Faculty of Engineering

Principles of Prioritization for Failure Mode and Effects Analysis with Missing Risk Ratings

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Principles of Prioritization for Failure Mode and Effects Analysis with Missing Risk Ratings

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DECLARATION

The thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

Name:

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ABSTRACT

Failure mode and effects analysis (FMEA) is a reliability engineering methodology used to define, identify, and eliminate known and /or potential failure modes, problems, errors, for a system, design, process, and /or service. FMEA adopts a risk priority number (RPN) model which considers three risk factors, i.e., Severity (S), Occurrence (O), and Detection (D), for prioritization of potential failure modes and corrective actions. However, most of the studies do not focus on the rationality of the prioritization outcomes, and assume that all risk ratings (i.e., S, O, and D ratings) are known *a priori*. In this thesis, three basic principles, i.e., *transitivity*, *pareto optimality,* and *independence of irrelevant alternatives*, are formulated to ensure rationality of prioritization of failure modes or corrective actions in FMEA, are analysed together with the rational properties of an RPN model. It is deduced that the monotonicity property is important for fulfilling the pareto optimality principle. Besides, a new tree model and a vector representation for potential failure modes and corrective actions are proposed. Moreover, an interval-valued RPN model to solve the problem with missing S, O, and/or D ratings by imputing the corresponding rational interval value(s) is further proposed. The proposed solution is evaluated analytically with respect to the aforementioned three principles. Empirical evaluations using benchmark information ascertain the usefulness of the proposed solution.

Keywords: Failure mode and effects analysis, transitivity, pareto optimality, independence and irrelevant alternatives, monotonicity property

Prinsip Keutamaan bagi Analisis Mod Kegagalan dan Kesan dengan Pengkadaran Risiko yang Hilang

ABSTRAK

Analisis Mod Kegagalan dan Kesan (Failure Mode and Effects Analysis, FMEA) adalah suatu metodologi kejuruteraan kebolehpercayaan untuk menentukan, mengenal pasti, dan menghapuskan mod kegagalan, masalah, atau kesilapan yang diketahui atau berpotensi bagi sesuatu sistem, reka bentuk, proses, atau perkhidmatan. FMEA menggunakan modal Nombor Keutamaan Risiko (Risk Priority Number, RPN) yang mempunyai tiga faktor risiko, iaitu Keparahan (Severity, S), Kejadian (Occurrence, O), dan Pengesanan (Detection, D), untuk menyusun mod kegagalan dan tindakan pembetulan. Walau bagaimanapun, kebanyakan kajian tidak mengutamakan kerasionalan hasil keutamaan dan mengandaikan bahawa semua pengkadaran risiko iaitu, pengkadaran S, O, dan D adalah diketahui dengan tepat. Dalam tesis ini, tiga prinsip asas iaitu transitiviti, optimalisasi pareto (pareto optimality), dan kebebasan alternatif yang tidak relevan (independence of irrelevant alternatives) telah dirumuskan bagi memastikan kerasionalan penyusunan mod kegagalan atau tindakan pembetulan di FMEA, serta menganalisis sifat rasional model RPN. Penyelidikan ini telah menyimpulkan sifat monoton (monotonicity) adalah penting untuk memenuhi prinsip optimalisasi pareto. Selain itu, satu rajah pohon dan perwakilan vektor bagi mod kegagalan dan tindakan pembetulan telah dicadangkan. Di samping itu, model RPN yang bernilai interval telah dicadangkan untuk mengatasi masalah pengkadaran S, O, dan D yang hilang dengan mengantikan pengkadaran tersebut dengan nilai interval yang rasional. Penyelesaian tersebut telah dinilai secara analitikal berdasarkan tiga prinsip asas *yang telah disebutkan di atas. Kegunaan penyelesaian telah dinilai berdasarkan analisis secara empirikal dengan penggunaan maklumat penanda aras.*

Kata kunci: *Analisis Mod Kegagalan dan Kesan, transitiviti, optimalisasi pareto, kebebasan alternatif yang tidak relevan, sifat monoton*

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LIST OF ABBREVIATIONS

CNF/1000	A cumulative number of component failures per 1000 components over the design life of the component under study
D	Detection
FMEA	Failure mode and effects analysis
FMECA	Failure mode, effects, and criticality analysis
HFMEA	Healthcare Failure Mode and Effect Analysis
MIL-P	Military Procedures document
NASA	National Aeronautics and Space Administration
O	Occurrence
RPN	Risk priority number
S	Severity
TODIM	An acronym in Portuguese of interactive and multicriteria decision making
US	United States

CHAPTER 1

INTRODUCTION

1.1 Background

Failure Mode and Effects Analysis (FMEA) was first developed as a formal design methodology in 1960s for use in the aerospace industry (Carlson, 2012). The main goal of FMEA is used to define, identify, and eliminate known and /or potential failure modes, problems, errors, for a system, design, process, and /or service (Stamatis, 2003). FMEA is also known as Failure Mode, Effect, and Criticality Analysis (FMECA), while focuses in failure modes prioritization (Certa, Hopps, Inghilleri, & La Fata, 2017). A failure mode occurs when a component, subsystem, part, or process fails to meet its intended purpose of function(s) (Carlson, 2012; Ireson, Coombs, & Moss, 1996; Press, 2003). A failure mode of a component can be a root cause for the failure mode(s) of another component. A root cause leads to the occurrence of a failure mode (Ireson et al., 1996; Press, 2003). Prevention method (s) for a root cause needs to be identified. For each failure mode, the effect(s) of the failure mode also needs to be determined. Corrective actions or failure modes are then prioritized using an RPN model. The RPN model takes into account three factors, i.e., Severity (S), Occurrence (O), and Detection (D). S is an assessment of the effect of a potential failure mode; O is the likelihood that a specific root cause occurs; while D is an assessment of the ability of the current control mechanism to detect a potential root cause (Ireson et al., 1996). Traditionally, an RPN score is obtained by direct multiplication of the

S, O and D ratings. Corrective actions or failure modes associated with higher RPN scores are usually given higher priorities.

Regardless of the popularity of FMEA, there are several issues related to the effective implementation of FMEA, i.e., it is a time consuming activity (Carlson, 2012; Mikulak, McDermott, & Beauregard, 2008; Press, 2003; Stamatis, 2003); issues pertaining to prioritization of failure modes or corrective actions (Bowles & Peláez, 1995; Franceschini & Galetto, 2001; Pillay & Wang, 2003; Tay & Lim, 2006b); it is difficult to use (Montgomery, Pugh, Leedham, & Twitchett, 1996; Signor, 2002); the FMEA form is complex (Montgomery et al., 1996; Signor, 2002); the three factors of S, O, and D are difficult to be precisely evaluated (Gargama & Chaturvedi, 2011; Liu, Liu, & Lin, 2013; Yang, Bonsall, & Wang, 2008). Indeed, many methods have been proposed to solve the prioritization issues of failure modes or corrective actions (Asan & Soyer, 2016; Gargama & Chaturvedi, 2011; Liu, Liu, & Liu, 2013; Liu, You, Li, & Su, 2016).

1.2 Problem Statements and Motivations

Most of the studies do not focus on the rationality of the prioritization outcomes, and assumes that all risk ratings (i.e., S, O, and D ratings) are known *a priori*. In addition, it is not clear, how rational prioritization outcomes could be obtained with missing risk ratings. These issues motivate the research into a principled analysis of FMEA.

1.3 Objectives of the Research

The research aim is three-fold: (i) three rational principles, inspired by the work in (Arrow & Raynaud, 1986), i.e., *transitivity*, *pareto optimality,* and *independence of irrelevant alternatives* are analysed for prioritization of failure modes or corrective actions in FMEA, together with the rational properties of an RPN model; (ii) a new tree model and a vector representation for potential failure modes and corrective actions are proposed for effective FMEA activities, and (iii) an interval method to tackle missing S, O, and/or D ratings is proposed based on the three principles.

1.4 Scopes of the Research

In this thesis, the interval-valued RPN model is viewed as a solution to the prioritization of failure modes and corrective actions in FMEA, which is subject to missing risk ratings. The scope of research is on the exploitation of the three rational principles for developing an interval-valued RPN model. Besides, a new out-tree model and a nested-vector representation are developed for effective evaluation, ranking, and prioritization of potential failure modes and corrective actions in an FMEA activity.

It is worth noting that the imprecise information such as probabilistic, linguistic and fuzziness are not considered in the imputation of missing risk ratings. In addition, the experiments are conducted with benchmark examples pertaining to the scope of design FMEA.

1.5 Research Methodology

The research methodology adopted in this research is depicted in Figure 1.1. Firstly, the background and literature review related to this research are reviewed. The three rational principles for prioritization of failure modes and corrective actions in FMEA are proposed and analysed together with rational properties of an RPN model. A tree model and a vector representation for failure modes and corrective actions in FMEA are developed. An interval method to handle missing risk ratings is proposed and analysed with aforementioned three principles. Finally, concluding remarks and suggestions for future works are presented.

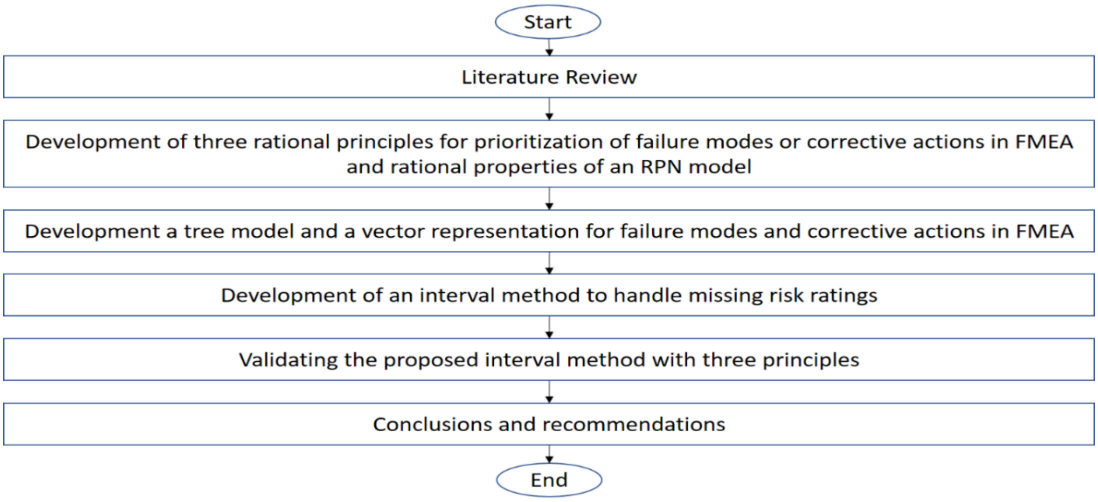

Figure 1.1: Research Methodology

1.6 Organization of Thesis

This thesis is organized as follows. In Chapter 1, the research background is first described. The problem statements and motivations are explained. The research objectives, scopes, and methodology are also presented. In Chapter 2, the preliminaries and background of FMEA, as well as the limitations and improvements of FMEA, are described. In Chapter 3, four benchmark examples of FMEA, as well as the three principles for prioritization of failure modes and corrective actions are presented together with analysis. In Chapter 4, a tree model and a vector representation for failure modes and corrective actions in FMEA are proposed. In Chapter 5, an interval method for handling missing risk ratings are presented and analysed. Moreover, two experimental studies with two sets of real information are conducted. Lastly, concluding remarks and suggestion on future works are introduced and discussed.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

In this chapter, the background studies on FMEA and its recent advances are presented. In Section 2.2, background, use, types, benefits and limitations of FMEA, and the FMEA procedure are then presented in details. Subsequently, improvement of FMEA related to missing risk ratings are further presented and discussed in Section 2.3. Finally, concluding remarks are presented.

2.2 Failure Mode and Effects Analysis (FMEA)

In general, FMEA is a reliability engineering methodology used to define, identify, and eliminate known and /or potential failure modes, problems, errors, for a system, design, process, and /or service (Stamatis, 2003). Begun in the 1940s, FMEA was formalized by the US military by the introduction of Military Procedures document (MIL-P)-1629, to classify failure according to their impact on mission success and personnel or equipment safety. FMEA was later developed in 1960s, and adopted as a formal design methodology for safety and reliability requirements by aerospace industry (Bowles & Peláez, 1995; Mikulak et al., 2008) in NASA’s National Aeronautics and Space Administration Apollo-Saturn program to mitigate risk. In the late 1970s, FMEA was used in the automotive industry, i.e., Ford Motor Company for safety and regulatory consideration after the Pinto affair. Since then, it has been proven to be a useful tool in assessing potential failure modes and preventing them from occurring (Carlson, 2012; Stamatis, 2003).

A failure mode occurs when a component, subsystem, part, or process fails to meet its intended purpose of function(s) (Carlson, 2012; Ireson et al., 1996; Press, 2003). A root cause leads to the occurrence of a potential failure mode (Ireson et al., 1996; Press, 2003). The prevention method(s) of a root cause needs to be identified too. For each failure mode, the effect(s) of the failure mode also needs to be determined. The potential failure modes are then prioritized using a Risk Priority Number (RPN) model. The RPN model takes into account three factors, i.e., Severity (S), Occurrence (O), and Detection (D). S is an assessment of the effect of a potential failure mode; O is the likelihood that a specific root cause occurs; while D is an assessment of the ability of the current control mechanism to detect a potential root cause. Traditionally, an RPN score is obtained by direct multiplication of the S, O, and D ratings. The potential failure modes associated with higher RPN scores are usually given higher priorities.

2.2.1 Types of FMEA

Generally, there are four types of FMEAs (Stamatis, 2003), i.e., system FMEA, design FMEA, process FMEA, and service FMEA. All these types of FMEA produce a list of failure modes or corrective actions with associated RPN scores to reduce failures or to improve failure detection.

System FMEA is a high level analysis of an entire system, made up of various subsystems in the early concept and design phase (Carlson, 2012; Stamatis, 2003). It analyzes the failure modes pertaining to system-related deficiencies, i.e., system safety, system integration, interactions between subsystems or with other systems, interactions with the surrounding environments, human interaction, services, and other systems (Carlson, 2012). A significant output of systems FMEA is a list of potential system functions that could detect potential failure modes.

Design FMEA focuses on product design at the subsystem or component level, typically failure modes caused by design-related deficiencies. A significant output of design FMEA is a list of design improvement actions with priority that could ensure product operation is reliable and safe during the useful life of the equipment (Carlson, 2012).

Process FMEA focuses on the manufacturing or assembly process. It helps to improve to the manufacturing process by ensuring that a product is built to design requirements in a safe manner with minimal failures (Carlson, 2012). A significant output of process FMEA is a list of corrective actions with priority to address the critical and significant characteristics of the process.

Service FMEA focuses on failure modes caused by system or process deficiencies (Stamatis, 2003). The services are analysed in service FMEA before they reach the customer (Stamatis, 2003). The significant outputs of service FMEA are a list of significant or critical tasks or process, a list of potential bottleneck processes or task, and a list of potential functions for monitoring system or process functions.

2.2.2 The FMEA Procedure

The procedure of FMEA involves several activities, as depicted in Figure 2.1. A description of the key activities is as follows.

i. Examine the process, design, product or service;

ii. Develop the scale tables for S, O, and D;

iii. Ascertain the potential failure modes of the process, design, product or service which include problems, concerns, and opportunities for improvement;

iv. Ascertain the effect of each failure mode to processes, products, operations, customers, and government regulations;

v. Assess the impact pertaining to the effect using the S scale table;

vi. Ascertain the root cause of each failure mode;

vii. Assess the occurrence frequency pertaining to the root cause using the O scale table;

viii. Identify the current prevention method pertaining to each root cause;

ix. Assess the effectiveness of each current prevention method using the D scale table;

x. Compute the RPN scores of each failure modes and prioritize the failure modes;

xi. Identify and implement corrective action to eliminate or reduce the high-risk failure modes;

xii. Re-assess risk with another FMEA cycle.

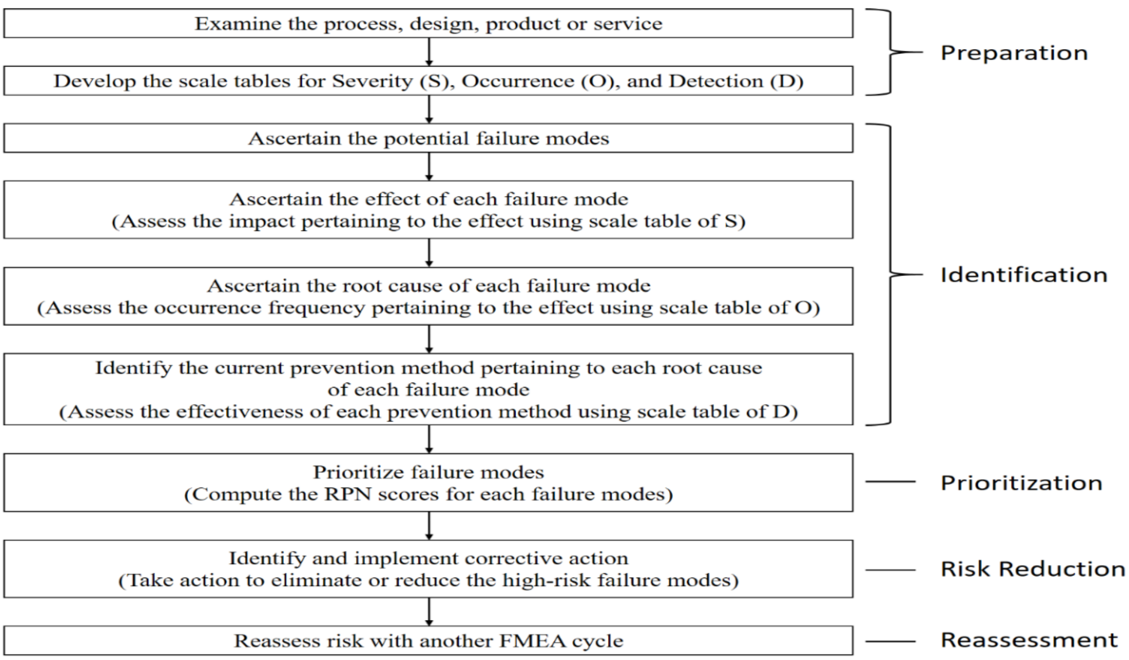

Figure 2.1: FMEA procedure

Figure 2.2 shows the common FMEA form. The basic items for FMEA are shown. Extra columns may be added depending on the need or requirement of systems. The guidelines for scale tables of D, S, and O also presented in Tables 2.1, 2.2, and 2.3, respectively. Note that all the guidelines and ratings may be vary to fit in different circumstances.

Type of FMEA ____________ FMEA date ____________

Prepared by ____________ Page ______ of ______ pages ______

System/ design/ process/ service function	Potential failure mode	Potential effect(s) of failure	∇	Potential cause(s) of failure	Detection method	O C C	S E V	D E T	R P N	Recommended action	Responsibility & completion date	Action results				
												Action taken	S E V	O C C	D E T	R P N

Figure 2.2: A generic FMEA from (Stamatis, 2003)

Table 2.1: Suggested Guideline for Detection Rating (Stamatis, 2003)

Detection	Rating	Criteria
Almost Certain	1	Current controls almost always will detect the failure. Reliable detection controls are known and used in similar processes.
Very High	2	Very high likelihood current controls will detect the failure.
High	3	Good likelihood current controls will detect the failure.
Moderately High	4	Moderately high likelihood current controls will detect the failure.
Medium	5	Medium likelihood current controls will detect the failure.
Low	6	Low likelihood current controls will detect the failure.
Slight	7	Slight likelihood current controls will detect the failure.
Very Slight	8	Very slight likelihood current controls will detect the failure.
Remote	9	Remote likelihood current controls will detect the failure.
Almost Impossible	10	No known controls available to detect the failure.

Table 2.2: Suggested Guideline for Severity Rating (Stamatis, 2003)

Effect	Rating	Criteria
No effect	1	No effect on product or subsequent processes.
Very slight effect	2	Customer more likely will not notice the failure. Very slight effect on product/process performance. Nonvital fault noticed sometimes.
Slight effect	3	Customer slightly annoyed. Slight effect on product or process performance. Nonvital fault noticed most of the time.
Minor effect	4	Customer experiences minor nuisance. Minor effect on product/process performance. Fault does not require repair. Nonvital fault always noticed.
Moderate effect	5	Customer experiences some dissatisfaction. Moderate effect on product/process performance. Fault on nonvital part requires repair.
Significant effect	6	Customer experiences discomfort. Product/process performance degraded, but operable and safe. Nonvital part inoperable.
Major effect	7	Customer dissatisfied. Major effect on process; rework/repairs on part necessary. Product/process performance severely affected but functionable and safe. Subsystem inoperable.
Extreme effect	8	Customer very dissatisfied. Extreme effect on process; equipment damaged. Product inoperable but safe. System inoperable.
Serious effect	9	Potential hazardous effect. Able to stop product without mishap; safety-related; time-dependent failure. Disruption to subsequent process operations. Compliance with government regulation is in jeopardy.
Hazardous effect	10	Hazardous effect. Safety-related—sudden failure. Noncompliance with government regulation.

