PARAMETRIC STUDY ON DUCTILITY OF RC BEAMS STRENGTHENED WITH EXTERNAL REINFORCEMENT

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Dedicated especially to

My beloved family
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<td>area of tension reinforcing steel</td>
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<tr>
<td>$A_e$</td>
<td>area of external reinforcing steel</td>
</tr>
<tr>
<td>$A_i$</td>
<td>area of internal reinforcing steel</td>
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<tr>
<td>$b$</td>
<td>breadth of the beam section</td>
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<td>$c_n$</td>
<td>depth of neutral axis (N.A.) when concrete crushes</td>
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<td>$c_y$</td>
<td>depth of N.A. when reinforcing steel yields</td>
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<td>$C_r$</td>
<td>compressive force of concrete block</td>
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<td>$C_{con}$</td>
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<td>concrete compressive force at crushing of concrete for original beam</td>
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<td>$C_{con,y}$</td>
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<td>$C_{con,y,0}$</td>
<td>concrete compressive force at yielding of internal steel for original beam</td>
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<tr>
<td>$C_{con,cr}$</td>
<td>depth of N.A. for strengthened beam when concrete crushes</td>
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<td>$C_{con,cr,0}$</td>
<td>depth of N.A. for original beam concrete crushes</td>
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<tr>
<td>$C_{con,y,cr}$</td>
<td>depth of N.A. for strengthened beam when internal reinforcing steel yields</td>
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<td>depth of N.A. for original beam when internal reinforcing steel yields</td>
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<td>$d_e$</td>
<td>effective depth of tension reinforcement</td>
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<td>$d_i$</td>
<td>effective depth of external reinforcing steel</td>
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<td>$d_r$</td>
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<td>$D_R$</td>
<td>ductility ratio</td>
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<td>$E_c$</td>
<td>modulus of elasticity of concrete</td>
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<td>$E_s$</td>
<td>modulus of elasticity of steel</td>
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<td>$e_c$</td>
<td>concrete strain</td>
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<td>$e_{cr,0}$</td>
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<td>$e_u$</td>
<td>ultimate concrete strain (compressive strain at crushing of concrete)</td>
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<td>$e_{max}$</td>
<td>concrete strain corresponding to its maximum stress for Todeschini’s stress-strain curve</td>
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<td>$e_o$</td>
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<td>$e_y$</td>
<td>yield strain of reinforcing steel</td>
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<td>$e_j$</td>
<td>yield strain of external reinforcing steel</td>
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<tr>
<td>$e_j$</td>
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<td>$f_c$</td>
<td>concrete stress</td>
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<td>$f'_{c,cr}$</td>
<td>maximum concrete stress of Todeschini’s stress-strain curve</td>
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<td>$f_{i,y}$</td>
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<td>$\phi$</td>
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<td>$\phi_s$</td>
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<tr>
<td>$\phi_{cr}$</td>
<td>curvature of beam at crushing of concrete</td>
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<td>$\phi_{cr,0}$</td>
<td>curvature of original beam at crushing of concrete</td>
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<td>curvature of original beam at yielding of reinforcing steel</td>
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<tr>
<td>$\phi_{o,y,0}$</td>
<td>curvature of original beam at yielding of internal reinforcing steel</td>
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<td>$h$</td>
<td>overall depth of the beam section</td>
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<td>$M_{bd}$</td>
<td>internal moment of resistance when concrete crushes</td>
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\( M_{Rc} \) = internal moment of resistance of strengthened beam at crushing of concrete
\( M_{Roa} \) = internal moment of resistance of original beam at crushing of concrete
\( M_{Rd} \) = internal moment of resistance when steel yields
\( M_{Rsy} \) = internal moment of resistance of strengthened beam at yielding of internal reinforcing steel
\( M_{Rsy} \) = internal moment of resistance of original beam at yielding of internal reinforcing steel
\( \rho \) = reinforcement ratio
\( \rho_{oa} \) = internal reinforcement ratio
\( SR \) = strengthening ratio
\( T_e \) = tensile force of reinforcing steel
\( T_{oa} \) = tensile force of external reinforcing steel
\( T_d \) = tensile force of internal reinforcing steel
\( w_c \) = unit weight of the hardened concrete
\( \omega \) = reinforcing index
\( \omega_{oa} \) = external reinforcing index
\( \omega_d \) = internal reinforcing index
\( x_e \) = \( \varepsilon_{oa} \) to \( \varepsilon_e \) ratio
\( x_{oa} \) = \( \varepsilon_{oa} \) to \( \varepsilon_{oa} \) ratio
\( x_{oa} \) = \( \varepsilon_{oa} \) to \( \varepsilon_{oa} \) ratio
\( \psi \) = relative reinforcing index
\( Z_1 \) = lever arm between \( C_{oa} \) and \( T_e \)
\( Z_{oa} \) = lever arm between \( C_{oa} \) and \( T_{oa} \)
\( Z_{oa} \) = lever arm between \( C_{oa} \) and \( T_d \)
\( Z_{oa} \) = lever arm between \( C_{oa} \) and \( T_{oa} \)
\( Z_{oa} \) = lever arm between \( C_{oa} \) and \( T_d \)
\( Z_{oa} \) = lever arm between \( C_{oa} \) and \( T_d \)
ABSTRACT

