Faculty of Computer Science and Information Technology

CONVECTION-DIFFUSION OF HEAT TRANSFER IN PERFUSED BIOLOGICAL TISSUE

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Bachelor of Computer Science with Honours
(Computational Science)

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ABSTRACT

Convection and diffusion to used describe how scalar quantity is being transport in a space. This convection diffusion equation is benefited many fields of engineering, chemistry as well as biology. Since, during oxygen supplies to living tissues through the microcirculation of blood, convection and diffusion play an important role hence, this convection-diffusion equation can apply as a model solution for this study to helps us to describe heat transfer in perfused biological tissue. For this problem of convection diffusion, Green's function can be used to solve it. Green's function is used here because it is a powerful tool of mathematical method used in solving some linear inhomogeneous Partial Differential Equations as well as Ordinary Differential Equations. With the properties of Dirac's function, this Green's function is derived by spatially development method of separation of variables. Hence, this method considered more efficient than the other known classical methods. As for the discretization to form weak form of Convection-Diffusion, Finite Difference Method of Central Differencing is use. The main reason for this selection is because of the simplicity when implement in our Matlab program.

ABSTRAK

Perolakan dan penyebaran digunakan untuk menggambarkan bagaimana kuantiti skalar diangkut di ruang. Persamaan perolakan penyebaran ini telah menyumbang dalam pelbagai bidang kejuruteraan, kimia mahupun biologi. Oleh kerana, semasa proses dimana oksigen dibekalkan ke tisu badan, perolakan penyebaran memainkan peranan yang amat penting ketika proses, jadi persamaan perolakan penyebaran boleh digunakan sebagai model solusi untuk kajian kerana dapat membantu menggambarkan bagaimana haba dipindahkan dalam tisu biologikal badan. Untuk masalah ini perolakan penyebaran, fungsi Green boleh digunakan untuk menyelesaikannya. Fungsi green digunakan disini kerana merupakan satu kaedah matematik yang digunakan dalam menyelesaikan permasalahan linear yang tidak bersifat homogen sama ada Persamaan Pembezaan Separa mahupun Persamaan Pembezaan Biasa. Dengan sifat-sifat fungsi Dirac, fungsi ini Green diperolehi dengan kaedah pembangunan spatial pemisahan pembolehubah. Oleh itu, kaedah ini dianggap lebih cekap daripada kaedah klasik lain diketahui. Untuk diskretisasi membentuk, bentuk lemah Konveksi-Difusi, Kaedah Perbezaan Terhingga Pembezaan Pusat adalah penggunaan. Sebab utama pemilihan ini adalah kerana mudah untuk diaplikasikan dalam program Matlab.

CHAPTER 1

OVERVIEW

1.1 Background of study

Convection-diffusion equation is a combination of both convection and diffusion equation that use mass balance approach. This convection diffusion equation also known as transport equation used to describe how scalar quantity being transport in a space. Convection-diffusion is a fundamental element in both natural and human-made processes.

As a universal phenomenon, this convection-diffusion has been applied in many specializations such as in mechanical engineering, chemical engineering as well as in biology. The phenomenon of oxygen supplies to living tissues through the microcirculation of blood is a very an example on how this principle of diffusion and convection is being used in the field of biology. In our body, oxygen enters our bloodstream through diffusion when oxygen molecules we breathe in diffuse into deoxygenated blood where there exists a different concentration level within the cell. We called this process as a mechanism for mass transport. During this process, underlying tissue oxygen will transport the oxygen within body. Here it is described as convection process which describe the movement of oxygen within the circulation taking place through bulk transport when there is a different density exist within body.

Through the representation with biological studies which is the heat transfer in perfused biological tissue as our key of attention, we will model a solution to helps us to get a better understanding the convection-diffusion equation for heat transfer in perfused biological tissue. The convection diffusion equation without the source term can be expressed as follows:

$$\frac{\partial c}{\partial t} = -\,U\,\frac{\partial c}{\partial x} + D\,\frac{\partial^2 c}{\partial x^2}$$

The subscript *t* and *x* stand for differentiation with respect to time and space respectively. *D* is the coefficient of diffusion, *c* is the concentration and *U* is the flow velocity. To develop a mathematical modelling that allows us to gain an analytical solution for this study, a few assumptions are consider:

i. All tissue is considered as solid.

ii. The concentration that we will mainly focus for this case is the concentration, *c* of temperature for heat transfer.