Table 2.3: Suggested Guideline for Occurrence Rating (Stamatis, 2003)

Occurrence of Failure	Rating	Criteria
Almost Never	1	Failure unlikely. History shows no failures.
Remote	2	Rare number of failures likely.
Very Slight	3	Very few failures likely.
Slight	4	Few failures likely.
Low	5	Occasional number of failures likely.
Medium	6	Moderate number of failures likely.
Moderately High	7	Frequent high number of failures likely.
High	8	High number of failures likely.
Very High	9	Very high number of failures likely.
Almost Certain	10	Failure almost certain. History of failures exists from previous or similar designs.

2.2.3 Use of FMEA

Today, FMEA has been widely used in a variety of industries, which include aerospace (Bowles & Peláez, 1995), agriculture (Jong, Tay, & Lim, 2013; Sang, Tay, Lim, & Nahavandi, 2018), automotive (Banduka, Veža, & Bilić, 2016; Stamatis, 2003), chemical (Garrick, 1988), electronic (Certa et al., 2017; Zafiropoulos & Dialynas, 2005), healthcare (Chanamool & Naenna, 2016; Jamshidi, Rahimi, Ait-kadi, & Ruiz, 2015; Kumru & Kumru, 2013; McNally, Page, & Sunderland, 1997), manufacturing (Can, 2017; Chen, 2017; P.-S. Chen & Wu, 2013; Huang, Li, & Liu, 2017; Selim, Yunusoglu, & Yılmaz Balaman, 2016; Tay & Lim, 2006b), mechanical (Korayem & Iravani, 2008; Liu, You, Ding, & Su, 2015; Xu, Tang, Xie, Ho, & Zhu, 2002), nuclear (Guimarães & Lapa, 2004; Song, Ming, Wu, & Zhu, 2013), and petroleum (Calixto, 2016).

2.2.4 Benefits and Limitations of FMEA

The benefits of conducting an FMEA have been identified for many reasons as follows (Ireson et al., 1996; Press, 2003; Stamatis, 2003).

i. It identifies potential and known failure modes at a system, design, process, or service that may adversely affect safety or compliance with government regulations so that corrective actions can be taken to mitigate its effect.
ii. It is a systematic approach which can provide the basis for the test program during development and final validation of the system, design, process, or service.
iii. It provides historical document for future reference to aid in the analysis of failures and consideration of design, process and service changes.
iv. It provides an opportunity for FMEA users from different expertise areas in an organization to cooperate, communicate, recommend and tracking corrective actions.

Even though FMEA is popular, it is susceptible to several limitations, as listed below and summarized in Table 2.4.

i. The relative significance of risk ratings is equally weighted. For example, a failure mode with a serious effect, low frequency of occurrence, and high detectability (e.g., S=9, O=5, and D=3, respectively) may have a lower RPN score (RPN=135) than one with all moderate ratings (e.g., S=5, O=6, and D=6 leads to RPN score of 180), even though it should have a higher priority for corrective action.
ii. Different combinations of risk ratings may obtain the same values of RPN score, but their hidden risk application may be totally different. For example, two different failure modes with the ratings of 4, 4, 5 and 10, 2, 4 for S, O, and D, respectively,

will have the same RPN score of 80. Yet, the hidden risk applications of the two failure modes may be different due to the different severities of the failure effect.

iii. A variation in one risk ratings may lead to significant variations on RPN scores. For example, if the ratings of S and O are both equal to 10, then a 1-point difference in D rating results in a 100-point difference in the RPN score.

iv. The RPN model only considers three risk factors in terms of safety which it ignores other factors such as production cost, quality, etc.

v. FMEA is difficult to use because the FMEA worksheet is hard to produce, hard to maintain, as well hard to read. FMEA requires a number of FMEA users to fill in a worksheet with a list of items, e.g., product/process of the system, their function, their potential failure modes, and the potential effects, root causes and current preventive method of the failure modes. Besides, the FMEA is static, as well as the RPN is often not updated with the emergence of new sources of failure risks.

vi. The FMEA form is complex due to the entries in a FMEA worksheet are voluminous and complex. For instance, it is unwieldly with much mouse scrolling required if FMEA is implemented in Microsoft Excel. Too much mouse scrolling prevents FMEA users from seeing the overall structure of the FMEA worksheet and detracts FMEA users' mental representation of the FMEA worksheet as a whole.

vii. It is a time-consuming activity because the analysis of complex systems that have multiple functions consisting of a number of components, subsystem, part, or process is tedious and difficult. Besides, FMEA do take time because it involves several FMEA users from different expertise and background to identify the failure modes, understand the failure modes, and prioritize the analysis.

viii. The three risk factors are difficult to be precisely evaluated. It is difficult to give a numerical and precise assessment for qualitative information such as S, O, and D.

ix. The conversion of ratings is different for the three risk factors. The relationship between O and the associated rating is non-linear while the D and associated rating having a linear relationship.

Table 2.4: Limitations of FMEA in FMEA Studies

No.	Reference	Limitations								
		i	ii	iii	iv	v	vi	vii	viii	ix
1	Wang et al. (2018)	*	*	*					*	
2	Yazdi, Daneshvar, & Setareh (2017)	*	*						*	
3	Certa et al. (2017)	*	*	*	*				*	
4	Huang et al. (2017)	*		*					*	
5	J. K. Chen (2017)	*	*						*	*
6	Can (2017)	*	*	*	*			*	*	
7	Bao, Johansson, & Zhang (2017)	*	*						*	
8	Mohsen & Fereshteh (2017)		*	*					*	
9	Chang, Pang, & Tay (2017)					*	*			
10	Chemweno et al. (2017)					*				
11	Liu, Chen, You, & Li (2016)	*	*						*	
12	Li, Xiao, Fei, Mahadevan, & Deng (2016)	*	*						*	
13	Guo (2016)	*	*						*	
14	Banduka et al. (2016)					*		*		
15	Lolli, Gamberini, Rimini, & Pulga (2016)	*	*						*	
16	Nguyen, Shu, & Hsu (2016)	*	*							
17	Liu, You, Chen, & Chen (2016)	*	*						*	
18	Liu, You, Li, et al. (2016)	*	*						*	
19	Liu, You, Shan, & Shao (2015)	*	*	*					*	
20	Liu, You, Lin, & Li (2015)	*	*	*					*	

Table 2.4 continued

No.	Reference	Limitations								
		i	ii	iii	iv	v	vi	vii	viii	ix
21	Liu, Li, You, & Chen (2015)	*	*						*	*
22	Liu, You, You, & Shan (2015)	*	*	*					*	
23	Bozdag, Asan, Soyer, & Serdarasan (2015)	*		*					*	
24	Lolli, Ishizaka, Gamberini, Rimini, & Messori (2015)	*	*						*	
25	Liu, You, Ding, et al. (2015)	*	*							
26	Mandal et al. (2015)	*	*						*	
27	Mentes & Ozen (2015)	*	*	*						
28	Liu, You, Fan, & Lin (2014)	*	*						*	
29	Liu, You, & You (2014)	*	*						*	
30	Liu, Fan, Li, & Chen (2014)	*	*						*	
31	Helvacioglu & Ozen (2014)	*	*							
32	Mandal & Maiti (2014)	*	*						*	
33	Liu, Liu, & Lin (2013)	*	*		*				*	*
34	Hsu, Lee, & Chao (2013)	*								
35	Kutlu & Ekmekçioğlu (2012)	*	*	*					*	
36	Liu, Liu, Liu, & Mao (2012)	*	*	*					*	
37	Carlson (2012)							*		
38	Liu et al. (2011)	*	*	*	*				*	*
39	Zhang & Chu (2011)	*	*							
40	Bradley & Guerrero (2011)	*	*							
41	Gargama & Chaturvedi (2011)	*	*	*					*	
42	Sharma & Sharma (2010)	*	*							
43	Tay & Lim (2010)	*	*							
44	Chin, Wang, Poon, & Yang (2009a)	*	*						*	
45	Chin, Wang, Poon, & Yang (2009b)	*	*	*	*				*	*
46	Keskin & Özkan (2009)	*	*							
47	Chang & Paul Sun (2009)	*	*							
48	Carmignani (2009)	*	*		*					*

Table 2.4 continued

No.	Reference	Limitations								
		i	ii	iii	iv	v	vi	vii	viii	ix
49	Wang, Chin, Poon, & Yang (2009)	*	*						*	
50	Sharma, Kumar, & Kumar (2008)	*	*							
51	Yang et al. (2008)	*	*	*	*				*	
52	von Ahsen (2008)	*	*							*
53	Mikulak et al. (2008)							*		
54	Sharma, Kumar, & Kumar (2007a)	*	*							
55	Sharma, Kumar, & Kumar (2007b)	*	*							
56	Tay & Lim (2006b)	*	*							
57	Seyed-Hosseini, Safaei, & Asgharpour (2006)	*								
58	Tay & Lim (2006a)	*	*							
59	Sharma, Kumar, & Kumar (2005)	*	*						*	
60	Shahin (2004)	*	*							
61	Pillay & Wang (2003)	*	*							
62	Stamatis (2003)							*		
63	Press (2003)							*		
64	Xu et al. (2002)								*	
65	Puente, Pino, Priore, & de la Fuente (2002)	*	*							*
66	Signor (2002)					*	*			
67	Franceschini & Galetto (2001)	*	*							
68	Montgomery et al. (1996)					*	*	*		
69	Bowles & Peláez (1995)	*								

Recap from Table 2.4, it is obvious that limitations of i, ii, and viii, which closely related to risk ratings and risk factors are occurs in most of the studies. In addition, most of the studies in the literature assume that all risk ratings (i.e., S, O and D values) are known *a priori*. This means that a complete set of risk ratings, which may be known precisely or imprecisely, is available. We argue that these risk ratings may not always be available in hand for analysis, e.g., see the examples in (Stamatis, 2003) (page 239), (Chin et al., 2009b) (pages 1774 & 1775), (Liu et al., 2011) (page 4412), and (Murray, 2013) (page 16). The risk ratings may be missing because (1) FMEA users fail to provide a complete assessment of all risk ratings; (2) assessment is completed partially where some risk ratings are ignored; (3) difficulty in achieving a consensus for some risk ratings owing to different expertise and background of FMEA users (Naude, Lockett, & Holmes, 1997); (4) mistakes in data entry and/or information transfer; (5) risk ratings are censored, i.e., not missing at random (NMAR) (García-Laencina, Sancho-Gómez, & Figueiras-Vidal, 2010).

2.3 Recent Advances on FMEA

Over the years, many methods have been proposed to improve the FMEA methodology and to overcome the limitations. Liu, Liu, & Liu (2013) claimed that a total of 75 papers pertaining risk evaluation of FMEA were reviewed and categorized into five main categories, i.e., multicriteria decision making methods, mathematical programming, artificial intelligence, hybrid methods and others. Bozdag et al. (2015) also pointed out that there are several theories to deal with the missing risk ratings in FMEA, i.e., fuzzy set theory (ordinary and intuitionistic), 2-tuple fuzzy linguistic representation, rough set theory, evidence theory, grey theory and interval type-2 fuzzy sets.

From the literature, studies that are closely related to the missing risk ratings issue of FMEA have been reported. Important findings that are closely related to this research are further discussed as follows.

2.3.1 Findings from Chin et al. (2009a)

Chin et al. (2009a) proposed a data envelopment analysis based FMEA to consider the relative significance of risk ratings, and able to prioritize the failure modes by considering incomplete and imprecise information from FMEA users. An incomplete set of risk ratings provided by five FMEA users, i.e., hypothetical ratings of 21 failure modes on three risk factors which adapted from Pillay and Wang (2003) was examined using the proposed method.

2.3.2 Findings from Chin et al. (2009b)

A group-based evidential reasoning method to capture diverse opinions from FMEA users, and to prioritize the failure modes under different types of uncertainties such as incomplete assessment, ignorance and intervals, was proposed. An incomplete set of risk ratings provided by five FMEA users, which is adapted from Pillay and Wang (2003), was examined using the proposed method to overcome the shortcomings of FMEA.

2.3.3 Findings from Liu et al. (2011)

A hybrid method using fuzzy evidential reasoning and grey theory to prioritize failure modes by taking into account incomplete and imprecise information was reported. The incomplete assessment is treated as fuzzy assessment and defuzzified into crisp numbers.

A numerical example is adapted from Wang et al. (2009) to demonstrate the usefulness of proposed method to handle incomplete and imprecise information from 5 FMEA users.

2.3.4 Findings from Bradley & Guerrero (2011)

A method to interpolate missing risk ratings was proposed to overcome the limitations of FMEA. The proposed method only required a partial set of risk scores to determine the missing risk scores using an interpolation method. An experimental result was examined to illustrate the capability of proposed method to solve the FMEA with missing risk ratings. Besides, Bradley & Guerrero (2011) also developed a spreadsheet tool that is simple and easy to motivate the use of FMEA in practice.

2.3.5 Findings from Liu et al. (2013)

An integrated method based on fuzzy evidential reasoning and belief rule-based methodology to prioritize the failure modes by considering missing information was proposed. The missing risk ratings are treated as interval to consider the confidence level of FMEA users in their assessment. A case study pertaining to an ocean going fishing vessel, which is adapted from Pillay and Wang (2003) was conducted using the proposed method to show the capability of the proposed method.

2.3.6 Findings from Liu et al. (2014)

A new risk priority (RPN) model by integrating D numbers and grey relational projection method, to prioritize failure modes under incompleteness was proposed. The proposed method also showed that it is capable to consider the relative significance of risk factors, various type of uncertainties, measurement scale, and double reference points compared to FMEA methodology. An application of rotor blades for an aircraft turbine which is adapted from Yang et al. (2011) was examined to illustrate the potential applications of the proposed method.

2.3.7 Findings from Bozdag et al. (2015)

Bozdag et al. (2015) proposed a novel fuzzy FMEA approach based on interval type-2 fuzz sets to prioritize the failure modes with incomplete information. Besides, several methods on FMEA have been compared in this paper with respect to their ability to deal with uncertainty. Bozdag et al. (2015) stated that 2-tuple fuzzy linguistic representation is outstanding while it is computing with words and tend to avoid information loss. An application of FMEA in an assembly process at a manufacturing facility operating in the automotive industry was examined to compare the results of the proposed method, FMEA methodology, and approach suggested by Kutlu & Ekmekçioğlu (2012) respectively.

2.3.8 Findings from Liu et al. (2015)

A novel FMEA approach combining interval 2-tuple linguistic variables with gray relational analysis to capture FMEA team members' diversity opinions and improve the effectiveness of the FMEA methodology, was presented. The diversity and uncertainty of FMEA users' opinions is modeled using interval 2-tuple linguistic variables. An empirical example of a C-arm X-ray machine was carried out to illustrate the capability of the proposed approach.

2.3.9 Remarks

A summary of methods from this sub-section is tabulated in Table 2.5. Recap from Table 2.5, all the discussed approaches are capable to handle missing risk ratings except the traditional RPN methodology. It is worth noting that all discussed approaches need a preliminary or additional information about data, i.e., assumptions, pre-defined functions, belief structures in evidential reasoning, pre-determined membership functions in fuzzy sets, or reference series in grey relational analysis to deal with missing risk ratings.

Except the traditional RPN methodology and the data elicitation method, most of the discussed approaches are capable to aggregate different assessments from a group of FMEA users and considers the confidence level of FMEA users in their subjective assessment completely or partially. In addition, those with prior information also can deal with different types of assessment information simultaneously.

Table 2.5: Comparisons of theories used in FMEA with respect to their ability to deal with missing risk ratings

Approach based on	Studies on FMEA	Can handle missing risk ratings	Can aggregate different assessments from a group of FMEA users	Considers the confidence level of FMEA users in their subjective assessment	Can deal with different types of assessment information simultaneously	Prior information
RPN methodology	Stamatis (2003), Mikulak et al. (2008)	No	No	No	No	Partly (equal weights assumptions)
Data envelopment analysis	Chin et al. (2009a)	Yes	Yes	Yes	Partly	Yes (pre-determined weight functions)
Data elicitation	Bradley & Guerrero (2011)	Yes	No	No	No	Partly (equal weights assumptions)
Evidential reasoning	Chin et al. (2009b), Liu et al. (2011), Liu et al. (2013)	Yes	Yes	Yes	Yes	Yes (belief structure, pre-determined functions)
D-numbers and grey relational projection	Liu et al. (2014)	Yes	Yes	Yes	Yes	Yes (reference series)
Interval type-2 fuzzy sets	Bozdag et al. (2015)	Yes	Yes	Partly	Partly	Yes (pre-determined membership functions)
Interval 2-tuple linguistic variables with gray relational analysis	Liu et al. (2015)	Yes	Yes	Partly	No	Yes (reference series)

2.4 Summary

In this chapter, the background and literatures related to FMEA methodology are presented. While, several limitations of FMEA methodology, i.e., it is a time-consuming activity, it is difficult to prioritize failure modes or corrective actions, risk factors are difficult to be precisely evaluated, it is difficult to use, the FMEA form is complex, and missing risk ratings, are discussed. Lastly, improvements of FMEA pertaining to missing risk ratings issue and their ability to deal with uncertainty are discussed and summarized in Table 2.5. Yet, most of the approaches is not evaluated analytically with the aforementioned three principles. In the next chapter, the principles for prioritization of failure modes and corrective actions in an FMEA will be presented.

CHAPTER 3

PRINCIPLES FOR PRIORITIZATION OF FAILURE MODES AND CORRECTIVE ACTIONS IN AN FMEA

3.1 Introduction

In the previous chapter, a review of literature was provided. Even though much study on prioritization issues of Failure Mode and Effect Analysis (FMEA) has been reported, investigations pertaining to rationality of the prioritization outcome is scarce. In this chapter, the focus is on the three rational principles, inspired by the work in Arrow and Raynaud (1986), i.e., *transitivity*, *pareto optimality*, and *independence of irrelevant alternatives* principles for prioritization of failure modes and corrective actions in an FMEA activity.

This chapter is organized as follows. Four FMEA examples, which are from Stamatis (2003), Murray (2013), Calixto (2016), and Guimarães and Lapa (2004) are presented in Section 3.2. In Section 3.3, the FMEA methodology is re-visited. The principles for prioritization of failure modes and corrective actions are presented in Section 3.4. In Section 3.5, the rational properties of RPN model are proposed. The mathematical analysis is presented in Section 3.6 and further discussed in Section 3.7. Finally, concluding remarks are presented.

3.2 FMEA examples

Four FMEA examples, i.e., from Stamatis (2003), Murray (2013), Calixto (2016), and Guimarães and Lapa (2004) are presented in this section.

3.2.1 Example 1: A Design FMEA for a Spectral Photometer System (Murray, 2013)

A design FMEA pertaining to spectral photometer from National Ecological Observatory Network Incorporated is presented (Murray, 2013). A spectral photometer is a tool used to measure properties of light over a specific portion of electromagnetic spectrum to identify materials. It was first invented in 1978 (Lubbers & Guilino, 1978) and further improved in 1979 (Lubbers, 1979). The spectral photometer emits light towards an object of interest, receives the light reflected back from the object of interest, and includes a timer or synchronizer operative for causing the spectral photometer to produce an output signal whose successive values correspond to the intensity of the spectrum derived from the object of interest at successive wavelengths (Lubbers, 1979). The spectral photometer output signal, also known as generated spectrum is displayed on the screen of oscilloscope.

The FMEA worksheet is presented in Appendix 1. Severity, occurrence, and detection scale tables for the FMEA are summarized in Tables 3.1, 3.2, and 3.3, respectively. Noted that the lower and upper bounds for severity, occurrence, and detection ratings are 1 and 5 respectively. The FMEA worksheet consists of 27 failure modes, labelled as F_1 to F_{27}. For example, F_1 is *part will not fit together / mount properly in location* with the effect, i.e., *inability to assemble or install correctly*. The effect of F_1 is measured based on the scale table for severity (see Table 3.1). The rating of severity for F_1 is 3. The same applies to ratings of occurrence and detection. The root cause of F_1 is *assembly components are* *undersized, oversized, mis located, or missing*. The rating of occurrence for F_1 given by the FMEA users is 1 based on the scale table for occurrence (see Table 3.2). Moreover, the current control of F_1 is implement *tolerance analysis and inspection procedures* to prevent the failure mode from occurring. According to the scale table for detection (see Table 3.3), the rating of detection for F_1 is given as 1. Lastly, the RPN score of F_1 is 3.