Recently, repair and rehabilitation of degrading and aging concrete structures have become more challenging. Bonding of steel plates or other advanced composite materials to the tension face of the beam is one of the methods for strengthening these deteriorated concrete structures. This may increase the strength and stiffness of the concrete structures, nevertheless the corresponding ductility will be affected at the same time. It is absolutely important for this parametric study to present how the ductility of reinforced concrete beam is being affected by increasing the strength of the beam using externally bonded steel plates. An analytical model was developed to obtain the ductility of the unstrengthened and strengthened beams, and hence the ductility ratio. MathCAD was used to develop a numerical simulation program for estimating the concrete strains when the internal reinforcing steel yields. The other parameters such as effective depth of internal reinforcing steel to beam overall depth ratio, reinforcement ratio, relative reinforcing index and grade of concrete were varied in this study to show the relationship between strengthening and ductility of RC beams. The analytical results for beams with effective depth of internal reinforcing steel to beam overall depth ratio of 0.8 showed that the beam can be further strengthened with external reinforcing steel as high as 4.16% to gain 623.66% in ultimate moment capacity. However, this led to a dramatically loss in ductility of about 93.10%. For the beams with internal reinforcement ratio of 0.020, the results indicated that the strengthened beam could gain its ultimate moment capacity up to 122.73%, and the corresponding loss in ductility of 63.35%. In general, the use of external reinforcing steel increases the flexural strength of the RC beams, but induces a reduction in ductility. Nomographs that relate among the ductility ratio, strengthening ratio and external reinforcing index were developed for RC beams with effective depth of internal reinforcing steel to beam overall depth ratio of 0.7, 0.8 and 0.9. These nomographs can be used as guidelines for checking of ductility of RC beams strengthened with external reinforcement.
Concrete structures have become more materials to the tension face of the beam for these structures. This may increase the corresponding ductility will be affected at study to present how the ductility of the beam using externally bonded fibre ductility of the unstrengthened and used to develop a numerical simulation of reinforcing steel yields. The other beam overall depth ratio, reinforcement in this study to show the relationship results for beams with effective depth of showed that the beam can be further to gain 623.66% in ultimate moment of about 93.10%. For the beams with the strengthened beam could gain its loss in ductility of 63.56%. In general, strength of the RC beams, but induces a ratio, strengthening ratio and external depth of internal reinforcing steel to beam be used as guidelines for checking of