1.2 Problem Statement

Convection and diffusion are used describe how scalar quantity is being transport in a space. In this problem, an inhomogeneous equation will be considered. Therefore, a solution by using Green's function and discretization using finite difference method (FDM) of Central Differencing will be proposed to solve the problem. This Green's function is derived by spatially development method of separation of variables with the properties of Dirac's function. Hence, this method of Green's function considered to be efficient than the other known classical method.

1.3 Scope

The main scope of this research is to focus on the theoretical investigation of solving convection-diffusion equation by taking account on how convection-diffusion is applied in the field of biological studies which we will mainly be focussing on heat transfer in perfused biological tissue. The other scope is listed as below:

i. Modelling the solution using Green's function and draw an explicit solution of convection-diffusion equation.
ii. Carrying out a discretization of convection-diffusion through Finite Difference Method (FDM) of Central Differencing and code it by using MATLAB software.

1.4 Objectives

The main objective of this research is to solve the problem of convection-diffusion equation through the representation with biological studies which is the heat transfer in perfused biological tissue as our key of attention. Other objectives towards accomplish the aim include:

i. To model the solution of convection-diffusion equation using Green's function.
ii. To discretize the convection-diffusion equation numerically using Finite Difference Method (FDM) of Central Differencing.
iii. To simulate the numerical computational of convection-diffusion equation using MATLAB software.

1.5 Brief Methodology

In this research, the methodology used is a modelling process. Stated below are how the process go through:

1. **Identify the Problem**

   The first step to begin with this research is to identify the problem. In this study, convection-diffusion equation is chosen due to its simplicity of one dimensional but contains the non-linear properties. Problem describing heat transfer in perfused biological tissue is said to be extremely complex due to existing difference vessel size, flow velocity and spatial distribution. By taking account of representation using heat transfer in perfused biological tissue a suitable solution of mathematical modelling can be conducted to enhance a better understanding for this convection-diffusion problem. On the hand, numerical method defines as the approximation of a solutions. Both Green's function and Finite Difference Method (FDM) is used to solve the problem theoretically and numerically to meet the aim of this study.

2. **Define Goals**

   In term of defining the goal, we can recall the objectives of this study which is to solve the problem of convection-diffusion equation for heat transfer in perfused biological tissue as our key of attention.

3. **Characterize the Model**

   To fulfil the objectives of this study, some assumptions need to be considered. First all tissue is considered as solid. Second is the concentration that we will mainly focus for this case is the concentration, c of temperature for heat transfer.

4. **Formulate Model**

   For this study, two method of computation will be applied to solve this convection-diffusion equation which is model solving it by using Green's function and discretize it numerically using Finite Difference Method (FDM) of Central Differencing.

   .

5. **Solve/ Simulate Model**

   To visualize the model solution later, MATLAB software will be used to analyse the result from the simulation.

6. **Analyse the Model**

   Once the model solution has been developed, the accuracy of the obtained result will be determined.

1.6 Significance of Project

In recent years, the research of this convection-diffusion equation had contributed to various achievement especially in both computer science and biological studies. Hence, the significant of this study are:

i. This research determined the accuracy of result obtained using numerical approach in convection-diffusion equation.

ii. This research provides other alternative to solve convection-diffusion equation for heat transfer in perfused biological tissue by using two different approach of numerical methods.