Table 3.1: Scale Table for Severity (Murray, 2013)

Rating	Criteria
5	**Critical:** Safety issue and/or non-compliance with a government regulation, failure may cause serious injury or death to the customer or an employee.
4	**Serious:** Failure results in a loss or reduction of primary function and renders the product inoperable causing a high degree of customer dissatisfaction or may cause minor injury to the customer or an employee.
3	**Moderate:** Failure results in a partial malfunction of the product, the performance/functionality loss causes customer dissatisfaction.
2	**Minor:** Failure may not be readily apparent and/or may create a minor nuisance to the customer but would have minor effects on the customer's satisfaction.
1	**Negligible:** No discernible effect, the failure would not be noticeable to the customer and would not affect the customer's process or product.

Table 3.2: Scale Table for Occurrence (Murray, 2013)

Rating	Criteria
5	**Frequent:** One occurrence every month
4	**Probable:** One occurrence every 1-12 months
3	**Occasional:** One occurrence every 12 months to 5 years
2	**Remote:** One occurrence every 5 to 10 years
1	**Improbable:** One occurrence in greater than 10 years

Table 3.3: Scale Table for Detection (Murray, 2013)

Rating	Criteria
5	**Very Remote:** chance the design control will detect a potential cause/mechanism and subsequent failure mode.
4	**Low:** chance the design control will detect a potential cause/mechanism and subsequent failure mode.
3	**Moderate:** chance the design control will detect a potential cause/mechanism and subsequent failure mode.
2	**High:** chance the design control will detect a potential cause/mechanism and subsequent failure mode.
1	**Almost certain:** chance the design control will detect a potential cause/mechanism and subsequent failure mode.

3.2.2 Example 2: A Design FMEA for a Cooling Fan Assembly (Stamatis, 2003)

A design FMEA related to cooling fan assembly is presented in this section (Stamatis, 2003). The cooling fan assembly is a part of vehicles which tends to reduce the temperature of coolant in radiator. It is also comprised of a cooling fan, a cooling fan motor, a fan shroud, and etc. The FMEA worksheet is shown in Appendix 2. The guidelines for severity, occurrence, and detection in design FMEA are summarized in Tables 3.4, 3.5, and 3.6, respectively. Noted that the lower and upper bounds for severity, occurrence, and detection ratings are 1 and 10 respectively. The FMEA worksheet consists of 13 failure modes, labelled as F_1 to F_{13}. For example, F_1 is *fan vibration from imbalance and axial T.I.R.* with the effect, i.e., *audible noise, vibration, and motor wear is increased*. Then, the effect of F_1 is measured based on the scale table for severity (see Table 3.4). In this term, the rating of severity for F_1 is 5. It is the same applied to ratings of occurrence and detection. The cause of F_1 is *fan centre of gravity off axis of rotation or axial T.I.R. causes 2-plane imbalance*. Then, the rating of occurrence for F_1 given by the FMEA users is 5 based on the scale table for occurrence (see Table 3.5). Moreover, the current control of F_1 is *design lightweight fan* *with minimum band mass, part thickness to favour uniform mould flow, and perform DV tests on vehicles to assess sensitivity to vibration inputs.* According to the scale table for detection (see Table 3.6), the rating of detection for F_1 is given as 4. Lastly, the RPN score of F_1 is 100.

Table 3.4: Severity Guideline for Design FMEA (Stamatis, 2003)

Rating	Effect	Criteria
10	Hazardous	Hazardous effect. Safety related – sudden failure. Noncompliance with government regulation.
9	Serious	Potential hazardous effect. Able to stop product without mishap – time-dependent failure. Compliance with government regulation is in jeopardy.
8	Extreme	Customer very dissatisfied. Produce inoperable but safe. System inoperable.
7	Major	Customer dissatisfied. Product performance severely affected but functionable and safe. Subsystem inoperable.
6	Significant	Customer experiences discomfort. Product performances degraded, but operable and safe. Nonvital part inoperable.
5	Moderate	Customer experiences some dissatisfaction. Moderate effect on product performance. Fault on nonvital part requires repair.
4	Minor	Customer experiences minor nuisance. Minor effect on product performance. Fault does not require repair. Nonvital fault always noticed.
3	Slight	Customer slightly annoyed. Slight effect on product performance. Nonvital fault noticed most of the time.
2	Very Slight	Customer not annoyed. Very slight effect on product performance. Nonvital fault noticed sometimes.
1	None	No effect.

Table 3.5: Occurrence Guideline for Design FMEA (Stamatis, 2003)

Rating	Occurrence	Criteria	CNF/1000
10	Almost certain	Failure almost certain. History of failures exists from previous or similar designs.	>316
9	Very high	Very high number of failures likely	316
8	High	High number of failures likely	134
7	Moderately high	Moderately high number of failures likely	46
6	Medium	Medium number of failures likely	12.4
5	Low	Occasional number of failures likely	2.7
4	Slight	Few failures likely	0.46
3	Very slight	Very few failures likely	0.0063
2	Remote	Rare number of failures likely	0.0068
1	Almost impossible	Failure unlikely. History shows no failures.	<0.00058

Table 3.6: Detection Guideline for Design FMEA (Stamatis, 2003)

Rating	Detection	Criteria
10	Almost impossible	No design technique available or known, and/or none is planned
9	Remote	Is unproven, or unreliable, or effectiveness is unknown
8	Very slight	Has lowest effectiveness in each applicable category
7	Slight	Has very low effectiveness
6	Low	Has low effectiveness
5	Medium	Has medium effectiveness
4	Moderately high	Has moderately high effectiveness
3	High	Has high effectiveness
2	Very high	Has very high effectiveness
1	Almost certain	Has the highest effectiveness in each applicable category

3.2.3 Example 3: A Design FMEA for a Seal Pump (Calixto, 2016)

A design FMEA for a seal pump for oil and gas industry during the development phases is considered (Calixto, 2016). The failures related to assembly configuration or material used in the seal pump is pinpointed in this analysis, e.g., sensor mount seal, hose connection, and heat transfer structure. The FMEA worksheet is presented in Appendix 3. The scale table for severity, occurrence, and detection are shown in Tables 3.7, 3.8, and 3.9, respectively. Noted that the lower and upper bounds of severity, occurrence, and detection ratings are 1 and 10 respectively.

There are 7 failure modes to be prioritized, labelled as F_1 to F_7. For example, F_4 is *crack, break, burst, bad seal, or poor hose quality* for the *hose connection* item. Noted that F_4 have three effects, three root causes, and three design controls, respectively. Each effect, root cause, and current design control is associated with its severity, occurrence, and detection rating, respectively. As an example, the first effect for F_4 is *leak*, which is caused by *over pressure* and it could be detected with *burst and validation pressure cycle*. In this term, the ratings of severity, occurrence, and detection are 7, 7, and 1, respectively. The second effect of F_4 is *failed mount*, which is caused by *vibration* and it could be controlled with *vibration with road tapes*. The severity, occurrence, and detection ratings are 4, 8, and 3, respectively. The third effect of F_4 is *hose leak*, which is caused by *over pressure* and it could be prevented with *burst and vibration pressure cycle with clamps*. The rating of severity, occurrence, and detection is 6, 5, and 3, respectively. Lastly, the RPN scores for F_4 are 49, 96, and 60, respectively.

Table 3.7: Scale Table of Severity (Calixto, 2016)

Severity Level	Severity Description	Rating
Hazardous without warning	Very high severity ranking when a potential failure mode affects safe system operation without warning	10
Hazardous with warning	Very high severity ranking when a potential failure mode affects safe system operation with warning	9
Very High	System inoperable with destructive failure without compromising safety	8
High	System inoperable with equipment damage	7
Moderate	System inoperable with minor damage	6
Low	System inoperable without damage	5
Very Low	System operable with significant degradation of performance	4
Minor	System operable with some degradation of performance	3
Very Minor	System operable with minimal interference	2
None	No effect	1

Table 3.8: Scale table of Occurrence (Calixto, 2016)

Frequency Qualification	Frequency	Rating
Very high: failure is almost inevitable	>1 in 2	10
	1 in 3	9
High: repeated failures	1 in 8	8
	1 in 20	7
Moderate: occasional failures	1 in 80	6
	1 in 400	5
	1 in 2,000	4
Low: relatively few failures	1 in 15,000	3
	1 in 150,000	2
Remote: failure is unlikely	<1 in 1,500,000	1

Table 3.9: Scale Table of Detection (Calixto, 2016)

Detection	Likelihood of Detection by Design Control	Rating
Absolute uncertainty	Design control cannot detect potential cause/mechanism and subsequent failure mode	10
Very remote	Very remote chance the design control will detect potential cause/mechanism and subsequent failure mode	9
Remote	Remote chance the design control will detect potential cause/mechanism and subsequent failure mode	8
Very low	Very low chance the design control will detect potential cause/mechanism and subsequent failure mode	7
Low	Low chance the design control will detect potential cause/mechanism and subsequent failure mode	6
Moderate	Moderate chance the design control will detect potential cause/mechanism and subsequent failure mode	5
Moderately high	Moderately high chance the design control will detect potential cause/mechanism and subsequent failure mode	4
High	High chance the design control will detect potential cause/mechanism and subsequent failure mode	3
Very high	Very high chance the design control will detect potential cause/mechanism and subsequent failure mode	2
Almost certain	Design control will detect potential cause/mechanism and subsequent failure mode	1

3.2.4 Example 4: A FMEA for a Pressurized Water Reactor in a Nuclear Power Plant (Guimarães & Lapa, 2004)

In this section, an FMEA activity related to chemical and volume control system of a pressurized water reactor in a nuclear power plant is discussed (Guimarães & Lapa, 2004). In the manuscript, it was also stated that the primary function of chemical and volume control system for pressurized water reactor westinghouse are to purify the reactor coolant, control reactor coolant system inventory; and to provide reactor coolant pump seal water injection. Besides, it also provides high-pressure safety injection, reactor coolant pump seal injection, and emergency boratian while the plant is in an emergency state. In this example, a fuzzy inference system -based FMEA is presented to allow users to interpret and to assess the information with vague and imprecise linguistic terms.

Part of the FMEA worksheet is presented in Appendix 4 (details see (Guimarães & Lapa, 2004)). Severity, occurrence, and detection scale tables are summarized in Table 3.10. Noted that the lower and upper bounds for severity, occurrence, and detection ratings are 1 and 10, respectively. In addition, the interpretation of the linguistic terms is presented in Table 3.11. The FMEA worksheet consists of 82 failure modes, labelled as F_1 to F_{82}. For example, F_1 is *let-down flow control valves* is *fails open* with the effect, i.e., *loss of redundancy, unable to terminate let-down flow*. The root cause of F_1 is *mechanical binding* which can be controlled by implementing *remote valve position indication, downstream flow and temperature indicators*. Then, the effect, cause, and prevention method of F_1 are measured based on the scale table for severity, occurrence, and detection (see Table 3.10). The RPN score and fuzzy RPN score of F_1 are 75 and 22, respectively.

Table 3.10: Severity, Occurrence, and Detection Scale Tables for Conventional RPN model (Guimarães & Lapa, 2004)

Occurrence (O) Severity (S)	Detection (D)	Rating	Possible failure rate (operating days)	Probability range (%)
Remote	Very High	1	<1:20,000	0-5/6-15
Low	High	2/3	1:20,000/1:10,000	16-25/26-35
Moderate	Moderate	4/5/6	1:2000/1:1000/1:200	36-45/46-55/56-65
High	Low	7/8	1:100/1:20	66-75/76-85
Very high	Remote	9/10	1:10/1:2	86-100

Table 3.11: Interpretations of the Linguistic Terms for Developing Fuzzy Rule System (Guimarães & Lapa, 2004)

Linguistic term	Probability of occurrence	Severity	Detection
Remote	It would be very unlikely for those failures to be observed even once	A failure that has no effect on the system performance, the operator probably will not notice	Defect remains undetected until the system performance degrades to the extent that the task will not be completed.
Low	Likely to occur once, but unlikely to occur more frequently	A failure that would cause slight annoyance to the operator, but that cause no deterioration to the system.	Defect remains undetected until system performance is severely reduced.
Moderate	Likely to occur more than once	A failure that would cause a high degree of operator dissatisfaction or that cause noticeable but slight deterioration in system performance	Defect remains undetected until system performance is affected
High	Near certain to occur at least once	A failure that causes significant deterioration in system performance and/or leads to minor injuries	Defect remains undetected until inspection or test is carried out
Very high	Near certain to occur several times	A failure that would seriously affect the ability to complete the task or cause damage, serious injury or death	Failure remains undetected; such a defect would almost certainly be detected during inspection or test

3.3 Definitions for FMEA

The FMEA procedure is revisited. Three definitions pertaining to FMEA are considered as follows.

Definition 3.1: Three risk factors in an FMEA activity, i.e., S, O, and D, are considered. The risk ratings are represented by $s, o,$ and d, i.e., $s \in \mathrm{S}, o \in \mathrm{O}$, and $d \in \mathrm{D}$, respectively. In addition, $x \in \{s, o, d\}$ is used, in which x is an element of $\{s, o, d\}$.

3.1.1. The lower and upper bounds of S, O, and D are represented by $\underline{s}$ and $\overline{s}$, $\underline{o}$ and $\overline{o}$, and $\underline{d}$ and $\overline{d}$, respectively;

3.1.2. x is a natural number, i.e., $x \subset \mathbb{N}$ and $\underline{x} \leq x \leq \overline{x}$ is always true;

3.1.3. m is a two-element subset of $\{s, o, d\}$, where n is excluded, i.e., $m, n \subset \{s, o, d\}$; $n \notin m$; and $\{m, n\} = \{s, o, d\}$ is always true;

3.1.4. A monotone order exists among the elements of S, O, and D;

3.1.4.1. The "seriousness" of effect pertaining to S with a rating of s_1 is greater than that of s_2, if $s_1 > s_2$, for $s_1, s_2 \in \mathrm{S}$.

3.1.4.2. The occurrence likelihood of the root cause(s) pertaining to O with a rating of o_1 is higher than that of o_2, if $o_1 > o_2$, for $o_1, o_2 \in \mathrm{O}$.

3.1.4.3. The efficiency of a prevention method pertaining to D with a rating of d_1 is lower than that of d_2 if $d_1 > d_2$, for $d_1, d_2 \in \mathrm{D}$.

Definition 3.2: The RPN space contains all possible RPN scores, i.e., $RPN \in$ RPN space.

3.2.1. The lower and upper bounds of the RPN space are denoted by $\underline{RPN}$ and $\overline{RPN}$, respectively, and $\underline{RPN} \leq RPN \leq \overline{RPN}$ is always true;

3.2.2. A monotone order exists among the elements of the RPN space, i.e., the priority of a failure mode or corrective action with an RPN score of RPN_1 is greater than that of RPN_2, if $RPN_1 > RPN_2$, for $RPN_1, RPN_2 \in$ RPN space.

Definition 3.3: The f_{RPN} function is a mapping of the risk factors, S, O, and D, to the RPN space, i.e., $f_{RPN}\colon \mathrm{S}, \mathrm{O}, \mathrm{D} \rightarrow$ RPN space.

Instead of merely quantitative assessments, the S, O, and D scale tables can be both quantitative and qualitative (Stamatis, 2003). In Example 1, S=5 may refer to a critical safety issue, which it is evaluated qualitatively. Besides, the scale tables may be nonlinear too (Pillay & Wang, 2003; Puente et al., 2002; von Ahsen, 2008). As an example, O=4 may not be double in occurrence frequency than that of O=2 (See Table 3.2). Such arguments are still in agreement with *Definition 3.1.4*, which states that a *monotone* order should exist in the S, O, and D scale tables. Using Table 3.1 as an example, the *monotone* order suggests that S=5 is more serious than that of S=4, which is followed by S=3, S=2, and finally S=1.

The same observation can be made using Example 2. In Table 3.5, the occurrence guideline can be evaluated qualitatively or quantitatively. For example, O=2 is referred to rare number of failures likely to happen or the CNF/1000 value is equal to 0.0068. Although both examples in design FMEA using different scale, it is still consistent with *Definition 3.1.4*, which states that a monotone order should exist in the S, O, and D scale tables. Using Table 3.6 as an example, the *monotone* order suggests that D=10 has the lowest effectiveness of detection than that of D=9, which is followed by D=8, D=7, …, D=1. It is interesting that the monotone order always exists in the S, O, and D scale tables although the scale in Example 1 and Example 2 are both different.

3.4 Principles for Prioritization of Failure Modes and Corrective Actions

Inspired by the work in (Arrow & Raynaud, 1986; Bradley & Guerrero, 2011; Chen, Hoyle, & Wassenaar, 2013), three useful principles for prioritization of failure modes and corrective actions using f_{RPN}, are presented and analysed.

Axiom 3.1:

3.1.1. Transitivity: if $f_{RPN}(s_1, o_1, d_1) > f_{RPN}(s_2, o_2, d_2)$, and $f_{RPN}(s_2, o_2, d_2) > f_{RPN}(s_3, o_3, d_3)$, then $f_{RPN}(s_1, o_1, d_1) > \mathrm{f}_{RPN}(s_3, o_3, d_3)$.

3.1.2. Pareto optimality: if $s_1 \geq s_2$, $o_1 \geq o_2$ and $d_1 \geq d_2$, then $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_2, o_2, d_2)$.

3.1.3. Independence of irrelevant alternatives: if $f_{RPN}(s_1, o_1, d_1) > f_{RPN}(s_2, o_2, d_2)$, then the relative ranking of $f_{RPN}(s_1, o_1, d_1)$ and $f_{RPN}(s_2, o_2, d_2)$ remains the same, when an additional $f_{RPN}(s_3, o_3, d_3)$ is introduced.

Axioms 3.1 is the principles for prioritization of failure modes and corrective actions in FMEA. In Example 1, *Axiom 3.1.1* suggests that if RPN_{12} is greater than RPN_{16}, and RPN_{16} is greater than RPN_1, then RPN_{12} is greater than RPN_1. *Axiom 3.1.2* suggests that $RPN_{12} \geq RPN_{16}$, because $s_{12} \geq s_{16}$, $o_{12} > o_{16}$ and $d_{12} > d_{16}$. *Axiom 3.1.3* suggests that RPN_{12} is always greater than RPN_1, and their relative ordering remains undisturbed, even with the introduction of new failure modes (e.g., RPN_{16} does not affect the relative ordering of RPN_{12} and RPN_1).

With Example 2, *Axiom 3.1.1* suggests that if RPN_{12} is greater than RPN_{13}, and RPN_{13} is greater than RPN_2, then RPN_{12} is greater than RPN_2. *Axiom 3.1.2* suggests that $RPN_{12} > RPN_2$, because $s_{12} > s_2$, $o_{12} > o_2$ and $d_{12} > d_2$. *Axiom 3.1.3* suggests that RPN_{12} is always greater than RPN_2, and their relative ordering remains undisturbed, even with the introduction of new failure modes (e.g., RPN_{13} does not affect the relative ordering of RPN_{12} and RPN_2).

Again, with Example 3, *Axiom 3.1.1* suggests that if RPN_6 is greater than RPN_5, and RPN_5 is greater than RPN_2, then RPN_6 is greater than RPN_2. *Axiom 3.1.2* suggests that $RPN_6 \geq RPN_5$, because $s_6 \geq s_5$, $o_6 \geq o_5$ and $d_6 > d_5$. *Axiom 3.1.3* suggests that RPN_6 is always greater than RPN_2, and their relative ordering remains undisturbed, even with the introduction of new failure modes (e.g., RPN_5 does not affect the relative ordering of RPN_6 and RPN_2).

Recap from Example 4, *Axiom 3.1.1* suggests that if RPN_8 is greater than RPN_1, and RPN_1 is greater than RPN_5, then RPN_8 is greater than RPN_5. *Axiom 3.1.2* suggests that $RPN_1 \geq RPN_5$, because $s_1 \geq s_5$, $o_1 = o_5$ and $d_1 = d_5$. *Axiom 3.1.3* suggests that RPN_8 is always greater than RPN_5, and their relative ordering remains undisturbed, even with the introduction of new failure modes (e.g., RPN_1 does not affect the relative ordering of RPN_8 and RPN_5). However, referring to fuzzy FMEA in Example 4, *Axiom 3.1.2* suggests that fuzzy RPN_1 should be greater than or equal to fuzzy RPN_5, because $s_1 \geq s_5$, $o_1 = o_5$ and $d_1 = d_5$. Yet, the fuzzy RPN scores obtained for F_1 and F_5 are 3 and 5, respectively, which it is contrast to *Axiom 3.1.2*. It seems illogical because the failure mode with "serious injury or death" (i.e., $s_1 = 5$) is rated lower than that of "minor injuries" (i.e., $s_5 = 4$). Therefore, it is showed that *Axiom 3.1* plays an important role to avoid confusing or misleading prioritization result.