**ABSTRAK**

Sejak kebekalangan ini, kerja-kerja pemulihan dan penyelenggaraan struktur konkrit yang lama dan usang telah menjadi semakin mencabar. Satu daripada cara untuk menguatkan struktur konkrit yang semakin usang ini adalah melaksanakan kepingan kelali atau bahkan komposit unggul yang lain kepada permukaan tegangan mereka. Ini akan memanfaatkan kekuatan dan lekakahan bagi struktur konkrit tenapi kemularannya juga turut dipengaruhi. Maka betapa pentingnya untuk manajalan penyelidikan secara parameter untuk mengkaji sejauh mana kemularan bagi rasaik konkrit berteutul digerakkan apabila rasaik tersebut dialaskan dengan kepingan kelali dilekatkan secara luaran. Satu model analisis akan dihasilkan untuk mendapatkan nilai kemularan bagi rasaik yang asal dan juga rasaik yang diperkuatkan, serta dinyatakan nilai nisbah kemularan dapat dihitung. Satu program simulasi berangka telah dibentukkan dengan menggunakan MathCAD untuk mengangkarkan terikan semasa tetutul konkrit dalam alam. Parameter lain seperti nisbah tinggi berkesan tetutul kelali dalaman kepada ketegangan rasaik, nisbah tetutul, indeks berteutul relatif dan konkrit gred akan diubahsuai untuk mengkaji hubungan di antara penguatan dan kemularan bagi rasaik konkrit berteutul. Keputusan analisis untuk rasaik yang nisbah tinggi berkesan tetutul kelali dalaman kepada ketegangan rasaik bersamaan 0.8 telah menunjukkan bahwa rasaik dapat diperkuatkan dengan laju tetutul kelali luaran yang setinggi 4.16% dan keupayaan momen maksimumnya telah bertambah sebanyak 623.66%. Walau bagaimanapun, kemularannya telah jatuh seterusnya 93.10%. Sementara itu, rasaik yang mempunyai 2% tetutul kelali dalaman telah memperlihatkan keupayaannya bahawa rasaik yang telah diperkuatkan mendapatkan 122.73% keupayaan momen maksimumnya, tenapi telah kehilangan 63.56% kemularannya pada masa yang sama. Secara amnya, penggunaan tetutul kelali luaran dapat meningkatkan kekuatan lenturan bagi rasaik konkrit berteutul tetapi menyebabkan penurunan bagi kemularannya. Carta-cartu yang bergabungkan antara nisbah kemularan, nisbah penguatan dan indeks tetutul luaran diberitahu untuk rasaik konkrit berteutul yang mempunyai nisbah tinggi berkesan tetutul kelali dalaman kepada ketegangan rasaik bersamaan dengan 0.7, 0.8 dan 0.9. Carta-cartu ini boleh dijadikan gairah panduan untuk menyemak kemularan bagi rasaik konkrit berteutul yang diperkuatkan dengan tetutul luaran.
CHAPTER 1
INTRODUCTION

1.1 PRESENT SITUATION

In statistical record of year 2000, National Bridges Inventory (NBI) showed that 14.3 percent or 87,801 of the bridges in United States were structurally deficient, while 13.0 percent or 79,860 were functionally obsolete (NBI, 2002). A structurally deficient bridge is one that has significant deterioration caused by severe exposure to adverse environments, fatigue due to overloading and unpredicted loads, faulty in design and construction, or changes in structural purpose. It requires immediate rehabilitation and may be restricted to light vehicles or completely closed down. However functionally obsolete bridge is one on which the deck geometry, load-carrying capacity, vertical and horizontal clearances or approach roadway alignment no longer meets the usual criteria or the present usage and situation. In short, it is about 27.3 percent of the bridges in United States need repair, rehabilitation or replacement.

Many researchers had tried various innovative and creative strengthening techniques to retrofit these deteriorating infrastructures. Providing additional beams or props was one of the techniques to increase the supporting elements in the structures. External post-tensioning technique was also used to strengthen concrete bridges subjected to increasing service loads. However, these techniques accommodated large installation space and also created cumbersome and troublesome procedures.

In the previous decades, the continual deterioration of the concrete infrastructures had led the researches to conduct more studies on structural adhesives. This effective and efficient technology exhibited the bonding of the external steel plates or other advanced composite materials to the tension face of the deteriorated concrete structures, thereby increasing their strength and stiffness. The advantages of using external plates include ease of installation, high strength to weight ratio and occupying small amount of space on the RC structures. Norris et al. (1997), and Malek and Saadatmanesh (1998a, 1998b) were among the researchers who carried out studies on externally bonded fibre reinforced polymer (FRP) for shear and flexural strengthening of reinforced concrete (RC) beams. Arduini and Nanni (1997) also carried out a parametric analysis to investigate the effects of FRP reinforcement on serviceability, strength and failure mechanisms of strengthened RC beams. El-Mihalmy et al. (2000) on the other hand, presented design nomograms to show the analytical procedure for evaluating the ultimate flexural capacity of FRP-strengthened RC flexural members.

1.2 RESEARCH SIGNIFICANCE AND OBJECTIVE

Bonding the reinforcing steel externally to the tension face of RC beam can effectively increase its corresponding strength and stiffness in one hand, on the other hand the ductility of the beam will be mutually affected. From the structural safety point of view, ductility is significant and crucial in giving early warning of impending structural failure, thus ensuring the safety of the structure. In general, FRP externally strengthen RC beam can significantly improve the flexural capacity and ductility of RC beam, thus increasing its safety and durability. The main objective of this research is to study the effects of various parameters on ductility of RC beam externally strengthened with FRP.