1.7 Project Schedule

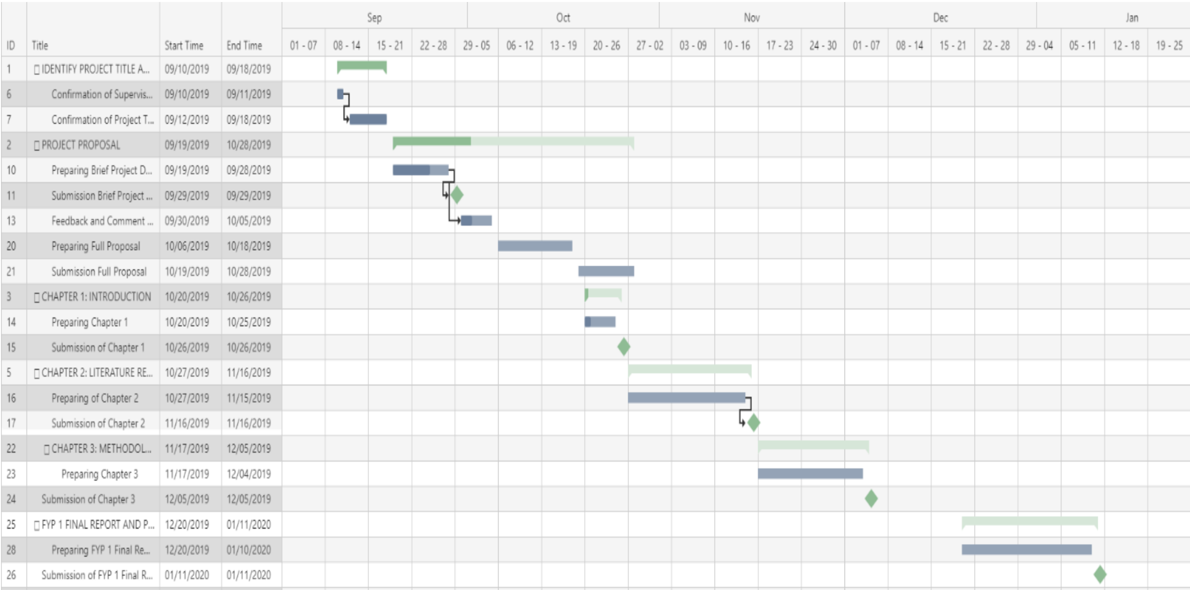

Figure 1.7.1: Gantt Chart of FYP 1

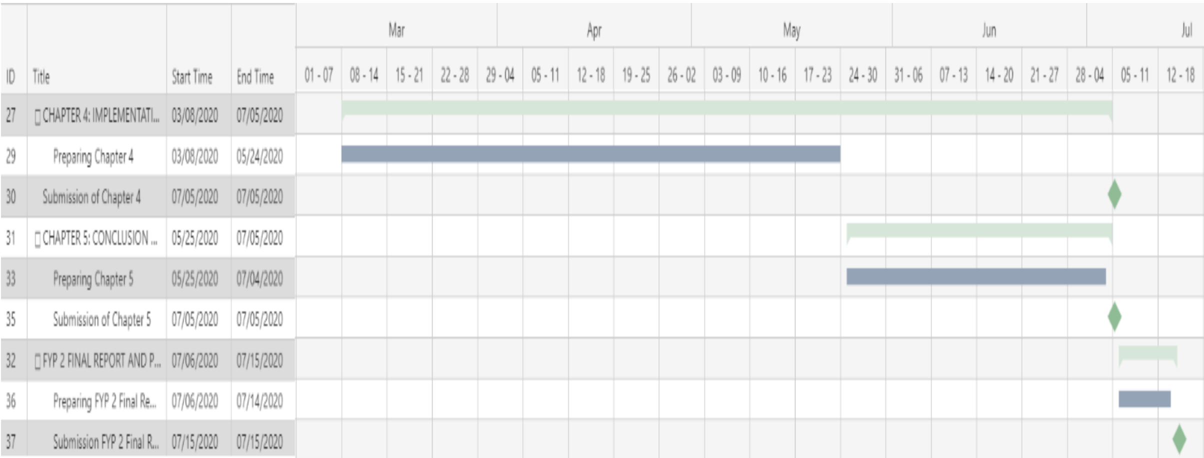

Figure 1.7.2: Gantt Chart of FYP 2

1.8 Expected Outcome

For this research, a theoretical investigation of solving convection-diffusion equation for heat transfer in perfused biological tissue will be solved. A solution using Green's function and discretization using finite difference method (FDM) of this convection-diffusion equation will be form and lastly a code using MATLAB software will be developed.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The main purpose of this chapter is to gain a relevant comprehensive literature review that related to this study. This review chapter will deal with comprehensive literature review related to various aspects of convection-diffusion equation that emphasis on phenomena of heat transfer in perfused biological tissue. It is important to gain idea and knowledge from different sources and materials in order to proceed study.