3.5 Properties of the RPN model

Four useful characteristics pertaining to f_{RPN} are introduced in *Property 3.1*, as follows.

Property 3.1:

3.1.1. $\underline{RPN} = f_{RPN}(\underline{s}, \underline{o}, \underline{\mathrm{d}})$.

3.1.2. $\overline{RPN} = f_{RPN}(\overline{s}, \overline{o}, \overline{d})$.

3.1.3. f_{RPN} is monotone if $f_{RPN}(m, n_1) \geq f_{RP\mathrm{N}}(m, n_2)$, when $n_1 > n_2$ (Jee, Tay, & Lim, 2015; Kerk, Tay, & Lim, 2015; Pang, Tay, & Lim, 2016; Tay & Lim, 2006a).

3.1.4. f_{RPN} is strong monotone if $f_{RPN}(m, n_1) > f_{RPN}(m, n_2)$, when $n_1 > n_2$.

Properties 3.1.1 and *3.1.2* are the boundary conditions for the RPN model. *Properties 3.1.3* and *3.1.4* are the (strong) monotonicity for an RPN model. *Property 3.1.4* is a subset of *Property 3.1.3*, i.e., if f_{RPN} is *strongly monotone*, it is always *monotone*, but, not the opposite.

Note that *Property 3.1.3* is closely related to the pareto optimality principle (i.e., *Axiom 3.1.2*). *Pareto optimality* is the key axiom in multi-criteria decision analysis (Arrow & Raynaud, 1986), as discussed in (Chen et al., 2013) and (Yager, 1981). The importance of *pareto optimality* in the context of FMEA has been highlighted in Franceschini & Galetto (2001). The monotonicity property of f_{RPN} is key to ensure the pareto optimality principle is satisfied, if the RPN scores are used for prioritization activities (see *Theorem 3.1*).

Theorem 3.1: If f_{RPN} is *monotone*, the *pareto optimality* principle is always satisfied, if the *RPN* scores are used for prioritization.

Proof of Theorem 3.1:

Consider two sets of $\{\mathrm{S}, \mathrm{O}, \mathrm{D}\}$, i.e., $\{s_1, o_1, d_1\}$ and $\{s_2, o_2, d_2\}$, where $s_1 \geq s_2$, $o_1 \geq o_2$ and $d_1 \geq d_2$. If f_{RPN} is *monotone*, $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_1, o_1, d_2)$ is always true, considering $m = \{s_1, o_1\}, n_1 = d_1$, and $n_2 = d_2$. Similarly, $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_1, o_2, d_1)$ and $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_2, o_1, d_1)$ are always true too. Again, if f_{RPN} is *monotone*, $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_1, o_2, d_2)$ is always true. The same applies to $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_2, o_1, d_2)$ and $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_2, o_2, d_1)$. It is obvious that $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_2, o_2, d_2)$ is always true. Therefore, if f_{RPN} is *monotone*, the *Pareto optimality* principle is always satisfied.

In the literature, there are a number of guidelines for prioritization of potential failure modes and correctives actions using the traditional RPN model, i.e., $f_{RPN} = s \times o \times d$. In general, the guidelines give the priority to high S (Carlson, 2012; Ireson et al., 1996; Press, 2003; Stamatis, 2003), high O (Stamatis, 2003), or high D (Stamatis, 2003) ratings, as well as explain how to implement an RPN threshold (Carlson, 2012; Ireson et al., 1996; Mikulak et al., 2008). It has been stated that RPN scores can hardly be a perfect measure of risk, and there is no perfect RPN model (Carlson, 2012). Therefore, the RPN scores should be used to only help the FMEA team in prioritization activities within the scope of a specific product or process (Carlson, 2012), instead of effective decision making.

The three principles complement the abovementioned guidelines and contribute toward developing an effective prioritization model. As an example, the principles are useful for an FMEA team to develop a non-linear RPN model, i.e., how to formulate f_{RPN} to give priority to high S, high O, and/or high D ratings while satisfying *Property 3.1*. The principles are useful for dealing with an FMEA activity that comprises a high number of failure modes and corrective actions.

3.6 Analysis

The following proposition is considered.

Proposition 3.1: If f_{RPN} is *monotone*, and the risk ratings are *complete*, *Axiom 3.1.1* is always satisfied.

Proof of Proposition 3.1:

If f_{RPN} is *monotone*, and the risk ratings are *complete*, i.e., $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_2, o_2, d_2)$ and $f_{RPN}(s_2, o_2, d_2) \geq f_{RPN}(s_3, o_3, d_3)$. Then, $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_3, o_3, d_3)$ is always true. Therefore, *Axiom 3.1.1* is always satisfied.

Proposition 3.2: If f_{RPN} is *monotone*, and the risk ratings are *complete, Axiom 3.1.2* is always satisfied.

Proof of Proposition 3.2:

From *Theorem 3.1*, if f_{RPN} is *monotone*, and the risk ratings are *complete*, *Axiom 3.1.2* is always satisfied.

Proposition 3.3: If f_{RPN} is *monotone*, and the risk ratings are *complete, Axiom 3.1.3* is always satisfied.

Proof of Proposition 3.3:

If f_{RPN} is *monotone*, and the risk ratings are *complete*, i.e., $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_2, o_2, d_2)$, then, the relative ranking of $f_{RPN}(s_1, o_1, d_1)$ and $f_{RPN}(s_2, o_2, d_2)$ remains the same even though an additional f_{RPN} is introduced. Therefore, *Axiom 3.1.3* is always satisfied.

In this section, *Propositions 3.1, 3.2,* and *3.3* suggest that *Property 3.1.3* is important to ensure the rationality of the risk evaluation outcomes.

3.7 Discussion

With Example 1, the usefulness of *Axiom 3.1* can be explained with F_3, F_5, and F_7. *Axiom 3.1.1* suggests if RPN_5 is greater than RPN_7, and RPN_7 is greater than RPN_3, then, RPN_5 should be greater than RPN_3. In this example, an f_{RPN} that satisfies *Property 3.1* is used, in which $\underline{RPN} = 1$ and $\overline{RPN} = 125$. Therefore, *Axiom 3.1.1* is satisfied, which is in agreement with *Proposition 3.1*. *Axiom 3.1.2* suggests that RPN_5 should be greater than RPN_3. This is because $s_5 \geq s_3$, $o_5 \geq o_3$, and $d_5 \geq d_3$. With an f_{RPN} that satisfies *Property 3.1, Axiom 3.1.2* is satisfied, which is in agreement with *Proposition 3.2*. *Axiom 3.1.3* suggests that the relative order of F_5 and F_7 remains the same, i.e., F_5 is greater than F_7, when a new failure mode, e.g., F_3, is added. Again, with an f_{RPN} that satisfies *Property 3.1, Axiom 3.1.3* is true, which is in agreement with *Proposition 3.3*.

Again, with Example 2, the usefulness of *Axiom 3.1* can be explained with F_8, F_{10}, and F_9. *Axiom 3.1.1* suggests if RPN_8 is greater than RPN_{10}, and RPN_{10} is greater than RPN_9, then, RPN_8 should be greater than RPN_9. In this example, an f_{RPN} that satisfies *Property 3.1* is used, in which $\underline{RPN} = 1$ and $\overline{RPN} = 1000$. Therefore, *Axiom 3.1.1* is satisfied, which is in agreement with *Proposition 3.1*. *Axiom 3.1.2* suggests that RPN_8 should be greater than RPN_9. This is because $s_8 \geq s_9$, $o_8 \geq o_9$, and $d_8 \geq d_9$. With an f_{RPN} that satisfies *Property 3.1*, *Axiom 3.1.2* is satisfied, which is in agreement with *Proposition 3.2*. *Axiom 3.1.3* suggests that the relative order of F_8 and F_9 remains the same, i.e., F_8 is greater than F_9, when a new failure mode, e.g., F_{10}, is added. Again, with an f_{RPN} that satisfies *Property 3.1*, *Axiom 3.1.3* is true, which is in agreement with *Proposition 3.3*.

3.8 Summary

In this chapter, a number of FMEA benchmark examples were presented. Besides, the three rational principles inspired by the work in Arrow and Raynaud (1986) for prioritization of failure modes and corrective actions in an FMEA activity were mathematically analysed with benchmark information. In Chapter 4, a tree model and a vector representation for potential failure modes and corrective actions in FMEA will be proposed.

CHAPTER 4

TREE MODEL AND VECTOR REPRESENTATIONS

4.1 Introduction

In this chapter, a tree model and a vector representation for potential failure modes and corrective actions in FMEA, are proposed. The tree model for a potential failure mode allows its root cause(s), effect(s) and corrective action(s), together with their Severity, Occurrence and Detection rating(s), to be represented as a three-layer tree model. The tree model for a corrective action with similar contents, is outlined too. The RPN model, together with its score, is represented as a node of the tree model. These tree models can also be represented as their associated equivalence layered-vector representations.

The concept of a tree, a connected graph without cycles, appeared implicitly in the work of Gustav Kirchho (1824-87), who employed graph-theoretical ideas in the calculation of currents in electrical networks. Later, trees were used by Arthur Cayley (1821-95), James Joseph Sylvester (1806-97), Georg Pólya (1887-1985), and others, in connection with the enumeration of certain chemical molecules. While working on a problem inspired by some work of Sylvester on “differential transformation and the reversion of *serieses*", (Cayley, 1857) was led to the enumeration of rooted trees. In general, a rooted tree is a directed tree having a distinguished vertex r, called the root, such that for every other vertex v, there is a directed r-v path (Gross, Yellen, & Zhang, 2013). It is also can be known as out-tree, branching, and arborescence.

This chapter is organized as follows. Preliminaries for tree model and vector representation of failure modes and corrective actions in FMEA are introduced in Section 4.2. In Section 4.3, four real examples, which are from Murray (2013), Stamatis (2013), Calixto (2016), and Guimarães and Lapa (2004) are presented to illustrate the usefulness of proposed method.

4.2 Preliminaries for Tree Model and Vector Representation

Two definitions are considered in this section.

Definition 4.1: An FMEA activity with *N* failure modes (F_i) or *N* corrective actions (C_i) to be prioritized is considered, where $i = 1, 2, \ldots, N$.

4.1.1. The effect(s), root cause(s), and control(s) or prevention method(s), for F_i or C_i, are denoted as E_i, RC_i, and PM_i, respectively;

4.1.2. Each E_i, RC_i, and PM_i is associated with $E_{i,a}, RC_{i,b}$, and $PM_{i,c}$, respectively;

4.1.3. $s_{i,a}$ is the S rating of $E_{i,a}$ for F_i or C_i, where $a = 1, 2, \ldots, u$;

4.1.4. $o_{i,b}$ is the O rating of $RC_{i,b}$ for F_i or C_i, where $b = 1, 2, \ldots, v$;

4.1.5. $d_{i,c}$ is the D rating of $PM_{i,c}$ for F_i or C_i, where $c = 1, 2, \ldots, w$. $w = 1$ for C_i;

4.1.6. $x_{i,k} \in \{s_{i,a}, o_{i,b}, d_{i,c}\}$ is used, in which $x_{i,k}$ is an element of $\{s_{i,a}, o_{i,b}, d_{i,c}\}$;

4.1.7. The RPN score for F_i or C_i is denoted as RPN_i.

In this section, F_i or C_i, together with E_i, RC_i, and PM_i, S, O, and D rating(s), as well as its RPN score are modelled as a three-layer *root tree*, or an *out-tree* (Gross, Yellen, & Zhang, 2013), as shown in Figure 4.1. Note that there is only one prevention method and D rating for the C_i model, i.e., $w = 1$, as shown in Figure 4.2. In the first layer, an F_i node or C_i node is the *root* of an F_i or C_i model, respectively. Each F_i node or C_i node is associated with an RPN_i value. There are three nodes in the second layer, i.e., E_i, RC_i, and PM_i. They are the children nodes of an F_i node or C_i node. Each set of E_i, RC_i, and PM_i is associated with s_i, o_i, and d_i, respectively. In the third layer, the children nodes of E_i, RC_i, and PM_i are $E_{i,a}, RC_{i,b}$, and $PM_{i,b}$, respectively. Each set of $E_{i,a}, RC_{i,b}$, and $PM_{i,b}$ is associated with $s_{i,a}, o_{i,b}$, and $d_{i,c}$, respectively.

The F_i or C_i model can also be represented as a nested vector, in the form of Equation (4.1). RPN_i can be represented as Equation (4.2), which can be further reduced to Equation (4.3). All s_i, o_i and d_i are obtained by aggregating $s_{i,a}, o_{i,b}$, and $d_{i,c}$ (i.e., a reduction of the tree), or by manual assignment from the FMEA user(s).

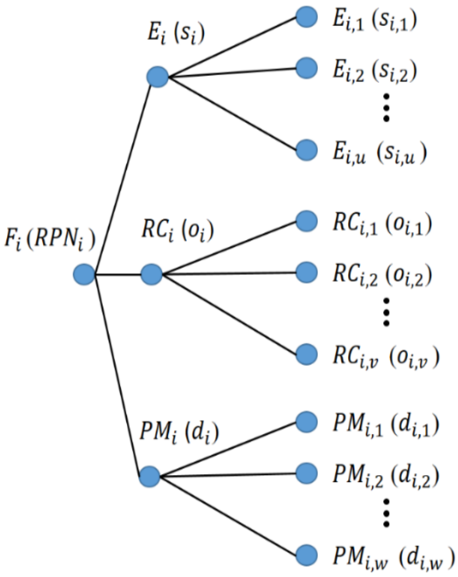

Figure 4.1: A F_i model

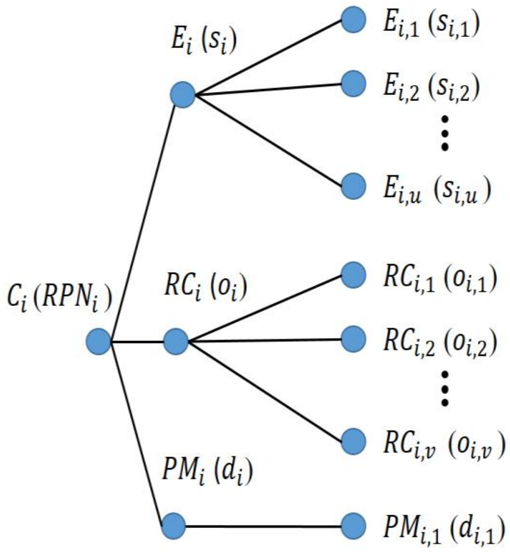

Figure 4.2: A C_i mode

$$F_i/C_i = \left[\left[E_{i,1}, E_{i,2}, \dots, E_{i,u}\right], \left[RC_{i,1}, RC_{i,2}, \dots, RC_{i,v}\right], \left[PM_{i,1}, PM_{i,2}, \dots, PM_{i,w}\right]\right] \quad (4.1)$$

$$RPN_i = \left[\left[s_{i,1}, s_{i,2}, \dots, s_{i,u}\right], \left[o_{i,1}, o_{i,2}, \dots, o_{i,v}\right], \left[d_{i,1}, d_{i,2}, \dots, d_{i,w}\right]\right] \quad (4.2)$$

$$RPN_i = [s_i, o_i, d_i] \quad (4.3)$$

Definition 4.2: Reduction and manual assignment of risk ratings for F_i or C_i.

4.2.1. $x_{i,k} \in \{s_{i,a}, o_{i,b}, d_{i,c}\}$ is *missing* if it is unknown;

4.2.2. $s_i, o_i,$ and d_i for F_i or C_i can be obtained by aggregating $s_{i,a}, o_{i,b},$ and $d_{i,c}$, as follows;

4.2.2.1. $s_i = max_{\forall a}(s_{i,\text{a}}) \middle| \forall s_{i,\text{a}}$ is known where $a = 1, 2, \dots, u$.

4.2.2.2. $o_i = max_{\forall b}(o_{i,b}) \middle| \forall o_{i,b}$ is known where $b = 1, 2, \dots, v$.

4.2.2.3. $d_i = max_{\forall c}(d_{i,c}) \middle| \forall d_{i,\text{c}}$ is known where $c = 1, 2, \dots, w$.

4.2.3. $x_i \in \{s_i, o_i, d_i\}$ is used, in which x_i is an element of $\{s_i, o_i, d_i\}$;

4.2.4. s_i, o_i, and d_i for F_i or C_i can also be obtained through manual assignment from the FMEA user(s);

4.2.5. The risk ratings are *complete* if all s_i, o_i, and d_i can be obtained by aggregation (Definition 4.2.2) or manual assignment (Definition 4.2.4); otherwise, it is incomplete.

4.3 Applications to Examples 1, 2, 3, and 4 from Chapter 3

In this section, there are four FMEA examples in several industries are presented to demonstrate the capability of the tree model and vector representation, e.g., automotive (Stamatis, 2003), manufacturing (Murray, 2013), and petroleum (Calixto, 2016).

4.3.1 Example 1 from Chapter 3

An FMEA worksheet from Murray (2013) (see Appendix 1) is used. A total of 27 failure modes are to be prioritized, i.e., $N = 27$. The tree model of F_1, F_{12}, F_{16} and F_{22} is depicted in Figure 4.3. The first layer of F_1 consists of its *root node* and it is associated to $RPN_1 = 3$. Again, the *root node* also denotes the RPN model together with RPN score. There are three nodes in the second layer, also the children for the *root node*, i.e., E_1, RC_1, and PM_1. E_1, RC_1, and PM_1 are associated with $s_1 = 3, o_1 = 1$, and $d_1 = 1$, respectively. In the third layer, the *children nodes* of E_1, RC_1, and PM_1 are $E_{1,1}, RC_{1,1}$, and $PM_{1,1}$, respectively. $E_{1,1}, RC_{1,1}$, and $PM_{1,1}$ are associated with $s_{1,1} = 3, o_{1,1} = 1$, and $d_{1,1} = 1$, respectively.

Moreover, F_{12} is a *root node* of its tree model and associated with $RPN_{12} = 48$. The *children nodes* of F_{12} in second layer of the tree model are E_{12}, RC_{12}, and PM_{12}. E_{12}, RC_{12}, and PM_{12} are associated with $s_{12} = 4, o_{12} = 4$, and $d_{12} = 3$, respectively. Again, the *children nodes* of E_{12}, RC_{12}, and PM_{12} in third layer of tree model are $E_{12,1}, RC_{12,1}$, and $PM_{12,1}$, respectively. $E_{12,1}, RC_{12,1}$, and $PM_{12,1}$ are associated with $s_{12,1} = 4, o_{12,1} = 4$, and $d_{12,1} = 3$, respectively.

Furthermore, the first layer of F_{16} consists only the *root node* and it is associated to $RPN_{16} = 16$. Again, the *root node* also denotes the RPN model, together with its RPN score. There are three nodes in the second layer, also the children for the *root node*, i.e., E_{16}, RC_{16}, and PM_{16}. E_{16}, RC_{16}, and PM_{16} are associated with $s_{16} = 4, o_{16} = 2$, and $d_{16} = 2$, respectively. In the third layer, the *children nodes* of E_{16}, RC_{16}, and PM_{16}, are $E_{16,1}$, $RC_{16,1}$, and $PM_{16,1}$, respectively. $E_{16,1}, RC_{16,1}$, and $PM_{16,1}$ are associated with $s_{16,1} = 4$, $o_{16,1} = 2$, and $d_{16,1} = 2$, respectively.

In this case, it is worth to mention the tree model of F_{22} which consists of missing risk ratings. F_{22} is a *root node* of its tree model and associated with $RPN_{22} = 0$. The *children nodes* of F_{22} in second layer of the tree model are E_{22}, RC_{22}, and PM_{22}. E_{22}, RC_{22}, and PM_{22} are associated with $s_{22} = 4, o_{22} = -$, and $d_{22} = -$, respectively. Again, the *children nodes* of E_{22}, RC_{22}, and PM_{22} in third layer are $E_{22,1}, RC_{22,1}$, and $PM_{22,1}$, respectively. $E_{22,1}, RC_{22,1}$, and $PM_{22,1}$ are associated with $s_{22,1} = 4, o_{22,1} = -$, and $d_{22,1} = -$, respectively. In this example, it can be clearly seen that s_i, o_i, and d_i are obtained through aggregation (Definition 4.2.2), except d_{12}, o_{22}, and d_{22}, where d_{12} is assigned manually by FMEA users and o_{22} and d_{22} are missing.