1.3 RESEARCH SCOPE

This research project focuses on bonding the reinforcing steel plate to the tension face of the RC beams strengthened with FRP, with constant effective thickness (d/h) and varying other parameters such as concrete compressive strength, reinforcing index (I), and ratio of the tensile strength of reinforcing steel compared to the design steel yield stress (f'Y/fu). In the definition of the word, reinforcing index represents the flexural capacity of the RC beams which exceed the FRP-strengthened RC flexural members.
s Inventory (NBI) showed that 14.3 were structurally deficient, while 13.0 (2002). A structurally deficient bridge by severe exposure to adverse predicted loads, faulty in design and requires immediate rehabilitation and closed down. However functionally, load-carrying capacity, vertical and no longer meets the usual criteria about 27.3 percent of the bridges in

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th (1998a, 1998b) were among the oned fibre reinforced polymer (FRP) concrete (RC) beams. Arduino and is to investigate the effects of FRP ure mechanisms of strengthened RC and, presented design monographs to s: ultimate flexural capacity of FRP-

1.3 RESEARCH SCOPE

This research project presents the parametric study on ductility of RC beams strengthened with external reinforcement, in which "external reinforcement" refers to the reinforcing steel plates only. The ductility of the RC beams is measured in term of curvature. A flexural analytical model will then be developed for this study. Hence, it can be carried out for different RC beam sections by setting some parameters as constant and varying other parameters accordingly to investigate respective effect on ductility of the RC beams strengthened with external reinforcing steel. Among them are (i) sections with constant effective depth of internal reinforcing steel to beam overall depth ratio (d/h) and varying internal reinforcement ratio (ρe); (ii) sections with constant ρe and varying d/h ratio and f'; (iii) sections with constant ρe and d/h ratio and varying f'; (iv) sections with constant d/h ratio and f', and varying relative reinforcing index (ψ); and (v) sections with constant ψ and varying d/h ratio and f'. In the definition of ductility ratio (DR), only the ductile failure will be considered in this study, in which the internal reinforcing steel yields before the concrete crushes at ultimate limit state. Charts of DR versus external reinforcement ratio (ρe), external reinforcing index (αe), relative reinforcing index and strengthening ratio (SR) are plotted to study the variation of ductility of the strengthened beams with respect to other parameters.

Vice face of RC beam can effectively in one hand, on the other hand the on the structural safety point of view, turning of impending structural failure,
CHAPTER 2
LITERATURE REVIEW

2.1 GENERAL

When a simply supported beam is subjected to various transverse loads downward, plain concrete beam is inefficient and insufficient in resisting the moment. The positive moment over the span of the beam will cause the beam deflecting in a sagging manner. The top face of the beam will be in compression and tension occurs at the bottom. Eventually, the plain concrete beam will fail in tension at low load due to small tensile strength of concrete. Internal tension reinforcing steel is then introduced to solve the problem. Now, fibre reinforced polymer/plastic (FRP) are being used in reinforced concrete beams. In addition, more researches were carried out on strengthening the RC beam externally by using reinforcing steel, FRP or other materials. Apparently, it is important to reinforce the concrete beam either internally or externally, however the ductility of a building design should not be neglected. This is to ensure that the tenants of a building have sufficient time to escape from the building if it collapses due to overload. In the parametric study on ductility of RC beam, the flexure theory becomes essential and closely related to it.

2.2 FLEXURE THEORY FOR RC BEAM

2.2.1 Flexural Behaviour of RC Beam

A simply supported RC beam as shown in Figure 2.1 is gradually loaded from zero to certain magnitude that will cause the beam to fail. Several different distinguishable stages can be clearly stated.

In the initial stage, the beam is uncracked as shown in Figure 2.1(a) since the load is relatively low. The strains are very small and the stress distribution over the depth of beam section is essentially linear.

When the load is further increased, the tensile stress of concrete at the bottom of the beam soon reaches its modulus of rupture and this leads to the tension cracking as shown in Figure 2.1(b). At this stage, the tensile force in concrete is transferred to the provided tensile reinforcing steel. Less concrete in tension face is effective in resisting moments.

At moderate load, if the concrete stresses do not exceed approximately $f'/2$, stresses and strains are closely proportional (Nislim & Winter, 1991), in which $f'$ refers to the concrete strength. When the load is further increased, stresses and strains rise correspondingly and no longer proportional. The distribution of concrete stresses on the compression side of the beam is of the same shape as the stress-strain curve for concrete.

Eventually, the reinforcement in steel increases and continues until the failure of the beam as shown in Figure 2.1(c).

![Figure 2.1 Behaviour of RC Beam](image)

2.2.2 Assumptions in Flexure Theory

The following assumptions were made:
1. Sections per remain plane.
2. The strain in the same level.
3. The stresses are the same using stress-strain curve.
4. The tensile steel.
5. Concrete is a material.
6. The compressive strength is rectangular, the

3
Eventually, the reinforcement reaches its yield point ($\varepsilon_y$). After yielding point, the strain in steel increases rapidly without the increase in stresses. This ductile behaviour of steel continues until the beam fails due to crushing of the concrete at the top face of the beam as shown in Figure 2.1(c).

