

Randomised Alpha-Cut Fuzzy Logic Hybrid Model in Solving 3-Satisfiability Hopfield Neural Network

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Abstract This paper proposes an innovative approach to improve the performance of 3SAT logic programming in the Hopfield neural network. The merged structures of the 3SAT and Hopfield network have specific weaknesses, one of which is that, at times, the system attained local minimum solutions rather than global minimum solutions. A new model of integration randomised alpha-cut fuzzy logic with 3SAT in the Hopfield network is built to convey information more effectively. 3SAT and fuzzy logic can work together to solve Hopfield networks' combinatorial optimisation issues. Procedures of fuzzifying and defuzzifying the neurons might ease the computational burden of determining the correct neuron states. Until the proper neuron state is established, unsatisfied neuron clauses will be modified using a randomised alpha-cut approach in the defuzzifier step. An incorporated design built a random approach to select the alpha-cut values of 0.1, 0.25, and 0.5. At this point, a fuzzy value switches into a crisp output back through the defuzzifier process. Based on the results obtained, the proposed hybrid strategy effectively improves the indicators of RMSE, SSE, MAE, MAPE, global minima and total computational time. A computer-generated data set was used to measure how well the hybridised techniques performed. The performance of the proposed network was trained and validated using Matlab 2020b. The results are significant because this model significantly affects how successfully Hopfield networks merged with fuzzy logic can tackle the 3SAT challenges. The obtained data and ideas will help to create novel approaches to data collection for upcoming logic programming exploration.

Keywords: 3-Satisfiability, Alpha-cut, Fuzzy logic, Hopfield Network, Logic programming.

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Introduction

A notable advance in artificial intelligence (AI) technology can be seen through merging neural networks, logic programming models, and the SAT issue into one unified hybrid system. The artificial neural network (ANN) is an advanced analytical managing prototype that has already been extensively researched and applied by professionals and researchers due to its capacity to control and represent complex issues. The Hopfield network (HNN), invented by Hopfield and Tank [1], is one of many types of neural networks. The HNN is a recurrent network that works like the human brain and can successfully solve a variety of mathematical complexities. Wan Abdullah made one of the characterised logic rules that created a symbolised representational rule that controls the information stored in the HNN [2], [3]. The proposed HNN model was significantly enhanced by Sathasivam, which has a low storage capacity, requires lengthy computations, and is commonly caught in local minima [4]. The networks frequently have restrictions on the data model, limiting the potential to take into account information collecting. The SAT problem is among the complexity of the algorithm exploration problems based on theory. Numerous approaches can be used to solve the SAT issue. In terms of logic programming, the SAT problem is like

a combinatorial optimisation problem. In other words, a SAT problem is a task that requires limitations, such as circuit layout and pattern reconstruction [5]. Since it may be seen as a combinatorial optimisation problem, SAT logic programming can be used on a neural network to produce the desired outcome. It is intended to reflect information effectively by combining SAT issues with HNN. The expansion of the existing network, fuzzy logic is presented in this project which introduces the hybridised network of 3SAT and randomised alpha-cut fuzzy logic to make the logic programming in HNN perform more effectively significantly. Fuzzy logic minimises the processing burden and enables the suggested strategy to accommodate higher accurate neurons since it uses a fuzzification and defuzzification mechanism to build the right neurons. In addition, until the ideal neuron state is identified, any unsatisfactory neuron clauses will be modified using the randomised alpha-cut method during the defuzzifying procedure. The link between fuzzy and crisp sets highly depends on the alpha-cut concept. An alpha-cut is a set with membership grades greater than or equal to the alpha value [0, 1]. In order to connect fuzzy and crisp sets, the alpha-cut concept is essential. Depending on the alpha value, sharpening generates a clean set. The alpha-cut method will modify neuron clauses during defuzzification until the proper neuron state is attained [6]. The breakdowns of the paper sections are as follows: In the first part, we briefly discussed the introduction of HNN, satisfiability problem and fuzzy logic. Whereby in Part 2, we reviewed the 3SAT logic in HNN. Next is the explanation of randomised alpha-cut fuzzy logic techniques. Then, the algorithm and workflow of executing 3SAT with randomised alpha-cut fuzzy logic in HNN are presented. Followed by results and discussion, and lastly, the research findings are concluded.

Materials and Methods

3-Satisfiability in Hopfield Neural Network

A mathematical logical rule called satisfiability (SAT) is made up of sentences with variables. In this study, we will concentrate on 3-satisfiability, also known as 3SAT. The 3SAT problem investigates whether a specific 3SAT rule has a value that assesses the formula to be true. The three components of basic SAT problems are as follows.

A group of l variables in SAT formula where all variables are related with function OR (\vee).

$$v_1, v_2, \dots, v_l, \text{ with every } v_i \in \{1, -1\} \tag{1}$$

The literals involved are either a variable (as a positive variable) or its negation (as a negative variable). A group of n different clauses joined by logical AND (\wedge).

$$S_1, S_2, \dots, S_n \tag{2}$$

3SAT is a clause that contains three literals per clause. As a result, the following are the general attributes of 3SAT clauses.

$$G = \bigwedge_{n=1}^{NC} S_n \tag{3}$$

where S_n implies the clauses and n indicates the number of clauses and NC is the overall number of clauses. The goal G must be justified in this investigation with $\{1, -1\}$ are the Boolean numbers.

The 3SAT formula with strictly three literals per clause is shown in Equation 4:

$$G_{3SAT} = (V_1 \vee \neg V_2 \vee \neg V_3) \wedge (V_4 \vee V_5 \vee \neg V_6) \wedge (V_7 \vee V_8 \vee V_9) \tag{4}$$

The logic programme's prime purpose is to identify the pattern's significance that fulfils the entire clause. Therefore, the fundamental challenge of converting Equation 4 into the negation of the formula G to identify the logical contradictions.

$$\neg G_{3SAT} = (\neg V_1 \wedge V_2 \wedge V_3) \vee (\neg V_4 \wedge \neg V_5 \wedge V_6) \vee (\neg V_7 \wedge \neg V_8 \wedge \neg V_9) \tag{5}$$

Meanwhile, the cost function for 3SAT is generated by the following formula:

$$E_{G_{3SAT}} = \frac{1}{2^3} \sum_{n=1}^3 \left(\prod_{i=1}^3 V_i \right) \tag{6}$$

with the corresponding formula

$$V_i = \begin{cases} (1 - NS) & \text{if } \neg V_i \\ (1 + NS) & \text{if } V_i \end{cases} \tag{7}$$

where NS is the state value of $V_1, V_2, V_3, \dots, V_i$.