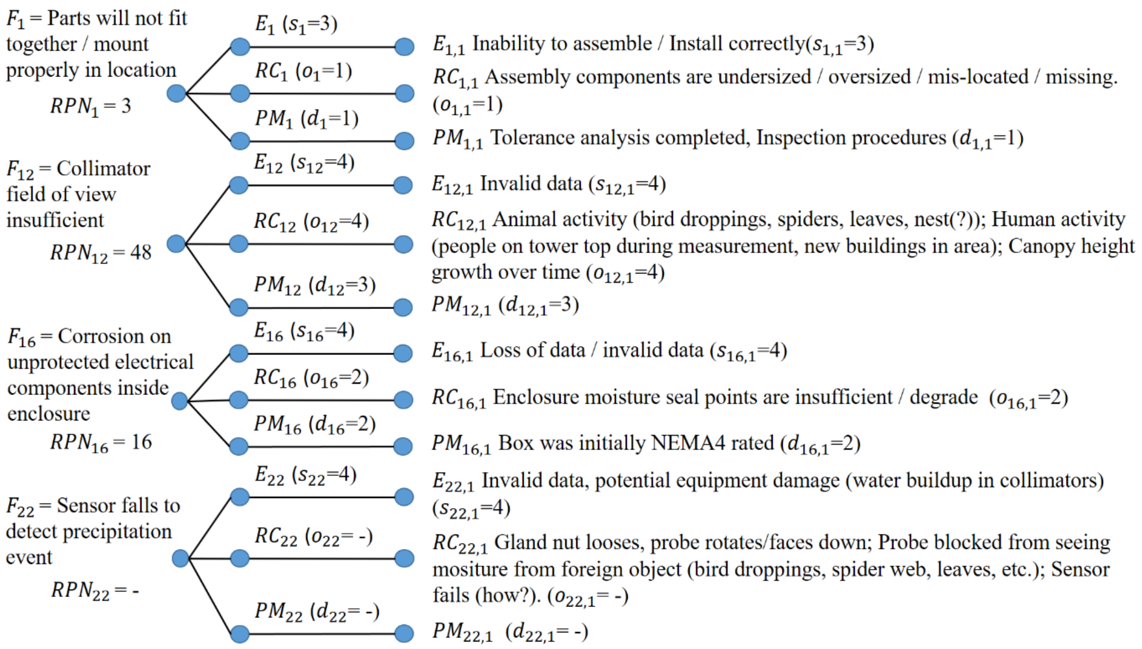

Figure 4.3: Three-layer rooted tree for F_1, F_{12}, F_{16} and F_{22}

The nested vector of F_1 is $F_1 = \left[[E_{1,1}], [RC_{1,1}], [PM_{1,1}]\right]$. RPN_1 can be represented as $RPN_1 = \left[[s_{1,1} = 3], [o_{1,1} = 1], [d_{1,1} = 1]\right]$, which can be further reduced to $RPN_1 = [s_1 = 3, o_1 = 1, d_1 = 1]$. It is the same applies to F_{12}, F_{16}, and F_{22}. The nested vector of F_{12}, F_{16}, and F_{22} is $F_{12} = \left[[E_{12,1}], [RC_{12,1}], [PM_{12,1}]\right]$, $F_{16} = \left[[E_{16,1}], [RC_{16,1}], [PM_{16,1}]\right]$, and $F_{22} = \left[[E_{22,1}], [RC_{22,1}], [PM_{22,1}]\right]$, respectively. RPN_{12} can be represented as $RPN_{12} = \left[[s_{12,1} = 4], [o_{12,1} = 4], [d_{12,1} = 3]\right]$, which can be further reduced to $RPN_{12} = [s_{12} = 4, o_{12} = 4, d_{12} = 3]$. RPN_{16} can be represented as $RPN_{16} = \left[[s_{16,1} = 4], [o_{16,1} = 2], [d_{16,1} = 2]\right]$, which can be further reduced to $RPN_{16} = [s_{16} = 4, o_{16} = 2, d_{16} = 2]$. Even F_{22} consists of two missing risk ratings, its RPN_{12} still can be represented as $RPN_{22} = \left[[s_{22,1} = 4], [o_{22,1} = -], [d_{22,1} = -]\right]$, which can be further reduced to $RPN_{22} = [s_{22} = 4, o_{22} = -, d_{22} = -]$.

4.3.2 Example 2 from Chapter 3

In this section, an FMEA worksheet from Stamatis (2003) (see Appendix 2) is used. A total of 13 failure modes are to be prioritized, i.e., $N = 13$. The tree model of F_4 and F_{11} is depicted in Figure 4.4. The first layer of F_4 consists of its *root node* and it is associated to $RPN_4 = 180$. Again, the *root node* also denotes the RPN model together with RPN score. There are three nodes in the second layer, also the children for the *root node*, i.e., E_4, RC_4, and PM_4. E_4, RC_4, and PM_4 are associated with $s_4 = 6, o_4 = 6$, and $d_4 = 5$, respectively. In the third layer, the *children nodes* of E_4, RC_4, and PM_4 are $E_{4,1}, E_{4,2}, RC_{4,1}, PM_{4,1}$, and $PM_{4,2}$, respectively. $E_{4,1}$ and $E_{4,2}$ are associated with $s_{4,1} = 6$ and $s_{4,2} = 6$ respectively. $RC_{4,1}$ is associated with $o_{4,1} = 6$. $PM_{4,1}$ and $PM_{4,2}$ are associated with $d_{4,1} = 3$ and $d_{4,2} = 5$ respectively. It is the same applies to F_{11}. F_{11} is a *root node* of its tree model and associated with $RPN_{11} = 147$. In second layer of the tree model, the *children nodes* of F_{11} are E_{11}, RC_{11}, and PM_{11}. E_{11}, RC_{11}, and PM_{11} are associated with $s_{11} = 7, o_{11} = 3$, and $d_{11} = 7$, respectively. Again, in the third layer, the *children nodes* of E_{11}, RC_{11}, and PM_{11} are $E_{11,1}, E_{11,2}, RC_{11,1}, RC_{11,2}, PM_{11,1}$, and $PM_{11,2}$, respectively. $E_{11,1}$ and $E_{11,2}$ are associated with $s_{11,1} = 5$ and $s_{11,2} = 7$ respectively. $RC_{11,1}$ and $RC_{11,2}$ are associated with $o_{11,1} = 3$ and $o_{11,2} = 2$ respectively. $PM_{11,1}$ and $PM_{11,2}$ are associated with $d_{11,1} = 4$ and $d_{11,2} = 7$ respectively.

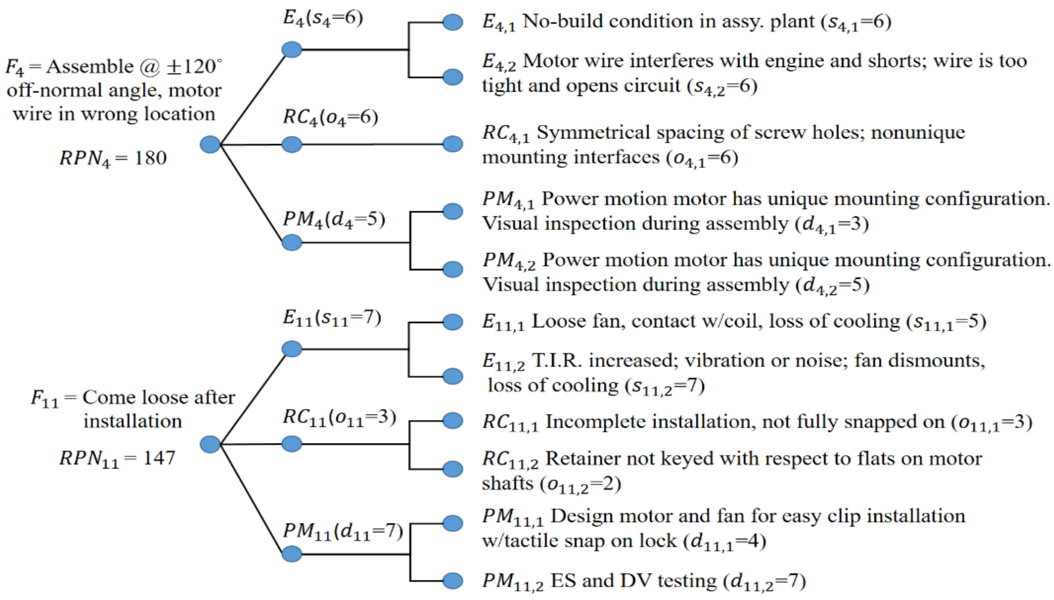

Figure 4.4: Three-layer rooted tree for F_4 and F_{11}

The nested vector of F_4 is $F_4 = \left[[E_{4,1}, E_{4,2}], [RC_{4,1}], [PM_{4,1}, PM_{4,2}]\right]$. RPN_4 can be represented as $RPN_4 = \left[[s_{4,1} = 6, s_{4,2} = 6], [o_{4,1} = 6], [d_{4,1} = 3, d_{4,2} = 5]\right]$, which can be further reduced to $RPN_4 = [s_4 = 6, o_4 = 6, d_4 = 5]$. It is the same applies to F_{11}. The nested vector of F_{11} is $F_{11} = \left[[E_{11,1}, E_{11,2}], [RC_{11,1}, RC_{11,2}], [PM_{11,1}, PM_{11,2}]\right]$. RPN_{11} can be represented as $RPN_{11} = \left[[s_{11,1} = 5, s_{11,2} = 7], [o_{11,1} = 3, o_{11,2} = 2], [d_{11,1} = 4, d_{11,2} = 7]\right]$, which can be further reduced to $RPN_{11} = [s_{11} = 7, o_{11} = 3, d_{11} = 7]$.

4.3.3 Example 3 from Chapter 3

An FMEA worksheet for a seal pump from Calixto (2016) is considered (see Appendix 3). A total of 7 failure modes need to be prioritized, i.e., N = 7. The tree model of F_4 and F_5. F_4 is depicted in Figure 4.5. The first layer of F_4 consists of its *root node*, also representing the RPN model and together with its RPN score, i.e., $RPN_4 = 168$. There are three nodes in the second layer, also the children for the *root node*, i.e., E_4, RC_4, and PM_4. E_4, RC_4, and PM_4 are associated with $s_4 = 7, o_4 = 8$, and $d_4 = 3$, respectively. In the third layer, the *children nodes* of E_4, RC_4, and PM_4 are $E_{4,1}, E_{4,2}, E_{4,3}$, $RC_{4,1}, RC_{4,2}, RC_{4,3}$, and $PM_{4,1}, PM_{4,2}, PM_{4,3}$, respectively. $E_{4,1}$, $E_{4,2}$, and $E_{4,3}$ are associated with $s_{4,1} = 7$, $s_{4,2} = 4$, and $s_{4,3} = 6$, respectively. $RC_{4,1}$, $RC_{4,2}$, and $RC_{4,3}$ are associated with $o_{4,1} = 7$, $o_{4,2} = 8$, and $o_{4,3} = 5$, respectively. $PM_{4,1}$, $PM_{4,2}$, and $PM_{4,3}$ are associated with $d_{4,1} = 1$, $d_{4,2} = 3$, and $d_{4,3} = 2$, respectively. It is the same goes to F_5. F_5 is a *root node* of its tree model and associated with $RPN_5 = 147$. In second layer of the tree model, the *children nodes* of F_5 are E_5, RC_5, and PM_5. E_5, RC_5, and PM_5 are associated with $s_5 = 7, o_5 = 6$, and $d_5 = 1$, respectively. Again, in the third layer, the *children nodes* of E_{11}, RC_{11}, and PM_{11} are $E_{5,1}, RC_{5,1}$, and $PM_{5,1}$, respectively. $E_{5,1}, RC_{5,1}$, and $PM_{5,1}$ are associated with $s_{5,1} = 7, o_{5,1} = 6$ and $d_{5,1} = 1$ respectively.

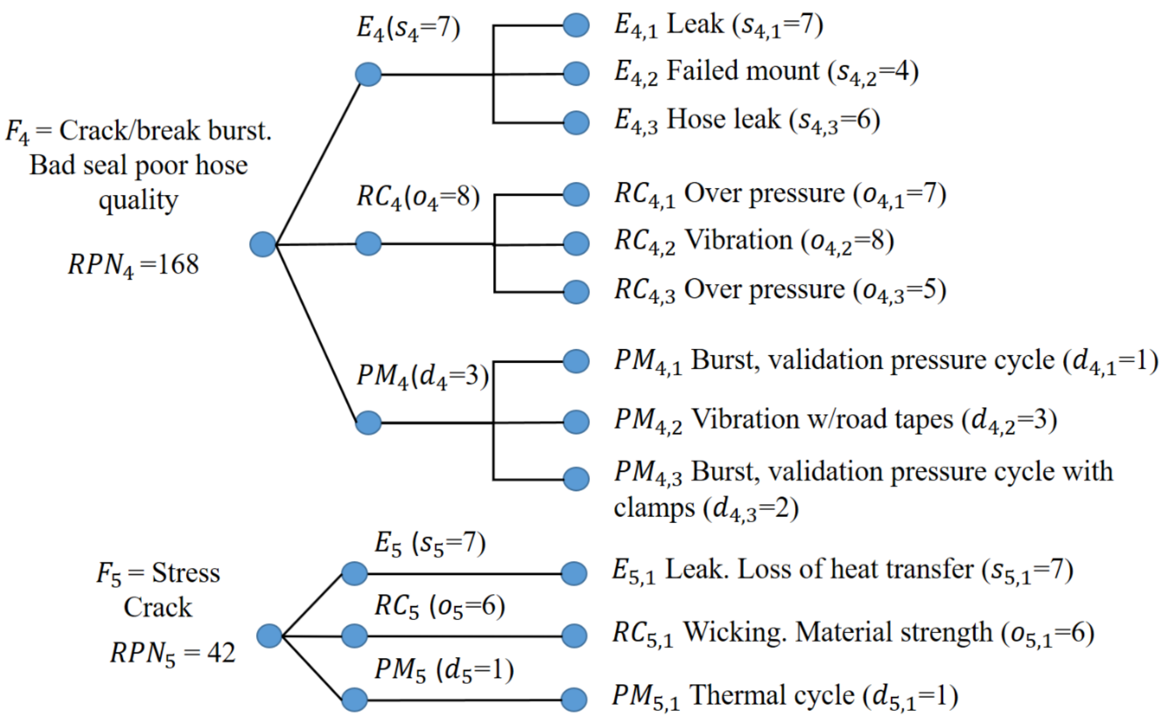

Figure 4.5: Three-layer rooted trees for F_4 and F_5

F_4 can be represented as a nested vector, i.e., $F_4 = \left[[E_{4,1}, E_{4,2}, E_{4,3}], [RC_{4,1}, RC_{4,2}, RC_{4,3}], [PM_{4,1}, PM_{4,2}, PM_{4,3}]\right]$. Besides, RPN_4 can be represented as $RPN_4 = \begin{bmatrix} [s_{4,1} = 7, s_{4,2} = 4, s_{4,3} = 6], [o_{4,1} = 7, o_{4,2} = 8, o_{4,3} = 5], \\ [d_{4,1} = 1, d_{4,2} = 3, d_{4,3} = 2] \end{bmatrix}$, which can be reduced to $RPN_4 = [s_4 = 7, o_4 = 8, d_4 = 3]$. The nested vector of F_5 is $F_5 = \left[[E_{5,1}], [RC_{5,1}], [PM_{5,1}]\right]$. RPN_5 can be represented as $RPN_5 = \left[[s_{5,1} = 7], [o_{5,1} = 6], [d_{5,1} = 1]\right]$, which can be further reduced to $RPN_5 = [s_5 = 7, o_5 = 6, d_5 = 1]$.

4.3.4 Example 4 from Chapter 3

An FMEA worksheet from Guimarães and Lapa (2004) (see Appendix 4) is used. A total of 82 failure modes are to be prioritized, i.e., $N = 82$. The tree model of F_1, F_3 and F_5 is depicted in Figure 4.6. The first layer of F_1 consists of its *root node* and it is associated to $RPN_1 = 75$. Again, the *root node* also denotes the RPN model together with RPN score. There are three nodes in the second layer, also the children for the *root node*, i.e., E_1, RC_1, and PM_1. E_1, RC_1, and PM_1 are associated with $s_1 = 5, o_1 = 3$, and $d_1 = 5$, respectively. In the third layer, the *children nodes* of E_1, RC_1, and PM_1 are $E_{1,1}, RC_{1,1}$, and $PM_{1,1}$, respectively. $E_{1,1}, RC_{1,1}$, and $PM_{1,1}$ are associated with $s_{1,1} = 5, o_{1,1} = 3$, and $d_{1,1} = 5$, respectively.

F_3 is a *root node* of its tree model and associated with $RPN_3 = 60$. The *children nodes* of F_3 in second layer of the tree model are E_3, RC_3, and PM_3. E_3, RC_3, and PM_3 are associated with $s_3 = 5, o_3 = 4$, and $d_3 = 3$, respectively. Again, the *children nodes* of E_3, RC_3, and PM_3 in third layer of tree model are $E_{3,1}, RC_{3,1}$, and $PM_{3,1}$, respectively. $E_{3,1}, RC_{3,1}$, and $PM_{3,1}$ are associated with $s_{3,1} = 5, o_{3,1} = 4$, and $d_{3,1} = 3$, respectively.

Furthermore, the first layer of F_5 consists only the *root node* and it is associated to $RPN_5 = 60$. Again, the *root node* also denotes the RPN model, together with its RPN score. There are three nodes in the second layer, also the children for the *root node*, i.e., E_5, RC_5, and PM_5. E_5, RC_5, and PM_5 are associated with $s_5 = 4, o_5 = 3$, and $d_5 = 5$, respectively. In the third layer, the *children nodes* of E_5, RC_5, and PM_5, are $E_{5,1}$, $RC_{5,1}$, and $PM_{5,1}$, respectively. $E_{5,1}, RC_{5,1}$, and $PM_{5,1}$ are associated with $s_{5,1} = 4$, $o_{5,1} = 3$, and $d_{5,1} = 5$, respectively.

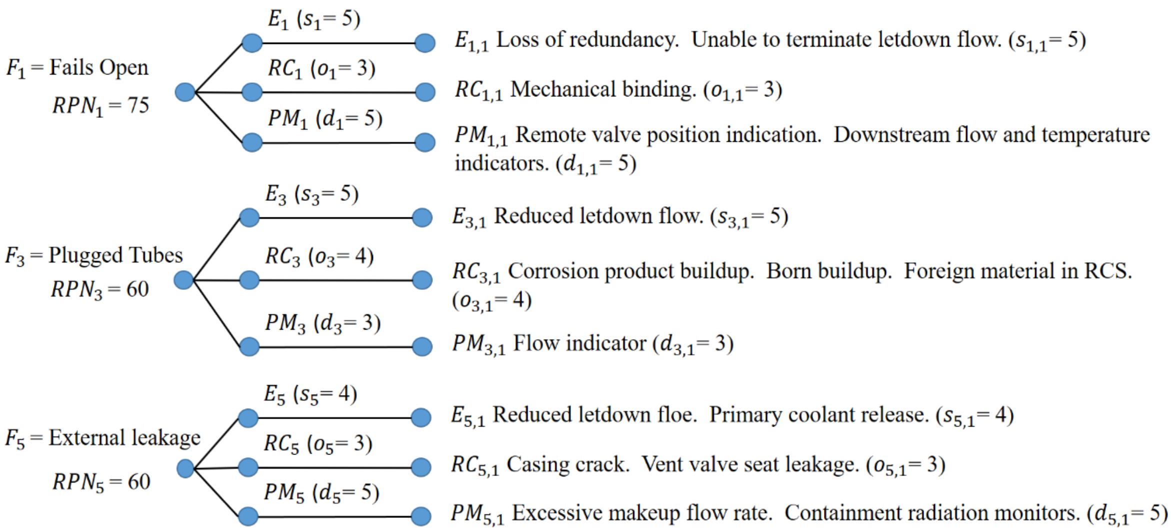

Figure 4.6: Three-layer rooted trees for F_1, F_3, and F_5

The nested vector of F_1 is $F_1 = \left[[E_{1,1}], [RC_{1,1}], [PM_{1,1}]\right]$. RPN_1 can be represented as $RPN_1 = \left[[s_{1,1} = 5], [o_{1,1} = 3], [d_{1,1} = 5]\right]$, which can be further reduced to $RPN_1 = [s_1 = 5, o_1 = 3, d_1 = 5]$. It is the same applies to F_3 and F_5. The nested vector of F_3 and F_5 is $F_3 = \left[[E_{3,1}], [RC_{3,1}], [PM_{3,1}]\right]$ and $F_5 = \left[[E_{5,1}], [RC_{5,1}], [PM_{5,1}]\right]$, respectively. RPN_3 can be represented as $RPN_3 = \left[[s_{3,1} = 5], [o_{3,1} = 4], [d_{3,1} = 3]\right]$, which can be further reduced to $RPN_3 = [s_3 = 5, o_3 = 4, d_3 = 3]$. RPN_5 can be represented as $RPN_5 =$

$\left[[s_{5,1} = 4], [o_{5,1} = 3], [d_{5,1} = 5]\right]$, which can be further reduced to $RPN_5 = [s_5 = 4, o_5 = 3, d_5 = 5]$.