Figure 2.1 Behaviour of reinforced concrete beam under increasing load (MacGregor, 1997)

2.2.2 Assumptions in Flexure Theory

The following assumptions are made for the analysis in the study (MacGregor, 1997):

1. Sections perpendicular to the axis of bending which are plane before bending remain plane after bending.
2. The strain in the reinforcement is equal to the strain in the concrete at the same level.
3. The stresses in the concrete and reinforcement can be computed from the strains using stress-strain curves for concrete and steel respectively.
4. The tensile strain of concrete is neglected in flexural strength calculations.
5. Concrete is assumed to fail when the compressive strain reaches a limiting value.
6. The compressive stress-strain relationship for concrete may be assumed to be rectangular, trapezoidal, parabolic, or any other shape.
2.3 STRESS-STRAIN CURVES

2.3.1 Concrete

Typical stress-strain curves for concrete of various strengths are shown in Figure 2.2. All curves depict very similar characteristics. They have an initial relatively straight elastic portion, which rise to a maximum stress at a strain about 0.002 to 0.003 for normal density concrete.

![Figure 2.2 Typical compressive stress-strain curves for normal density concrete with unit weight, \( w_c = 145 \text{pcf} \) or 2320 kg/m³ (Nilson & Winter, 1991)](image)

For concrete in the strength range to about 6000 psi or 40 MPa, the modulus of elasticity \( E_c \) in MPa can be computed with reasonable accuracy from the empirical equation found in the ACI Code (1995):

\[
E_c = 38 \left( \frac{w_c}{16.02} \right)^{1.5} \sqrt{f'_{c}} \times 6.895 \times 10^{-3}
\]  

(2-1)

where \( w_c \) = unit weight of the hardened concrete in \( \text{kg/m}^3 \); and \( f'_c \) = concrete strength in MPa.

For normal density concrete with \( w_c = 2320 \text{ kg/m}^3 \), \( E_c \) may be taken as

\[
E_c = 57,000 \sqrt{f'_{c}} \times 6.895 \times 10^{-3}
\]  

(2-2)

The two most common representations of the stress-strain curve proposed by Hognestad and Todeschini (MacGregor, 1997) are shown in Figure 2.3.

![Figure 2.3 Analytical approximations of stress-strain curves for concrete with unit weight, \( w_c = 145 \text{pcf} \) or 2320 kg/m³ (Nilson & Winter, 1991)](image)

The stress-strain curve in 2.3(b) is more common as a working function. The following is suggested:

\[
f''_c = \frac{2f'_c}{1 + (\frac{f'_c}{f''_c})^4}
\]

where the highest point on the curve is

\[
f''_c = 0.9 f'_c
\]

and the strain corresponding to it is

\[
\varepsilon_s = \frac{1.17 f'_c}{E_c}
\]
The stress-strain curve proposed by Todeschini (MacGregor, 1997) as shown in Figure 2.3(b) is more convenient for use in this analytical study because it is a continuous function. The following properties are defined for Figure 2.3(b).

The stress-strain curve can be represented by the following expression:

\[
f_r = \frac{2f''_c}{1 + \left( \frac{\varepsilon_r}{\varepsilon_s} \right)^2}
\]

where the highest point in the curve is given by

\[
f''_c = 0.9f'_c
\]

and the strain corresponding to maximum stress is expressed by

\[
\varepsilon_s = \frac{1.17f'_c}{E_c}
\]

![Diagram of stress-strain curves](image)

(a) Modified Hoggestad (1951)  
(b) Todeschini (1964)

Figure 2.3 Analytical approximations to the compressive stress-strain curve for concrete (MacGregor, 1997)
2.3.2 Steel

The important mechanical properties of reinforcing steel are displayed in their stress-strain curves as shown in Figure 2.4(a). The initial tangent modulus of elasticity \( E_s \), for all reinforcing steel can be taken as 29,000 ksi or 200,000 MPa. For simplicity in the analysis, the stress-strain curve of reinforcing steel is always idealized as elastoplastic behaviour as shown in Figure 2.4(b).

![Figure 2.4 Typical stress-strain for reinforcing steels](image)

2.4 ANALYSIS OF RC BEAMS

2.4.1 Stress and Strain Compatibility and Equilibrium

Two basic requirements of static have to be fulfilled throughout the analysis of RC beams. These include (i) stress & strain compatibility, and (ii) equilibrium.