Convection-diffusion is benefited in applications of different disciplines such as environmental engineering, mechanical engineering, soil science, petroleum engineering, chemical engineering as well as in biology (Mazaheri et al, 2013).

2.2 Discussion of Literature Review

2.2.1 Problem with Convection-Diffusion Equation

Bajellan (2015) proposed a study of computation convection-diffusion equation by fourth-order compact finite difference. This study was to develop a one-dimensional convection-diffusion equation with constant coefficient. The techniques used are based on the explicit finite difference schemes in space and first-order explicit schemes in time. From his findings, it is stated that the method used in the simulation is applicable for the solution of the convection-diffusion equation.

The main idea behind higher order finite difference methods for acquiring the solution of a given partial differential equation is to approximate the derivatives appearing in the equation by a set of values of the function at a selected number of points. Taylor series is the most usual way to generate these approximations. This approach permits the simple determination of the theoretical order of accuracy allowing methods to be compared with one another. It is also possible to eliminate the dominant error terms associated with the finite difference equations contain free parameters (weights) from the truncation error of the modified equivalent equation, thus leading to more accurate methods (Dehghan, 2004).

Through the demonstration of methods, some model problems of convection-diffusion equation are applied to this study. This model problems of the convection-diffusion equation with the initial and boundary conditions are then compared with the exact solutions. Courant (Cr) number is an important non-dimensional parameter in numerical analysis. This parameter gives the fractional distance relative to the grid spacing travelled due to advection in a single time step, $Cr = u\,\frac{\Delta t}{\Delta x}$. It is possible to show using a Fourier error analysis that for a forward difference in time approximation, no matter what approximation is used for the spatial derivatives, that the transport equation is stable for values of the $Cr \leq 1$. This stability constraint for explicit transport equations states that one cannot advent the concentration more than one grid cell in a single time step (Sari et al, 2010). Another important non-dimensional parameter is the Peclet number which compares the characteristic time for dispersion and diffusion given a length scale with the characteristic time for advection. In numerical analysis, one normally refers to a grid Peclet number, $Pe = u\,\frac{\Delta x}{D}$ where, u is the flow velocity, Δx is the characteristic length scale is given by the grid spacing and D is the diffusion coefficient. The literature suggests that for stable solution $Pe \le 5$.

From the numerical illustrations, comparison between numerical solutions of different schemes and the exact solution, it was found that the most accurate method is the Fourth-Order Compact Difference Schemes (FTC4S). Hence, it can be said that there is an excellent agreement between FTC4S and exact solutions. For each of the finite difference schemes investigated, the modified equivalent partial differential equation is employed which permits the order of accuracy of the numerical methods to be determined.

2.2.2 Simulation of Perfused Tissue with Hybrid Equation

A study for the simulation of perfused tissue during thermal treatment using a hybrid equation was reported by Wren et al. (Wren et al., 2001). The equation depends on different thermal characteristic related with small, intermediate, and large vessels size with the possibilities of the modelling these vessels using an effective thermal conductivity in combination with heat sink. This hybrid equation includes both bio-heat equation (BHE) of Pennes equation and k_{eff} equation. When describing the thermal effects of vessels with different diameters and geometrical configurations these two equations are commonly used. k_{eff} equation is found to be best described for small vessels in thermal equilibrium. Pennes on his study used the assumption that all temperature equilibration takes places in the capillaries. This assumption was rejected by several authors that argue the BHE of Pennes is fundamentally incorrect in some aspect. An experimental study by Lemons et al. also concluded that the BHE of Pennes cannot describe the influence of vascular architecture. Despite the critism against BHE of pennes and k_{eff} equation, without doubt these equations are still the two most widely used bio-heat equations for modelling regional thermal treatment of perfused tissue continuum. The main reason is their simplicity which can be implemented in standard thermal simulation programmes.

For this simulation of perfused tissue with hybrid equation using BHE of Pennes and the k_{eff} single coefficients (β) was includes in order to describe the relative importance of increasing in thermal conductivity and heat sink terms. Compared to stand-alone BHE of Pennes and k_{eff} equations, this proposed hybrid equation should be more accurate because stand-alone Pennes's equation and k_{eff} equation are associated with modelling of only relatively large and small vessels respectively whereas, the hybrid equation can account for vessels of all sizes.