4.4 Summary

In this chapter, a new method to represent the failure mode(s) and corrective action(s) in a nested vector and a tree model is presented. Four benchmark examples are presented to illustrate the usefulness of proposed representations. Note that there is a case (see Section 4.3.1) that failure mode(s) and corrective action(s) could not be ordered or prioritized because of the missing S, O, and/or D rating. Thus, an interval method to tackle missing S, O, and/or D rating, will be presented based on the three principles in the next chapter. Four possible orderings among interval-valued risk ratings and RPN score will be introduced and analysed to prioritize the failure modes and corrective actions.

CHAPTER 5

IMPUTATION OF MISSING RISK RATINGS

5.1 Introduction

In this chapter, an interval method to tackle missing S, O, and/or D rating in an FMEA activity, is proposed based on the three principles, i.e., *transitivity*, *pareto optimality*, and *independence of irrelevant alternatives*. All risk ratings are transformed into interval values to obtain interval-valued RPN score. Then, four possible orderings among interval-valued risk ratings and RPN score are introduced and analysed. Besides, a benchmark FMEA example which consists of missing S, O, and/or D rating (see Example 1 in Section 3.2.1) is presented and analysed.

This chapter is organized as follows. In Section 5.2, preliminaries for the interval method to handle missing risk ratings in an FMEA activity are presented. The orderings among interval-valued risk ratings and RPN score are developed in Section 5.3. The mathematical analysis is presented in Section 5.4. In Section 5.5, the effectiveness of proposed method is demonstrated with two benchmark examples from Murray (2013) and Kerk et al. (2015). The result shows the usefulness of the proposed interval method in handling risk ratings which are complete or missing. Finally, a concluding remark is presented.

5.2 Preliminaries

In this section, the interval-valued risk ratings and RPN scores are defined, as follows.

Definition 5.1: Consider $x_{i,k}$, x_i and RPN_i intervals.

5.1.1. $x_{i,k}$ is an interval value and is denoted as $\left[x_{i,k}^L, x_{i,k}^U\right]$, where $x_{i,k} \in \left\{s_{i,a}, o_{i,b}, d_{i,c}\right\}$;

5.1.2. x_i is an interval value and is denoted as $[x_i^L, x_i^U]$, where $x_i \in \{s_i, o_i, d_i\}$;

5.1.3. RPN_i is an interval value and is denoted as $[RPN_i^L, RPN_i^U]$;

5.1.4. Z_i is an interval value and is denoted as $[Z_i^L, Z_i^U]$, where $Z_i \in \left\{x_{i,k}, x_i, RPN_i\right\}$. $Z_i^L \leq Z_i^U$ is always true.

$x_{i,k}$, x_i and RPN_i are obtained using Equations (5.1), (5.2), and (5.3) respectively. Four possible orderings among $x_{i,k}$, x_i and RPN_i intervals i.e., *equivalence*, *strong order*, *weak order*, and *containment order* (Zapata, Kreinovich, Joslyn, & Hogan, 2013), are summarized in next section. To ease the explanation, $Z_i \in \left\{x_{i,k}, x_i, RPN_i\right\}$ is used as an example. $Z_1 = [Z_1^L, Z_1^U]$ and $Z_2 = [Z_2^L, Z_2^U]$ are considered.

$$x_{i,k} = \begin{cases} \left[x_{i,k}, x_{i,k}\right], & \text{if } x_{i,k} \text{ is known} \\ \left[max_{\forall k}\left(x_{i,k}, \underline{x}\right), \overline{x}\right], & \text{if } x_{i,k} \text{ is missing} \end{cases} \tag{5.1}$$

$$x_i = \begin{cases} \left[max_{\forall k}\left(x_{i,k}^L\right), max_{\forall k}\left(x_{i,k}^U\right)\right], & \text{if } x_i \text{ is not assigned manually} \\ [x_i, x_i], & \text{if } x_i \text{ is assigned manually} \end{cases} \tag{5.2}$$

$$RPN_i = \left[f_{RPN}(s_i^L, o_i^L, d_i^L), f_{RPN}\left(s_i^U, o_i^U, d_i^U\right)\right] \tag{5.3}$$

5.3 Four Possible Orderings among Interval-valued Risk Ratings and RPN Score

To ease the explanation, $Z_i = [Z_i^L, Z_i^U]$ is used, in which $Z_i \in \{x_{i,k}, x_i, RPN_i\}$. In this example, $Z_1 = [Z_1^L, Z_1^U]$ and $Z_2 = [Z_2^L, Z_2^U]$ are considered.

Definition 5.2: Four possible orderings among Z_i.

5.2.1. Z_1 and Z_2 are of equivalence, i.e., $Z_1 = Z_2$, if and only if, $Z_1^L = Z_2^L$ and $Z_1^U = Z_2^U$.

5.2.2. Z_1 and Z_2 are of weak order, i.e., $Z_1 > Z_2$, if and only if, $Z_2^L \leq Z_1^L$ and $Z_2^U \leq Z_1^U$, in which, the equivalence ordering (see Definition 5.2.1), i.e., $Z_1^L = Z_2^L$ and $Z_1^U = Z_2^U$, is omitted.

5.2.2.1. Z_1 and Z_2 are of strong order, i.e., $Z_1 > Z_2$, if and only if, $Z_2^L \leq Z_2^U < Z_1^L \leq Z_1^U$.

5.2.3. Z_1 and Z_2 are of containment order, i.e., $Z_1 > Z_2$ if and only if, $Z_1^L < Z_2^L$ and $Z_2^U < Z_1^U$.

We consider *strong order* (Definition 5.2.2.1) as a subset of *weak order* (Definition 5.2.2). *Containment order* is used to order interval values, in which $Z_1 > Z_2$, if Z_1 contains Z_2. *Containment order* is suitable for FMEA applications, because there exits element(s) in Z_1, which is greater than all elements of Z_2.

5.4 Analysis

In this section, an analysis of Equations (5.1), (5.2), and (5.3) is formulated as *Theorem 5.1*. Besides, an analysis of *Axioms 3.1.1, 3.1.2,* and *3.1.3* with interval-valued $s, o,$ and d is formulated as *Propositions 5.1, 5.2,* and 5.*3*, respectively.

Theorem 5.1: Analysis for Equations (5.1), (5.2), and (5.3).

5.1.1. $\underline{x} \leq x_{i,k}^{L} \leq x_{i,k}^{U} \leq \overline{x}$ is always satisfied.

5.1.2. $\underline{x} \leq x_{i}^{L} \leq x_{i}^{U} \leq \overline{x}$ is always satisfied.

5.1.3. $\underline{RPN} \leq RPN_{i}^{L} \leq RPN_{i}^{U} \leq \overline{RPN}$ is always satisfied.

5.1.4. $f_{RPN}(m, n_1) \geq f_{RPN}(m, n_2)$, if $n_1 > n_2$ where $m, n \subset \{s, o, d\}$; $n \notin m$.

Proof of Theorem 5.1.1:

Equation (5.1) is used. If $x_{i,k}$ is known, then, $x_{i,k}^{L} = x_{i,k}^{U} = x_{i,k}$, and $\underline{x} \leq x_{i,k} \leq \overline{x}$, therefore, $\underline{x} \leq x_{i,k}^{L} \leq x_{i,k}^{U} \leq \overline{x}$ is always true. If all $x_{i,k}$, $\forall k$, are missing, then, $x_{i,k}^{L} = \underline{x}$ and $x_{i,k}^{U} = \overline{x}$. Therefore, $\underline{x} \leq x_{i,k}^{L} \leq x_{i,k}^{U} \leq \overline{x}$ is always true. If not all $x_{i,k}$ are missing, then, $x_{i,k}^{L} = max_{\forall k}(x_{i,k})$ and $x_{i,k}^{U} = \overline{x}$. Therefore, $\underline{x} \leq x_{i,k}^{L} \leq x_{i,k}^{U} \leq \overline{x}$ is always true. In summary, $\underline{x} \leq x_{i,k}^{L} \leq x_{i,k}^{U} \leq \overline{x}$ is always satisfied, for all cases.

Proof of Theorem 5.1.2:

Equation (5.2) is used. If x_i is assigned manually, then, $x_{i}^{L} = x_{i}^{U} = x_i$ and $\underline{x} \leq x_i \leq \overline{x}$. Therefore, $\underline{x} \leq x_{i}^{L} \leq x_{i}^{U} \leq \overline{x}$ is always true. If x_i is not assigned manually, then, $x_{i}^{L} = max_{\forall k}(x_{i,k}^{L})$, $x_{i,j}^{U} = max_{\forall k}(x_{i,k}^{U})$, and $\underline{x} \leq x_{i,k}^{L} \leq x_{i,k}^{U} \leq \overline{x}$. Therefore, $\underline{x} \leq x_{i}^{L} \leq x_{i}^{U} \leq \overline{x}$ is always true. In summary, $\underline{x} \leq x_{i}^{L} \leq x_{i}^{U} \leq \overline{x}$ is always satisfied, for all cases.

Proof of Theorem 5.1.3:

From *Theorem 5.1.2*, $\underline{x} \leq x_i^L \leq x_i^U \leq \overline{x}$ is always true. Then, $\underline{s} \leq s_i^L \leq s_i^U \leq \overline{s}$, $\underline{o} \leq o_i^L \leq o_i^U \leq \overline{o}$, and $\underline{d} \leq d_i^L \leq d_i^U \leq \overline{d}$. Therefore, $f_{RPN}(\underline{s}, \underline{o}, \underline{d}) \leq f_{RPN}(s_i^L, o_i^L, d_i^L) \leq f_{RPN}(s_i^U, o_i^U, d_i^U) \leq f_{RPN}(\overline{s}, \overline{o}, \overline{d})$ is always true. In summary, $\underline{RPN} \leq RPN_i^L \leq RPN_i^U \leq \overline{RPN}$ is always satisfied.

Proof of Theorem 5.1.4:

Three possibilities of $n_1 > n_2$ are considered. If $n_1 > n_2$ is a *strong order*, then $n_2^L \leq n_2^U < n_1^L \leq n_1^U$. Therefore, $f_{RPN}(m^L, n_2^L) < f_{RPN}(m^L, n_1^L)$ and $f_{RPN}(m^U, n_2^U) < f_{RPN}(m^U, n_1^U)$ is true. As such, $[f_{RPN}(m^L, n_2^L), f_{RPN}(m^U, n_2^U)] < [f_{RPN}(m^L, n_1^L), f_{RPN}(m^U, n_1^U)]$ is a *strong order*. $f_{RPN}(m, n_1) \geq f_{RPN}(m, n_2)$ is always true. If $n_1 > n_2$ is a *weak order*, then, $n_2^L \leq n_1^L$ and $n_2^U \leq n_1^U$. Next, $f_{RPN}(m^L, n_2^L) \leq f_{RPN}(m^L, n_1^L)$ and $f_{RPN}(m^U, n_2^U) \leq f_{RPN}(m^U, n_1^U)$ is true. As such, $[f_{RPN}(m^L, n_2^L), f_{RPN}(m^U, n_2^U)] < [f_{RPN}(m^L, n_1^L), f_{RPN}(m^U, n_1^U)]$ is a *weak order*. $f_{RPN}(m, n_1) \geq f_{RPN}(m, n_2)$ is always true too. If $n_1 > n_2$ is a *containment order*, then, $n_1^L < n_2^L \leq n_2^U < n_1^U$. Next, $f_{RPN}(m^L, n_1^L) < f_{RPN}(m^L, n_2^L)$ and $f_{RPN}(m^U, n_2^U) < f_{RPN}(m^U, n_1^U)$ is always true. As such, $[f_{RPN}(m^L, n_2^L), f_{RPN}(m^U, n_2^U)] < [f_{RPN}(m^L, n_1^L), f_{RPN}(m^U, n_1^U)]$ is a *containment order*. Therefore, $f_{RPN}(m, n_1) \geq f_{RPN}(m, n_2)$ is always true. In summary, if $n_1 > n_2$, $f_{RPN}(m, n_1) \geq f_{RPN}(m, n_2)$ is always satisfied, for all cases.

Proposition 5.1: *Axiom 3.1.1* is always satisfied.

Proof of Proposition 5.1:

Three sets of RPN_i, i.e., RPN_1, RPN_2 and RPN_3, with $RPN_1 > RPN_2$ and $RPN_2 > RPN_3$, are considered. All three possibilities of orderings for each $RPN_1 > RPN_2$ and $RPN_2 > RPN_3$, are considered, i.e., strong order, weak order, and containment order, resulting in nine combinations of $RPN_1 > RPN_2$ and $RPN_2 > RPN_3$, as shown in Figure 5.1 and illustrated in Table 5.1.

1. If $RPN_1 > RPN_2$ and $RPN_2 > RPN_3$ are of *strong order*, then, $RPN_3^L \leq RPN_3^U < RPN_2^L \leq RPN_2^U < RPN_1^L \leq RPN_1^U$. Therefore, $RPN_1 > RPN_3$ is always true and it is a *strong order*.
2. If $RPN_1 > RPN_2$ is a *strong order* and $RPN_2 > RPN_3$ is a *weak order*, then, $RPN_3^L \leq RPN_2^L$, $RPN_3^U \leq RPN_2^U$ and $RPN_2^L \leq RPN_2^U < RPN_1^L \leq RPN_1^U$. As such, $RPN_3^L \leq RPN_3^U < RPN_1^L \leq RPN_1^U$ is always true. Therefore, $RPN_1 > RPN_3$ is always true and it is a *strong order*.
3. If $RPN_1 > RPN_2$ is a *strong order* and $RPN_2 > RPN_3$ is a *containment order*, then $RPN_2^L < RPN_3^L \leq RPN_3^U < RPN_2^U$ and $RPN_2^L \leq RPN_2^U < RPN_1^L \leq RPN_1^U$. As such, $RPN_3^L \leq RPN_3^U < RPN_1^L \leq RPN_1^U$ is always true. Therefore, $RPN_1 > RPN_3$ is true and it is a *strong order*.
4. If $RPN_1 > RPN_2$ is a *weak order*, and $RPN_2 > RPN_3$ is a *strong order*, then, $RPN_2^L \leq RPN_1^L$, $RPN_2^U \leq RPN_1^U$ and $RPN_3^L \leq RPN_3^U < RPN_2^L \leq RPN_2^U$. As such, $RPN_3^L \leq RPN_3^U < RPN_1^L \leq RPN_1^U$ is always true. Therefore, $RPN_1 > RPN_3$ is true and it is a *strong order*.

5. If $RPN_1 > RPN_2$ and $RPN_2 > RPN_3$ are of *weak order*, then $RPN_3^L \leq RPN_2^L \leq RPN_1^L$ and $RPN_3^U \leq RPN_2^U \leq RPN_1^U$. As such, $RPN_3^L \leq RPN_1^L$ and $RPN_3^U \leq RPN_1^U$ are always true. Therefore, $RPN_1 > RPN_3$ is always true. It is a *weak order* if $RPN_3^L \leq RPN_1^L$ and $RPN_3^U \leq RPN_1^U$, or it is a *strong order* if $RPN_3^L \leq RPN_3^U < RPN_1^L \leq RPN_1^U$.
6. If $RPN_1 > RPN_2$ is a *weak order* and $RPN_2 > RPN_3$ is a *containment order*, then, $RPN_2^L \leq RPN_1^L$, $RPN_2^U \leq RPN_1^U$ and $RPN_2^L < RPN_3^L \leq RPN_3^U < RPN_2^U$. Therefore, $RPN_1 > RPN_3$ is always true. It is a *weak order*, if $RPN_3^L \leq RPN_1^L$ and $RPN_3^U \leq RPN_1^U$. It is a *containment order* if $RPN_1^L < RPN_3^L \leq RPN_3^U < RPN_1^U$. It is a *strong order*, if $RPN_3^L \leq RPN_3^U < RPN_1^L \leq RPN_1^U$.
7. If $RPN_1 > RPN_2$ is a *containment order* and $RPN_2 > RPN_3$ is a *strong order*; then $RPN_1 > RPN_3$ is always true. It is a *weak order*, if $RPN_3^L \leq RPN_1^L$ and $RPN_3^U \leq RPN_1^U$. It is a *containment order* if $RPN_1^L < RPN_3^L \leq RPN_3^U < RPN_1^U$. It is a *strong order*, if $RPN_3^L \leq RPN_3^U < RPN_1^L \leq RPN_1^U$.
8. If $RPN_1 > RPN_2$ is a *containment order* and $RPN_2 > RPN_3$ is a *weak order*, then $RPN_1 > RPN_3$ is always true. It is a *weak order*, if $RPN_3^L \leq RPN_1^L$ and $RPN_3^U \leq RPN_1^U$. It is a *containment order* if $RPN_1^L < RPN_3^L \leq RPN_3^U < RPN_1^U$.
9. If $RPN_1 > RPN_2$ and $RPN_2 > RPN_3$ are *containment order*; then $RPN_1^L < RPN_2^L < RPN_3^L$ and $RPN_3^U < RPN_2^U < RPN_1^U$. Therefore, $RPN_1 > RPN_3$ is a *containment order* and it is true.

Considering these nine combinations, *Axiom 3.1.1* is always satisfied.