2.4.2 Flexural Failure Modes

Flexural failures of a RC beam under load may occur in three different ways. The failure mode of the beam greatly depends on the provided amount of tensile reinforcement.

**Balanced failure.** Concrete crushes in compression and steel yields simultaneously. This simply implies that steel reaches its yield point \( \varepsilon_y = \varepsilon_s \) and the concrete reaches its ultimate strain \( \varepsilon_{cu} \) at the same time as shown in Figure 2.5(b). From the strain distribution diagram, the depth of neutral axis (N.A.) can be calculated as follows:

\[
\varepsilon_n = \frac{\varepsilon_{cu} - \varepsilon_y}{\varepsilon_{cu} + \varepsilon_y}
\]  
(2-6)

in which

\[
\varepsilon_y = \frac{f_y}{E_s}
\]

**Compression failure.** Steel. A large area of the steel is stressed into failure since the beam with such a distribution diagram expression:

\[
c_u > \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y}
\]

**Tension failure.** Concrete. If the beam is overloaded, failure of the concrete. The following section.

![Figure 2.5](image)

Note: \( b \) = width of reinforcing steel
Steel are displayed in their stress-strain curve. For simplicity, reinforcing steel is always idealized as linear elastic.

\[ \varepsilon_y = \frac{f_y}{E_s} \]  

(2-7)

**Compression failure.** Crushing of concrete in compression occurs before yielding of steel. A large area of steel is used such that strain in concrete reaches its ultimate before the steel is stressed to its yield point. This mode of failure is also identified as brittle failure since the crushing of concrete happens in a sudden without any warning. A beam with such a large steel area is called over-reinforced. Therefore from the strain distribution diagram in Figure 2.5(c), the depth of N.A. can be depicted by the following expression:

\[ c_u > \frac{e_{cu}}{e_{cu} + e_y} d \]  

(2-8)

**Tension failure.** In this mode of flexural failure, yielding of steel is followed by concrete compression failure. When the steel starts to yield, the concrete has not crushed. After the yielding point, the steel undergoes great strain without any increase in stress. Thus, the beam deflected extensively and developed wide cracks to provide ample warning of impending failure. Occupants of the building have an opportunity to leave the building before the final collapse. This is identified as ductile behaviour. A beam failing in such a ductile mode is called under-reinforced. Similarly, the strain distribution diagram in Figure 2.5(d) gives the expression for the depth of N.A. as follows:

\[ c_u < \frac{e_{cu}}{e_{cu} + e_y} d \]  

(2-9)

Therefore, it is a good practice to have designs in such a manner that should they be overloaded, failure would be initiated by yielding of the steel rather than by crushing of the concrete. Therefore, only under-reinforced sections will be considered in the following sections of the study.

![Figure 2.5 Strain distribution for various modes of flexural failure](image)

Note: \( b = \text{width of the beam section; } h = \text{overall depth of the beam section; } d = \text{effective depth of reinforcing steel} \)
2.5 ORIGINAL OR UNSTRENGTHENED RC BEAM

The term original or unstrengthened RC beam used in the following sections refers to the concrete beam, which is reinforced internally only.

A singly rectangular RC beam shown in Figure 2.6(a) has a width of \( b \), overall depth of \( h \) and an effective depth of \( d \). The section is provided with the area of tension reinforcing steel of \( A_s \).

Hence, the reinforcement ratio is given as

\[
\rho = \frac{A_s}{bd} \tag{2-10}
\]

and the reinforcing index is given as

\[
\omega = \frac{A_s f_y}{b d f'_c} = \frac{f_y}{f'_c} \tag{2-11}
\]

When the beam is loaded and reaches its yielding point as shown in Figure 2.6(b), this implies that the reinforcing steel reaches its yield strain, \( \varepsilon_y \).

\[\text{Figure 2.6 Stress and strain distributions at yielding of reinforcing steel}\]

From Figure 2.6, the curvature of beam at yielding of steel is defined as

\[
\varphi_y = \frac{E_c}{E_y} = \frac{\varepsilon_y}{d - \varepsilon_y} \tag{2-12}
\]

The compressive force of concrete is given as

\[C_e = \text{shaded area} \times b \tag{2-13}\]

The tensile force of reinforcing steel is given as

\[T_s = A_s f_y \tag{2-14}\]

The moment of resisting

\[M_{R_s} = T_s Z\]

where \( Z \) is the lever arm.

