_r'', initially, the trend is increasing until frequency reach 8.5GHz, the trend started to decrease.

Fitting linear models were proposed to suit the permittivity trends and extrapolate the trend to higher frequency range. Every MUT in this study was analyzed by a specific proposed linear equation for extrapolation. The linear models as well as calculated relaxation frequency and relaxation time were shown in Table 2.

V. DISCUSSION

As the initial, the experiment took place in a normal laboratory environment, it was consider appropriate for the preliminary phase of the quantitative study to be conducted in a similar environment. Further work is planned to widen the sample and element to encompass different environments in temperature and pressure.

Sugar is playing an important role in conducting the mixture. As the amount of sugar increase, the conductivity of the mixture will increase and it is shown by Figure 5. Relationship between conductivity and loss factor is based on equation (2). Turning point happened at 8.5GHz shows that it is where the conductivity of the mixture is ignored due to the increasing of frequency. Starting from this turning point, the concentration of sugar in mixture will not affecting the loss factor.

Every mixture has a specific proposed fitting linear model. In order to obtain relaxation frequency and relaxation time from this linear model, permittivity is needed to fit into the model for calculation. Permittivity for each of the MUT can be obtained through the intersection point where $\varepsilon_r' = \varepsilon_r''$ from the extrapolation graph (Figure 6).

As a pure element was added by other element to become mixture, the relaxation time need for that particular mixture to reach steady state is longer than pure substance. In short, the amount of sugar in a mixture is affecting the relaxation period. As the amount of sugar increasing, a longer period of relaxation time is needed.

Imagination is no limit and therefore, there is a lot we can do with this study. One but not the only one straight forward application is this real-like brain phantom can act as clinical study or medical instrument for disease or tumor research. It also can be applied on the incredible thing likes replacing wax as the main material for actual-size human modeling since the proposed model gives a more natural and real life-like sensing of touching.

VI. CONCLUSION

Fitting linear models were proposed and successfully extrapolate the incomplete graph for predicting the relaxation time. The relaxation time was investigated in this study in order to set up a stable mixture for human brain phantom modeling in future. The study accomplished all the objectives stated and had a surprisingly good start. The findings indicate that sugar element able to stabilize and optimize the trends of slope but sugar also affect the relaxation time of the mixture. Hence, amount of sugar in a mixture is needed to be considered deeply for an accurate phantom model. All the results are presented in graphical form for better visual of revealed information.

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