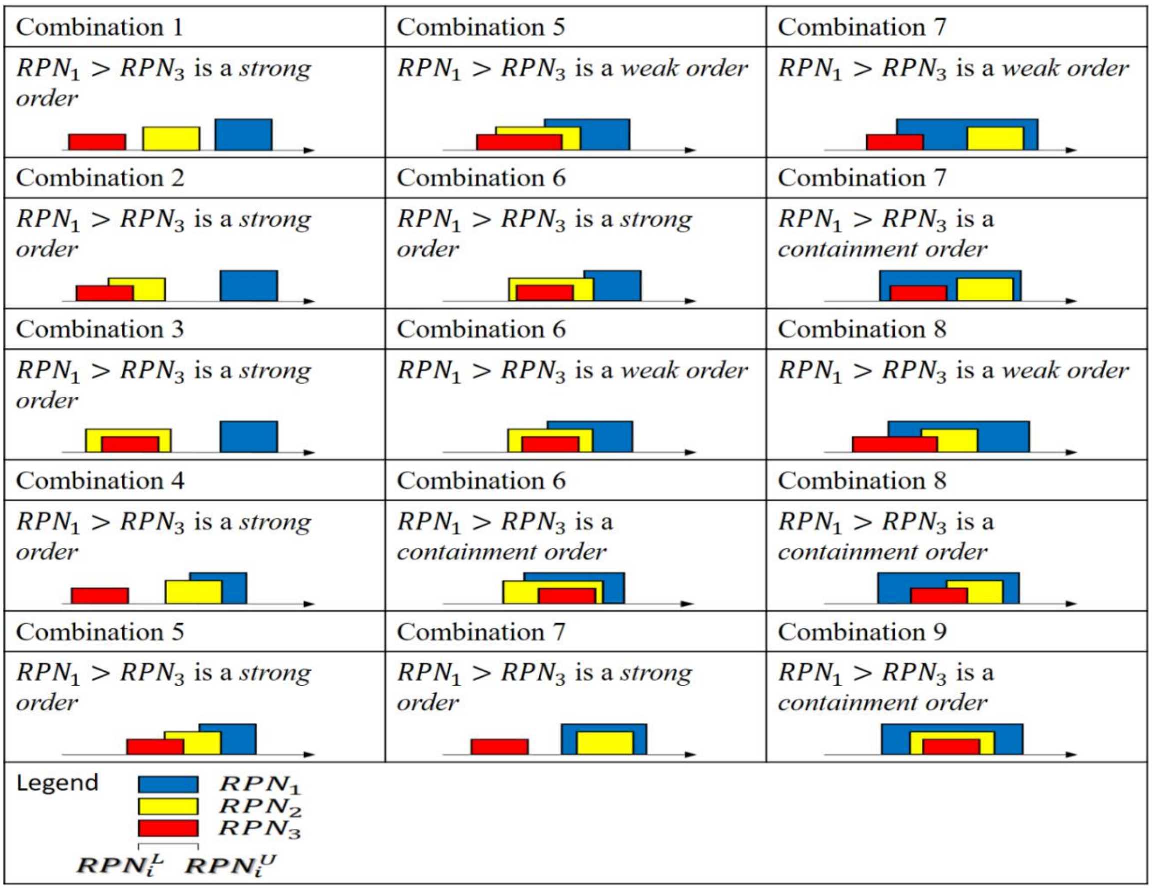

Figure 5.1: An analysis for Axiom 3.1.1

Table 5.1: An analysis for Axiom 3.1.1

Com.	$RPN_1 > RPN_2$	$RPN_2 > RPN_3$	Remarks
1.	strong order	strong order	$RPN_1 > RPN_3$, strong order
2.	strong order	weak order	$RPN_1 > RPN_3$, strong order
3.	strong order	containment order	$RPN_1 > RPN_3$, strong order
4.	weak order	strong order	$RPN_1 > RPN_3$, strong order
5.	weak order	weak order	$RPN_1 > RPN_3, \begin{cases} strong\ order, \text{if } RPN_3^L \leq RPN_3^U < RPN_1^L \leq RPN_1^U \\ weak\ order, \text{if } RPN_3^L \leq RPN_1^L \text{ and } RPN_3^U \leq RPN_1^U \end{cases}$
6.	weak order	containment order	$RPN_1 > RPN_3, \begin{cases} strong\ order, \text{if } RPN_3^L \leq RPN_3^U < RPN_1^L \leq RPN_1^U \\ weak\ order, \text{if } RPN_3^L \leq RPN_1^L \text{ and } RPN_3^U \leq RPN_1^U \\ containment\ order, \text{if } RPN_1^L < RPN_3^L \leq RPN_3^U < RPN_1^U \end{cases}$
7.	containment order	strong order	$RPN_1 > RPN_3, \begin{cases} strong\ order, \text{if } RPN_3^L \leq RPN_3^U < RPN_1^L \leq RPN_1^U \\ weak\ order, \text{if } RPN_3^L \leq RPN_1^L \text{ and } RPN_3^U \leq RPN_1^U \\ containment\ order, \text{if } RPN_1^L < RPN_3^L \leq RPN_3^U < RPN_1^U \end{cases}$
8.	containment order	weak order	$RPN_1 > RPN_3, \begin{cases} weak\ order, \text{if } RPN_3^L \leq RPN_1^L \text{ and } RPN_3^U \leq RPN_1^U \\ containment\ order, \text{if } RPN_1^L < RPN_3^L \leq RPN_3^U < RPN_1^U \end{cases}$
9.	containment order	containment order	$RPN_1 > RPN_3$, containment order

Proposition 5.2: Axiom *3.1.2* is always satisfied

Proof of Proposition 5.2:

From *Theorem 5.1.4*, if $n_1 > n_2$, $f_{RPN}(m, n_1) \geq f_{RPN}(m, n_2)$ is always true. As such, $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_2, o_1, d_1)$, $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_1, o_2, d_1)$, and $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_1, o_1, d_2)$. Again, if f_{RPN} is *monotone*, then, $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_1, o_2, d_2)$ is always true. The same goes to $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_2, o_1, d_2)$ and $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_2, o_2, d_1)$. Similarly, $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_2, o_2, d_2)$ is always true. Therefore, *Axiom 3.1.2* is always satisfied.

Proposition 5.3: *Axiom 3.1.3* is always satisfied.

Proof of Proposition 5.3:

If $f_{RPN}(s_1, o_1, d_1) \geq f_{RPN}(s_2, o_2, d_2)$, then, the relative ordering of $f_{RPN}(s_1, o_1, d_1)$ and $f_{RPN}(s_2, o_2, d_2)$ remains the same even though there is an additional failure mode(s) or corrective action(s). Therefore, *Axiom 3.1.3* is always satisfied.

5.5 Case Study

In this section, two benchmark examples which from (Murray, 2013) and (Kerk et al., 2015) are used to illustrate the capability of proposed method.

5.5.1 Example 1 from Chapter 3

In this section, Example 1 is used to illustrate the usefulness of the proposed formulation and methodology. The S, O and D scale tables are summarized in Tables 3.2, 3.3, and 3.4, respectively. Note that the traditional RPN model, i.e., $RPN = s \times o \times d$, is used, in which $\underline{x} = 1$ and $\overline{x} = 5$. There are 27 potential failure modes to be prioritized, i.e., $N = 27$. In this experiment, the risk ratings are *incomplete*, since not all s_i, o_i, and d_i can be obtained by aggregation (Definition 4.2.2) or through manual assignment (Definition 4.2.4). Notice that o_{22}, d_{22}, o_{23}, and d_{23} are missing. Using the traditional RPN model, it is not possible to conclude that the prioritization of F_i. To solve this problem, Equations (5.1) and (5.2) are used to transform all s_i, o_i, and d_i (including missing risk ratings) into interval values. Then, RPN_i for each F_i is computed using Equation (5.3). All RPN_i are ordered using four types of orderings, as in Definition 5.2.

Table 5.2 summarizes the ordering outcomes. Column "F_i" shows the failure modes, with the respective names in column "*Failure modes*". Column "Traditional RPN model" shows the prioritization outcomes from traditional RPN model. Sub-column "$[s_i, o_i, d_i]$" shows the respective S, O, and D ratings associated with each F_i, with the resulting RPN calculation in sub-column "RPN_i", in which $RPN_i = s_i \times o_i \times d_i$. As an example, for F_1, $s_1 = 3, o_1 = 1, d_1 = 1$, and $RPN_1 = 3$. Note that not all risk ratings are known. As an example, o_{22} and d_{22} are missing, and RPN_{22} could not be determined. Column "Interval-valued RPN model" shows the prioritization results from interval-valued RPN model. Sub-columns "s_i", "o_i", and "d_i" list the interval-valued s_i, o_i, and d_i ratings, which are obtained in Equations (5.1) and (5.2), with their interval-valued RPN scores in sub-column "RPN_i". Since not all F_i can be ordered using traditional RPN model, interval valued RPN model is used instead. The resulting ordering is indicated in column r_i, in which high RPN_i appears first. For F_1, $s_1 = [3,3], o_1 = [1,1], d_1 = [1,1]$, $RPN_1 = [3,3]$, and $r_1 = 8$. Using traditional RPN model, F_{22}, F_{23} can not be ordered. But, using interval-valued RPN model, $r_{22} = r_{23} = 1$, i.e., F_{22} and F_{23} are ordered to have the first priority.

In traditional RPN model, the usefulness of *Axiom 3.1* can be explained with $RPN_1 = [3,1,1]$, $RPN_2 = [5,2,1]$, and $RPN_4 = [4,2,1]$. The RPN scores for F_1, F_2, and F_4 are $RPN_1 = 3$, $RPN_2 = 10$, and $RPN_4 = 8$, respectively. *Axiom 3.1.1* suggests if RPN_2 is greater than RPN_4, and RPN_4 is greater than RPN_1, then, RPN_2 should be greater than RPN_1. In this example, an f_{RPN} that satisfies *Property 3.1* is used, in which $\underline{RPN} = 1$ and $\overline{RPN} = 125$. Therefore, *Axiom 3.1.1* is satisfied, which is in agreement with *Proposition 3.1*. *Axiom 3.1.2* suggests that RPN_2 should be greater than RPN_1. This is because $s_2 \geq s_1$, $o_2 \geq o_1$, and $d_2 \geq d_1$. With an f_{RPN} that satisfies *Property 3.1*, *Axiom 3.1.2* is satisfied, which is in agreement with *Proposition 3.2*. *Axiom 3.1.3* suggests that the relative order of F_2 and F_4 remains the same, i.e., RPN_2 is greater than RPN_4, when a new failure mode, e.g., F_1, is added. Again, with an f_{RPN} that satisfies *Property 3.1*, *Axiom 3.1.3* is true, which is in agreement with *Proposition 3.3*. Even through *Axiom 3.1* is useful, it does not explain F_{22} and F_{23}. Therefore, the proposed interval method needs to be employed to tackle this limitation.

In interval-valued RPN model, the usefulness of *Axiom 3.1* can be explained with $RPN_8 = \big[[4,4],[2,2],[3,3]\big]$, $RPN_{12} = \big[[4,4],[4,4],[3,3]\big]$, and $RPN_{22} = \big[[4,4],[1,5],[1,5]\big]$, as well as $RPN_8 = [24,24]$, $RPN_{12} = [48,48]$, and $RPN_{22} = [4,100]$. *Axiom 3.1.1* suggests if RPN_{22} is greater than RPN_{12} (a *containment order*), and RPN_{12} is greater than RPN_8 (a *strong order*), then, RPN_{22} should be greater than RPN_8. In this example, the four possible orderings among RPN_i are considered, in which RPN_{22} is greater than RPN_8 (a *containment order*). Therefore, *Axiom 3.1.1* is satisfied, which is in agreement with *Proposition 5.1*. *Axiom 3.1.2* suggests that RPN_{22} should be greater than RPN_{12}. This is because $s_{22} \geq s_{12}$, $o_{22} \geq o_{12}$, and $d_{22} \geq d_{12}$. With an f_{RPN} that satisfies *Property 3.1*, *Axiom 3.1.2* is satisfied, which is in agreement with *Proposition 5.2*. *Axiom 3.1.3* suggests that the relative order of F_{12} and F_8 remains the same, i.e., RPN_{12} is greater than that of RPN_8, when a new failure mode, e.g., F_{22}, is added. Similarly, with an f_{RPN} that satisfies *Property 3.1*, *Axiom 3.1.3* is true, which is in agreement with *Proposition 5.3*.

Table 5.2: Failure modes ordering outcomes with the example from (Murray, 2013)

F_i	Failure modes	Traditional RPN model		Interval-valued RPN model				
		$[s_i, o_i, d_i]$	RPN_i	s_i	o_i	d_i	RPN_i	r_i
F_1	Parts will not fit together/mount properly in location	[3,1,1]	3	[3,3]	[1,1]	[1,1]	[3,3]	8
F_2	Hardware falls from tower	[5,2,1]	10	[5,5]	[2,2]	[1,1]	[10,10]	6
F_3	Parts will not fit together/mount properly in location	[3,1,1]	3	[3,3]	[1,1]	[1,1]	[3,3]	8
F_4	Assembly not properly aligned or damaged	[4,2,1]	8	[4,4]	[2,2]	[1,1]	[8,8]	7
F_5	Assembly/components fall from tower	[5,2,1]	10	[5,5]	[2,2]	[1,1]	[10,10]	6
F_6	Parts will not fit together/mount properly in location	[3,1,1]	3	[3,3]	[1,1]	[1,1]	[3,3]	8
F_7	Assembly not properly installed / aligned; Motor assembly internally fails/damaged; Cables damaged	[4,2,1]	8	[4,4]	[2,2]	[1,1]	[8,8]	7
F_8	Robot parks in undesirable state (not at nadir)	[4,2,3]	24	[4,4]	[2,2]	[3,3]	[24,24]	3
F_9	Assembly/component falls from tower	[5,2,1]	10	[5,5]	[2,2]	[1,1]	[10,10]	6
F_{10}	Parts will not fit together/mount properly in location	[3,1,1]	3	[3,3]	[1,1]	[1,1]	[3,3]	8

Table 5.2 continued

F_i	Failure modes	Traditional RPN model		Interval-valued RPN model				
		$[s_i, o_i, d_i]$	RPN_i	s_i	o_i	d_i	RPN_i	r_i
F_{11}	Sensor broken/internally fails	[4,3,2]	24	[4,4]	[3,3]	[2,2]	[24,24]	3
F_{12}	Collimator field of view insufficient	[4,4,3]	48	[4,4]	[4,4]	[3,3]	[48,48]	2
F_{13}	Sensor head/component falls from tower	[5,1,2]	10	[5,5]	[1,1]	[2,2]	[10,10]	6
F_{14}	Parts will not fit together/mount properly in location	[3,1,1]	3	[3,3]	[1,1]	[1,1]	[3,3]	8
F_{15}	Assembly falls from tower	[5,2,1]	10	[5,5]	[2,2]	[1,1]	[10,10]	6
F_{16}	Corrosion on unprotected electrical components inside enclosure	[4,2,2]	16	[4,4]	[2,2]	[2,2]	[16,16]	4
F_{17}	Spectral photometer doesn't GoSun correctly	[3,2,2]	12	[3,3]	[2,2]	[2,2]	[12,12]	5
F_{18}	Daughter interface board becomes disconnected	[4,3,1]	12	[4,4]	[3,3]	[1,1]	[12,12]	5
F_{19}	Battery charge insufficient (4.8V)	[4,3,1]	12	[4,4]	[3,3]	[1,1]	[12,12]	5
F_{20}	Known' bug on Cimel units model CE318, LCD window displays only "noise" and the unit is not operational	[4,3,1]	12	[4,4]	[3,3]	[1,1]	[12,12]	5
F_{21}	Internal storages fill up	[4,3,1]	12	[4,4]	[3,3]	[1,1]	[12,12]	5
F_{22}	Sensor fails to detect precipitation event	$[4,-,-]$	-	[4,4]	[1,5]	[1,5]	[4,100]	1
F_{23}	Sensor incorrectly reports precipitation event	$[4,-,-]$	-	[4,4]	[1,5]	[1,5]	[4,100]	1
F_{24}	Parts will not fit together/mount properly in location	[3,1,1]	3	[3,3]	[1,1]	[1,1]	[3,3]	8
F_{25}	Assembly falls from tower	[5,2,1]	10	[5,5]	[2,2]	[1,1]	[10,10]	6
F_{26}	Bridge damaged/broken	[4,2,1]	8	[4,4]	[2,2]	[1,1]	[8,8]	7
F_{27}	Splitter damage broken	[4,2,1]	8	[4,4]	[2,2]	[1,1]	[8,8]	7

5.5.2 An FMEA activity from Kerk et al. (2015)

In this section, an FMEA activity from (Kerk et al., 2015) (page 8) is used to demonstrate the effectiveness of the proposed formulation and methodology. The S, O, and D scale tables are summarized in Tables 5.3, 5.4, and 5.5, respectively. Note that the interval fuzzy inference system RPN model (see Section IV in (Kerk et al., 2015)), is used, in which $\underline{x} = 1$ and $\overline{x} = 10$. There are 15 potential failure modes to be prioritized, i.e., $N = 15$. In this experiment, the risk ratings are *complete*, since all s_i, o_i, and d_i can be obtained by aggregation (Definition 4.2.2) or through manual assignment (Definition 4.2.4). Since all RPN_i are interval values, then all F_i are ordered using four types of orderings, as in Definition 5.2.

Table 5.6 summarizes the ordering outcomes of Experiment 5 in (Kerk et al., 2015). Failure modes is shown in column "F_i". Columns "s_i", "o_i", and "d_i" shows the respective S, O, and D ratings associated with each F_i. The prioritization outcomes from the interval fuzzy inference system RPN model in (Kerk et al., 2015) is shown in column "RPN_i". The resulting ordering of interval fuzzy inference system RPN model is listed in column "Ordering with interval fuzzy inference system RPN model". Then, the prioritization results from our proposed method, i.e., interval-valued RPN model, is shown in column "Interval-valued RPN model". Both resulting ordering indicates that high RPN_i is appear first. For F_1, $s_1 = [1,1], o_1 = [2,2], d_1 = [1,1]$, $RPN_1 = [94,179]$. Also. $r_1 = 11$ in interval fuzzy inference system RPN model and $r_1 = 14$ in interval-valued RPN model.

Table 5.3: Scale Table for Severity (Kerk et al., 2015)

Ranking	Criteria
10	**Very high (liability)**: Failures will affect safety or compliance to law.
8-9	**High (reliability/reputation)**: Customer impact or major reliability excursions.
6-7	**Moderate (quality/convenience)**: Impacts customer yield or wrong package/par/marking.
2-5	**Low (special handling)**: Yield hit, cosmetic.
1	**None (unnoticed)**: Unnoticed.

Table 5.4: Scale Table for Occurrence (Kerk et al., 2015)

Ranking	Criteria
9-10	**Very High**: Many/shift, many/day.
7-8	**High**: Many/week, few/week
4-6	**Moderate**: Once/week, several/month
3	**Low**: Once/month
2	**Very Low**: Once/quarter
1	**Remote**: Once ever

Table 5.5: Scale Table for Detection (Kerk et al., 2015)

Ranking	Criteria
10	**Extremely low**: No control available.
9	**Very low**: Control probably will not detect.
7-8	**Low**: Control may not detect excursion until reach next functional area.
5-6	**Medium**: Controls are able to detect within the same functional area.
3-4	**High**: Controls are able to detect within the same machine/module.
1-2	**Very high**: Prevent excursion from occurring.

Table 5.6: Failure modes ordering outcomes with the example from (Kerk et al., 2015)

F_i	s_i	o_i	d_i	RPN_i	Ordering with interval fuzzy inference system RPN model	Ordering with interval-valued RPN model
F_1	[1,1]	[2,2]	[1,1]	[94,179]	11	14
F_2	[1,1]	[2,2]	[3,3]	[105,190]	10	13
F_3	[1,1]	[3,3]	[1,1]	[219,385]	9	12
F_4	[1,1]	[5,5]	[1,1]	[488,543]	8	11
F_5	[2,2]	[2,2]	[1,1]	[415,638]	8	10
F_6	[1,1]	[7,7]	[1,1]	[532,736]	7	9
F_7	[4,4]	[2,2]	[1,1]	[511,765]	7	7
F_8	[5,5]	[2,2]	[1,1]	[557,770]	6	6
F_9	[7,7]	[1,1]	[1,1]	[514,758]	7	8
F_{10}	[2,2]	[9,9]	[1,1]	[891,981]	5	5
F_{11}	[2,2]	[10,10]	[1,1]	[976,996]	4	4
F_{12}	[2,2]	[10,10]	[4,4]	[976,996]	4	4
F_{13}	[7,7]	[10,10]	[10,10]	[996,1000]	2	2
F_{14}	[9,9]	[10,10]	[9,9]	[982,1000]	3	3
F_{15}	[10,10]	[10,10]	[10,10]	[999,1000]	1	1

In traditional RPN model, the usefulness of *Axiom 3.1* can be explained with $RPN_1 = \left[[1,1],[2,2],[1,1]\right]$, $RPN_3 = \left[[1,1],[3,3],[1,1]\right]$, and $RPN_4 = \left[[1,1],[5,5],[1,1]\right]$, as well as $RPN_1 = [94,179]$, $RPN_3 = [219,385]$, and $RPN_4 = [488,543]$. *Axiom 3.1.1* suggests if RPN_4 is greater than RPN_3 (a *strong order*), and RPN_3 is greater than RPN_1 (a *strong order*), then, RPN_4 should be greater than RPN_1. In this example, an f_{RPN} that satisfies *Property 3.1* is used, in which $\underline{RPN} = 1$ and $\overline{RPN} = 1000$. Therefore, *Axiom 3.1.1* is satisfied, which is in agreement with *Proposition 5.1*. *Axiom 3.1.2* suggests that RPN_3 should be greater than RPN_1. This is because $s_3 \geq s_1$, $o_3 \geq o_1$, and $d_3 \geq d_1$. With an f_{RPN} that satisfies *Property 3.1*, *Axiom 3.1.2* is satisfied, which is in agreement with *Proposition 5.2*. *Axiom 3.1.3* suggests that the relative order of F_3 and F_4 remains the same, i.e., F_4 is greater than F_3, when a new failure mode, e.g., F_1, is added. Again, with an f_{RPN} that satisfies *Property 3.1*, *Axiom 3.1.3* is true, which is in agreement with *Proposition 5.3*. Even through *Axiom 3.1* is useful, it does not explain F_6, F_7, and F_9 have the same prioritization outcomes in interval fuzzy inference system RPN model even the RPN_6, RPN_7, and RPN_9 are not the same. Therefore, the proposed interval method needs to be employed to tackle this limitation. Using interval-valued RPN model, $r_6 = 9$, $r_7 = 7$, and $r_9 = 8$, i.e., F_7 are ordered to have the higher priority than F_9, followed by F_6.