After yielding of the steel, the compression reaches greater than its yield.

\[C_e = \alpha_s f'_c \]

In common, the same symbols are used for all purposes. From Figures 2.7 (MacGregor, 1997)

\[C_e = \alpha_s f'_c \]

in which

\[a_s = \begin{cases} 0.85 & \text{if } \frac{0.85}{0.85 - 0.05} \leq \frac{0.85}{0.85 - 0.05} \\ 0.85 & \text{otherwise} \end{cases} \]

\[\beta_s = \begin{cases} 1.05 & \text{if } \frac{1.05}{1.05 - 0.65} \leq \frac{1.05}{1.05 - 0.65} \\ 0.65 & \text{otherwise} \end{cases} \]

From the equilibrium

\[C_e = \frac{T_s}{\alpha_s f'_c} \]
The moment of resistance at yielding of reinforcing steel can then be calculated from

\[ M_{Ry} = T_e Z - C_e Z \]  
(2-15)

where \( Z \) is the lever arm between \( C_e \) and \( T_e \).

After yielding of reinforcing steel, the steel is further deformed until the concrete in compression reaches its ultimate strain \( (\epsilon_u) \). In this state, the strain of steel is much greater than its yielding strain \((\epsilon_y)\), as shown in Figure 2.7.

![Figure 2.7 Stress and strain distributions at ultimate flexure limit state](image)

In common, the simplified stress block as shown Figure 2.7(d) is used for the design purposes. From Figure 2.7(d), the compressive force of concrete is expressed by (MacGregor, 1997)

\[ C_e = \alpha_f f'_c \beta_e \epsilon_u b \]  
(2-16)

in which

\[ \alpha_f = \left\{ \begin{array}{ll}
0.85 & \text{for } f'_c \leq 8000 \text{ psi or } 55.16 \text{ N/mm}^2 \\
0.85 - \frac{f'_c - 55.16}{344.75} & \geq 0.73 \text{ for } f'_c > 55.16 \text{ N/mm}^2
\end{array} \right. \]  
(2-17)

\[ \beta_e = \left\{ \begin{array}{ll}
0.85 & \text{for } 0 < f'_c \leq 4000 \text{ psi or } 27.58 \text{ N/mm}^2 \\
1.85 - \frac{f'_c}{137.9} & \text{for } 27.58 \text{ N/mm}^2 < f'_c \leq 55.16 \text{ N/mm}^2 \\
0.65 & \text{for } f'_c > 55.16 \text{ N/mm}^2
\end{array} \right. \]  
(2-18)

From the equilibrium equation, the depth of N.A. is then found as

\[ \epsilon_u = \frac{T_e}{\alpha_f f'_c \beta_e b} \]  
(2-19)
The curvature of beam at crushing of concrete can be expressed by
\[ \phi_u = \frac{e_{cm}}{e_u} \]  
(2-20)

The moment of resistance at ultimate is given as
\[ M_{Ru} = T_z Z = C_z Z \]  
(2-21)
in which the lever arm between \( T_z \) and \( C_z \) is found as
\[ Z = d - 0.5 \beta I e_u \]  
(2-22)
One of the conventional ductility measurement methods (see Section 2.7) for RC beam is calculating its curvature ductility which can be defined as
\[ \phi = \frac{\phi_u}{\phi_f} \]  
(2-23)

2.6 EXTERNALLY STRENGTHENED RC BEAM

Demand on structural retrofit and repair work on aging and deteriorated RC infrastructures and buildings has increased widely in the construction industry. These structures are either structurally deficient or functionally obsolete. This can be attributed to the fatigue of the structures subjected to cycling loading, exposure to unpredicted loads, increase in loading or overloading, faulty design and construction, and changes in structural purpose. One of the effective methods to rehabilitate these RC structures is bonding either reinforcing steel or FRP plates to the structures externally.

2.6.1 Reinforcing Steels

Reinforcing steels are traditional materials used in construction industry and primarily provided as internal reinforcements to carry the tensile forces in the RC beams. The behaviour and strength of the reinforcing steel are taken to be the same in tension and compression, hence it is agreeably to be used as external reinforcements. Two types of the reinforcing steel that most frequently used are mild steel and high yield steel.

2.6.2 FRP Composites

Fibre reinforced polymer/plastic (FRP) composites are composed of fibres embedded in a polymeric matrix. Fibres provide the composites with high tensile strength and rigidity along their longitudinal axis (Mayo et al., 1999). Matrix consists of a polymer or resin that bind and hold the reinforcing fibres together. It acts as a load transfer medium and resists the fibres from deterioration due to harsh environments.

Common high performance fibres for structural application are carbon, aramid and glass. Carbon fibres are the strongest, stiffest, and most durable. Aramid is an organic
fibre offers excellent impact resistance, and glass produces a common, low-cost reinforcing fibre (Mayo et al., 1999a).

Three types of commonly available resins are epoxy, vinyl ester and phenolic. Epoxy resins are the most common and have excellent structural properties and adhesion characteristics. Vinyl ester resins are lower cost matrix materials with good durability characteristics, but have lower structural performance and low resistance to heat. Whereas, phenolic resins are similar to vinyl ester but have a higher resistance to heat and low smoke generation (Mayo et al., 1999a).

FRP composites are used to strengthen the RC structures in various forms and systems. FRP can be made into sheets, which are thin and flexible fabric-like materials. Stacking one or more layers of sheets using resin into a desired thickness can form laminates. Unidirectional sheets refer to sheets of fibre that are all aligned in a common direction. Multidirectional sheets are similar to unidirectional sheets except fibres running in multiple directions are woven together. The fibres used can be different materials to form hybrid FRP laminates (Mayo et al., 1999a).

Normally, FRP composites vary significantly from one manufacturer to another. However, generally, FRP composites are light weight, high strength, resistance to corrosion, non-magnetic, fatigue resistance, ease of handling and erection, provide more versatile strengthening system, and require little or no maintenance. Due to these excellent characteristics, FRP have been widely and extensively used in strengthening or upgrading the existing RC and prestressed concrete (PC) structures.

2.7 DUCTILITY

Generally, ductility can be defined as an ability of a metal or other materials to deform plastically without any significant breaking or fracturing. This ability comes from the sufficient cohesive force remaining between the molecules to hold them together.

The ductility of a beam is described as its ability to undergo inelastic deformation without loss in load-carrying capacity or significant loss in resistance, prior to failure (Grace et al., 1999). It is a very important consideration in structural point of view to emphasise the safety, permit the moment redistribution or design the structures for seismic loading.

Ductility can be expressed in terms of deformation or energy. In the structure that shows clear elastic and plastic deformation, for example steel-reinforced beam, ductility can easily be determined by using deformation method. Conventionally, deformation can be referred to strain ($\varepsilon$), rotation ($\theta$), curvature ($\phi$) or deflection ($\delta$). Thus, ductility can be defined mathematically as the ratio of ultimate deformation to the deformation at yielding state, which is given by

$$\phi = \frac{\varepsilon_u}{\varepsilon_y}$$

or

$$\delta = \frac{\varepsilon_u}{\varepsilon_y}$$
\[ \phi = \frac{\theta_x}{\theta_y} \]  

or Eq. (2-23) or

\[ \phi = \frac{\delta_x}{\delta_y} \]  

The beam strengthened with FRP materials usually reaches its ultimate state without exhibiting material yielding point. Therefore, the conventional definition of ductility is not applicable for FRP-strengthened beams. Alternatively, using energy method, ductility may be described as a ratio relating any two of the inelastic, elastic, and total energies. For instance, Grace et al. (1999) proposed a new ductility definition as energy ratio, which is the ratio of the inelastic energy to the total energy. In general, energy method provides a more common basis in calculating ductility.

2.8 PREVIOUS RESEARCHES

Previous researches showed that more and more studies were carried out in the field of structural adhesives, in which some reinforcing materials are bonded to the structure externally for strengthening purposes. For the relevant references, studies on (i) beam strengthening, and (ii) both beam strengthening and ductility, are highlighted here.

2.8.1 . Studies on Beam Strengthening

2.8.1.1 Norris et al. (1997)

Norris et al. (1997) conducted an experimental and analytical study on epoxy-bonded carbon fibre reinforced plastic (CFRP) of damaged or understrength RC beams. Nineteen RC beams were cast and precracked prior to retrofitting to more closely represent the field conditions. These beams were retrofitted with three different CFRP systems bonded to their tension face and web to increase the corresponding flexural and shear strengths. The results showed an increase in strength of 20% to 100% over the control beam. It was found that the measured results tallied quite reasonably with the predicted values obtained from a computer program.

2.8.1.2 Mayo et al. (1999)

Mayo et al. (1999a, 1999b) had proposed the project to increase the flexural strength of the existing structurally deficient and functionally obsolete bridges in United States using externally bonded carbon fibre reinforced polymer (CFRP). The demonstration of the CFRP strengthening technology was done for Bridge G-270. Verification on the effectiveness of the strengthening system was accomplished by laboratory testing of two full-scale beams and in-situ field tests of the actual bridge before and after strengthening. The results showed that the externally bonded CFRP sheets can effectively produce an enhancement in flexural capacity and stiffness of the structure. The average deflection measurements after strengthening were 94% of the original.