In interval-valued RPN model, the usefulness of *Axiom 3.1* can be explained with $RPN_1 = \left[[1,1],[2,2],[1,1]\right]$, $RPN_3 = \left[[1,1],[3,3],[1,1]\right]$, and $RPN_6 = \left[[1,1],[7,7],[1,1]\right]$, as well as $RPN_1 = [94,179]$, $RPN_3 = [219,385]$, and $RPN_6 = [532,736]$. *Axiom 3.1.1* suggests if RPN_6 is greater than RPN_3 (a *strong order*), and RPN_3 is greater than RPN_1 (a *strong order*), then, RPN_6 should be greater than RPN_1. In this example, the four possible orderings among RPN_i are considered, in which RPN_6 is greater than RPN_1 (a *strong order*). Therefore, *Axiom 3.1.1* is satisfied, which is in agreement with *Proposition 5.1*. *Axiom 3.1.2* suggests that RPN_6 should be greater than RPN_1. This is because $s_6 \geq s_1$, $o_6 \geq o_1$, and $d_6 \geq d_1$. With an f_{RPN} that satisfies *Property 3.1*, *Axiom 3.1.2* is satisfied, which is in agreement with *Proposition 5.2*. *Axiom 3.1.3* suggests that the relative order of F_6 and F_3 remains the same, i.e., RPN_6 is greater than that of RPN_3, when a new failure mode, e.g., F_1, is added. Similarly, with an f_{RPN} that satisfies *Property 3.1*, *Axiom 3.1.3* is true, which is in agreement with *Proposition 5.3*.

5.6 Comparison among Other Approaches

In this section, Table 5.7 shows the comparison among other approaches, i.e., the traditional RPN methodology (Stamatis, 2003), (Mikulak et al., 2008) and the data elicitation approach (Bradley & Guerrero, 2011) with our proposed methodology because the discussed approaches is based on crisp assessment. The traditional RPN model could not handle missing risk ratings, but the data elicitation method and the proposed method do. Besides, the proposed method is better than that of the data elicitation method, for able to handle confidence level and different types of assessment information simultaneously.

It is also worth noting that the benchmark examples from Murray, 2013 and Kerk et al., 2015 does not provide additional (imprecise) information. Thus, other methods tabulated in Table 2.5, are not comparable.

5.7 Summary

In this chapter, a new method to prioritize the interval-valued RPN score of failure modes and corrective actions that may consist of missing risk rating(s) is presented. The ordering of interval-valued risk ratings and RPN score is analysed to ensure it always satisfied the principles for prioritization of failure modes and corrective actions. Besides, two benchmark examples from (Murray, 2013) and (Kerk et al., 2015) are presented to demonstrate the strength of proposed method. In the next chapter, concluding remarks will be presented.

Table 5.7: Comparisons of theories used in FMEA with respect to their ability to deal with missing risk ratings

Approach based on	Studies on FMEA	Can handle missing risk ratings	Can aggregate different assessments from a group of FMEA users	Considers the confidence level of FMEA users in their subjective assessment	Can deal with different types of assessment information simultaneously	Prior information
RPN methodology	(Stamatis, 2003), (Mikulak et al., 2008)	No	No	No	No	Partly (equal weights assumptions)
Data elicitation	(Bradley & Guerrero, 2011)	Yes	No	No	No	Partly (equal weights assumptions)
Proposed methodology	-	Yes	No	Partly	Partly	Partly (equal weights assumptions)

CHAPTER 6

CONCLUSION AND SUGGESTIONS FOR FUTURE WORKS

6.1 Conclusion

In a nutshell, the objectives of this research are achieved. Firstly, three principles to prioritize failure modes and corrective actions, i.e., *transitivity*, *pareto optimality*, and *independence of irrelevant alternatives*, have been introduced, initially for crisp risk ratings, and extended for interval-valued risk ratings. Two important properties for the RPN model, i.e., boundary conditions and (strong) monotonicity property, have been analysed and discussed. The monotone property of the RPN model is important in order to satisfy the *pareto optimality* axiom.

Secondly, a new method to represent failure modes and corrective actions for FMEA to be prioritized, as a nested vector or a tree model, have been proposed. Four benchmark examples from Murray (2013), Stamatis (2003), Calixto (2016), and Guimarães & Lapa (2004) are presented to illustrate the usefulness of proposed representations.

An interval method to solve the missing S, O, and/or D ratings in FMEA is presented based on the three principles for prioritization of failure modes and corrective actions. Methods to prioritize interval-valued risk ratings and RPN score have been described and analysed to ensure it is always satisfied the three principles. In addition, two case studies with benchmark information from Murray (2013) and Kerk et al. (2015) has been conducted, and the results ascertain the effectiveness of our proposed method pertaining to rationality in prioritization of failure modes.

6.2 Suggestions for Future Work

For future work, extension of the principles to various types of rough and fuzzy risk ratings (Can, 2017; Chanamool & Naenna, 2016; Song, Ming, Wu, & Zhu, 2014) will be examined. In addition, application of the proposed method to real-world FMEA case studies, e.g., Healthcare Failure Mode and Effect Analysis (HFMEA) and Failure Mode, Effects, and Criticality Analysis (FMECA) will be conducted.

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APPENDICES

Appendix 1: An Example of A Design FMEA in (Murray, 2013) (pages 12 – 16)

F_i	Item Description / Function	Potential Failure Mode	Effects of Failure	S	Potential Causes(s) of Failure	O	Control	D	RPN
F_1	Assembly Spectral Photometer System, CD03060000, Highest level assembly of the Spectral Photometer	Parts will not fit together / mount properly in location	Inability to assemble / install correctly	3	Assembly components are undersized / oversized / mis located / missing.	1	Tolerance Analysis Completed, Inspection Procedures	1	3
F_2	Assembly Spectral Photometer System, CD03060000, Highest level assembly of the Spectral Photometer	Hardware falls from tower	Safety issue / Loss or invalid data	5	1) Insufficient Material Strength; 2) Service / Installation Over Torque-Breakage, Under torque – Loss of hardware); 3) Corrosion; 4)Stress Fatigue / Wear; 5) Mechanical damage to assembly – environmental.	2	1) Material Analysis & Callout; 2) Torques / pattern specified on drawing; 3) Material compatibility analysis; 4) Material analysis, inspection; 5) Upon failure, hardware may be secured by cable / locking hardware.	1	10
F_3	Assembly, Spectral Photometer Control Mounting System, CD03060300, Mount for sensor control enclosure	Parts will not fit together / mount properly in location	Inability to assemble / install correctly	3	Assembly components are undersized / oversized / mis-located / missing.	1	Tolerance Analysis Completed, Inspection Procedures	1	3

	and associated hardware (w/ East shield) (10)								
F_4	Assembly, Spectral Photometer Control Mounting System, CD03060300, Mount for sensor control enclosure and associated hardware (w/ East shield) (10)	Assembly not properly aligned or damaged	Loose hardware, cable strain, data loss	4	1) Assembly components are undersized / oversized / mis-located / missing; 2) Hardware failure (strength, torque, corrosion, fatigue); 3) Mechanical damage to assembly – environmental.	2	1) Tolerance Analysis Completed, Inspection Procedures; 2) Material compatibility and strength analysis; 3) Procedures in place for install / remove / maintenance.	1	8
F_5	Assembly, Spectral Photometer Control Mounting System, CD03060300, Mount for sensor control enclosure and associated hardware (w/ East shield) (10)	Assembly / components fall from tower	Loss or invalid data / Safety Issue	5	1) Insufficient Material Strength; 2) Service / Installation Over Torque-Breakage, Under torque – Loss of hardware); 3) Corrosion; 4)Stress Fatigue / Wear; 5) Mechanical damage to assembly – environmental.	2	1) Material Analysis & Callout; 2) Torques / pattern specified on drawing; 3) Material compatibility analysis; 4) Material analysis, inspection; 5) Upon failure, hardware may be secured by cable / locking hardware.	1	10
F_6	Assembly, Spectral Photometer Robots, Mount, CD03060200, Corner mount arm / bracket and azimuth robot (20)	Parts will not fit together / mount properly in location	Inability to assemble / install correctly	3	Assembly components are undersized / oversized / mis-located / missing.	1	Tolerance Analysis Completed, Inspection Procedures	1	3

F_7	Assembly, Spectral Photometer Robots, Mount, CD03060200, Corner mount arm / bracket and azimuth robot (20)	1) Assembly not properly installed / aligned; 2)Motor Assembly internally fails / damaged; 3)Cables damaged	Data loss / invalid data	4	1) Assembly components are undersized / oversized / mis-located / missing; 2) Hardware failure (strength, torque, corrosion, fatigue); 3) Mechanical damage to assembly – environmental; 4) Improper routing of cabling.	2	1) Tolerance Analysis Completed, Inspection Procedures; 2) Material compatibility and strength analysis; 3) Procedures in place for install/removal/maintenance; 4) Cables routed away from potential pinch/shear points and anchored. Strain relief in place.	1	8
F_8	Assembly, Spectral Photometer Robots, Mount, CD03060200, Corner mount arm / bracket and azimuth robot (20)	Robot parks in undesirable state (not at nadir)	Precipitation / animal contaminatio n enters collimators (Calibration traceability issue)	4	1) Power failure to assembly during automatic routine; 2) Disconnection / intermittent connection of power cables.	2	1, 2) Loss of power to a sensor/grape/POE switch *may be* detected by a SOH scheme. What will be checked and what will not be checked is TBD.	3	24
F_9	Assembly, Spectral Photometer Robots, Mount, CD03060200, Corner mount arm / bracket and azimuth robot (20)	Assembly/compo nent falls from tower	Loss or invalid data / Safety Issue	5	1) Insufficient Material strength; 2) Service / Installation (Over torque-Breakage, Under torque - Loss of hardware); 3) Corrosion; 4) Stress Fatigue/Wear; 5) Mechanical damage to assembly - environmental	2	1) Material Analysis & Callout; 2) Torques/pattern specified on drawing; 3) Material compatibility analysis; 4) Material analysis, inspection;	1	10

							5) Upon failure, components may be secured by locking hardware / cable		
F_{10}	Sensor Acsry CIMEL Head, 0303660002, Spectral Photometer head (collimator, zenith motor, optics) (30)	Parts will not fit together / mount properly in location	Inability to assemble / install correctly	3	1, 2) Damage to mounting surfaces; 3) Sensor head is not keyed when placed in clamp - can be mis-installed	1	1) COTS part, assembly and fit would have been checked during MFG. 2) Components packaged securely for shipping.	1	3
F_{11}	Sensor Acsry CIMEL Head, 0303660002, Spectral Photometer head (collimator, zenith motor, optics) (30)	Sensor broken / internally fails	Loss of data / invalid data	4	1) Cable migration / pinching; 2) Micro-switch for 'Park' position misadjusted; 3) Drive belts too loose (backlash gear adjustment); 4) Back-up nuts for motor arms loosen; 5) Inconsistent filter quality (supplier/lot effects).	3	COTS part, "Full Swap Out" Any field serviceable activities?	2	24
F_{12}	Sensor Acsry CIMEL Head, 0303660002, Spectral Photometer head (collimator, zenith motor, optics) (30)	Collimator field of view insufficient	Invalid data	4	1) Animal activity (bird droppings, spiders, leaves, nest(?)); 2) Human activity (people on tower top during measurement, new buildings in area); 3) Canopy height growth over time.	4		3	48

F_{13}	Sensor Acsry CIMEL Head, 0303660002, Spectral Photometer head (collimator, zenith motor, optics) (30)	Sensor head / component falls from tower	Loss or invalid data / Safety Issue	5	1) Insufficient Material strength; 2) Service/Installation (Over torque-Breakage, Under torque - Loss of hardware); 3) Corrosion; 4) Stress Fatigue/Wear; 5) Mechanical damage to assembly – environmental.	1	1) Material Analysis & Callout; 2) Torques/pattern specified on drawing; 3) Material compatibility analysis; 4) Material analysis, inspection; 5) Upon failure, components may be secured by locking hardware / cable.	2	10
F_{14}	Assembly, Spectral Photometer Control System, CD03060310, Enclosure and components for Spectral Photometer control and DAQ (40)	Parts will not fit together / mount properly in location	Inability to assemble / install correctly	3	Assembly components are undersized / oversized / mis-located / missing.	1	Tolerance Analysis Completed, Inspection Procedures	1	3
F_{15}	Assembly, Spectral Photometer Control System, CD03060310, Enclosure and components for Spectral Photometer control and DAQ (40)	Assembly falls from tower	Loss or invalid data / Safety Issue	5	1) Insufficient Material strength; 2) Service/Installation (Over torque-Breakage, Under torque - Loss of hardware); 3) Corrosion; 4) Stress Fatigue/Wear; 5) Mechanical damage to assembly – environmental.	2	1) Material Analysis & Callout; 2) Torques/pattern specified on drawing; 3) Material compatibility analysis; 4) Material analysis, inspection; 5) Upon failure, components may be secured by locking hardware / cables.	1	10

F_{16}	Assembly, Spectral Photometer Control System, CD03060310, Enclosure and components for Spectral Photometer control and DAQ (40)	Corrosion on unprotected electrical components inside enclosure	Loss of data / invalid data	4	Enclosure moisture seal points are insufficient / degrade	2	1) Box was initially NEMA4 rated;	2	16
F_{17}	Sensor Acsry CIMEL Electronic Box, 0303660003, Control box for Spectral photometer (white)	Spectral Photometer doesn't GoSun correctly	Will it still use the Quadcell to track and find the sun even if grossly out of position?	3	1) Incorrect lat/long setting; 2) Incorrect Day/time setting.	2	1) Lat /long will be verified as part of acceptance. What happens with replacement boxes? 2) LC sets time of day - should always be correct to 1 second	2	12
F_{18}	Sensor Acsry CIMEL Electronic Box, 0303660003, Control box for Spectral photometer (white)	Daughter interface board becomes disconnected	Loss of signal/control of instrument. Loss of data/damage to sensor	4	Thermal cycling	3	1) Unplug / re-plug in sensor cable and connector panel	1	12
F_{19}	Sensor Acsry CIMEL Electronic Box, 0303660003, Control box for Spectral photometer (white)	Battery charge insufficient (4.8V)	Loss of data	4	Internal battery failure	3	1) Preventive Maintenance during Annual Calibration	1	12

F_{20}	Sensor Acsry CIMEL Electronic Box, 0303660003, Control box for Spectral photometer (white)	Known' bug on Cimel units model CE318, LCD window displays only `noise' and the unit is not operational.	Sensor won't function, loss of data	4		3	1) Swap out Control box	1	12
F_{21}	Sensor Acsry CIMEL Electronic Box, 0303660003, Control box for Spectral photometer (white)	Internal Storage fills up	Missing data	4	1) Extended loss of power to DAS; 2) DAS comm. issue.	3	1) External Site generator can be brought to site	1	12
F_{22}	Assembly, Spectral Photometer Wetness probe, CD03060400, detects wetness to prevent photometer collimator from filling with precipitation (60)	Sensor fails to detect precipitation event	invalid data, potential equipment damage (water build-up in collimators)		1) Gland nut looses, probe rotates/faces down2) Probe blocked from seeing moisture from foreign object (bird droppings, spider web, leaves, etc.)3) Sensor fails (how?)				0
F_{23}	Assembly, Spectral Photometer Wetness probe, CD03060400, Detects wetness to prevent photometer collimator from filling with precipitation (60)	Sensor incorrectly reports precipitation event	Loss of data (sensor never attempts data acquisition)	4	1) Excessive humidity / condensation in area (Is probe generally exposed to line of sight of sun?)				0

F_{24}	Assembly, Shield West Spectral photometer, CD03200000, radiation shield for control enclosure (50)	Parts will not fit together / mount properly in location	Inability to assemble / install correctly	3	Assembly components are undersized / oversized / mis-located / missing.	1	Tolerance Analysis Completed, Inspection Procedures	1	3
F_{25}	Assembly, Shield West Spectral photometer, CD03200000, radiation shield for control enclosure (50)	Assembly falls from tower	Exposure of Control Enclosure to direct sunlight / Safety issue	5	1) Insufficient Material strength; 2) Service/Installation (Over torque-Breakage, Under torque - Loss of hardware); 3) Corrosion; 4) Stress Fatigue/Wear; 5) Mechanical damage to assembly - environmental	2	1) Lanyard secures shield to main assembly	1	10
F_{26}	Bridge, Ethernet to Serial with PoE, XXXXXXXXXX, converts Serial to Ethernet (short term solution until Grape is enabled to do this)	Bridge damaged / broken	Loss of data	4	1) Internal failure (MTBF)	2	1) COTS part; 2) MFG testing.	1	8
F_{27}	PoE splitter, 48VDC output to both Grape and Bridge	Splitter damaged broken	Loss of data	4	1) Internal failure (MTBF)	2	1) COTS part; 2) MFG testing.	1	8

Appendix 2: A Design FMEA for Cooling Fan Assembly in (Stamatis, 2003) (pages CD-98 – CD-100)

F_i	Part name/ part no.	Product function	Potential failure mode	Potential effect(s) of failure mode	Potential cause(s) of failure	Existing conditions				
						Current controls	Occurrence	Severity	Detection	RPN
F_1	Motor	Provide mechanical power to fans; position fans within shroud	Fan vibration from imbalance and axial T.I.R.	Audible noise, vibration; increased motor wear	Fan centre of gravity off axis of rotation; axial T.I.R. causes 2-plane imbalance	Design lightweight fan with min. band mass; part thickness to favour uniform mold flow. DV tests on vehicles to assess sensitivity to vibration inputs	5	5	4	100
F_2			Motor burnout, bearing or brush failure	Loss of cooling and A/C function	Overheating, lack of air circulation	Vent holes in motor case; fins in fan hub pull air through ES, durability tests	2	5	5	50
F_3			Misassemble to shroud, off-center or cooked	Loss of cooling function	Fan contact shroud, noise or motor burn-out	Design for easy assembly, accurate positioning in shroud	2	7	3	42
F_4			Assemble @ $\pm 120^{\circ}$ off-normal angle, motor wire in wrong location	No-build condition in assy. plant	Symmetrical spacing of screw holes; nonunique mounting interfaces	Power motion motor has unique mounting configuration. Visual inspection during assembly	6	6	3	108

				Motor wire interferes with engine and shorts; wire is too tight and opens circuit		Power motion motor has unique mounting configuration. Visual inspection during assembly	6	6	5	180
F_5			Fan retainer compression too high	Retainer loose, lost, loose fan and lost cooling function	"e" clip too close to shaft and slot	Maintain dimensional capability within limits of tolerance stack-up	3	6	5	90
F_6			Fan retainer compression too low or nonexistent	Retainer loose, excess play between slot and "e" clip; breakage; lost cooling	"e" clip too far from shaft and slot	Maintain dimensional capability within limits of tolerance stack-up	3	6	5	90
F_7			Backplate not swaged securely	Motor & fan loose; fan touches shroud, stalls; fan damages coil; loss of cooling, coolant	Designed interference of swaged tangs not sufficient; swaging process not capable	Swage method revised; pull out tests; DV testing; ES tests	2	6	7	84
F_8	Screws-motor mount	Attach motor to shroud	Loosen after extended service	Loose motor allows fan contact with shroud, stalling motor	Plastic creep relieves screw tension, allows back-out	Maintain dimensional capability within limits of tolerance stack-up	2	7	5	70
F_9			Misapplication	Loose motor allows fan contact with shroud, stalling motor	Drive in crooked, miss hole or not fully tight	Counterbore in screw-hole, torque-to-fail much higher than pre-vailing	2	7	3	42
F_{10}			Plastic threads strip out	Loose motor, fan contact with shroud	Shroud mount boss splits	Counterbore, optimize boss diameter and height	2	7	4	56

F_{11}	Fan retainer	Attach fan and torque plate to motor	Come loose after installation	Loose fan, contact w/coil, loss of cooling	Incomplete installation, not fully snapped on	Design motor and fan for easy clip installation w/tactile snap on lock	3	5	4	60
				T.I.R. increased; vibration or noise; fan dismounts, loss of cooling	Retainer not keyed with respect to flats on motor shaft	ES and DV testing	2	7	7	98
F_{12}	Torque plate	Transmit torque from motor to fa	Motor shaft deforms flats in hole	Fan does not reach full RPM, fan bore wears. Loss of cooling, vibration from loose fan	Torque plate hole oversized; steel not proper thickness or hardness	Maintain dimensional capability within limits of tolerance stack-up, ES tests	3	6	7	126
F_{13}			Torque plate not fully seated	Engagement of flats is reduced torque plate hole enlarged from stress	Close tolerances of fan pockets and legs of plate	Make assy. retainer on unseated plate difficult	3	5	7	105

Appendix 3: A FMEA for a Welding Process from (Calixto, 2016) (page 166)

F_i	Item / Function	Potential Failure Mode(s)	Potential Effect(s) of Failure	S	Potential Cause(s) / Mechanism(s) of Failure	O	Current Design Controls	D	RPN	Recommended Action(s)	Responsibility and Target Completion Date
	Seals										
F_1	Sensor mount. seal	Loosen during sensor assembly / service	Leakage	6	Fitting not held in place	1		1	6	New fitting design. Prototype validation	Reliability engineer
F_2	Sensor mount. seal	Damaged internal thread	Cannot install sensor	6	Damaged during installation or transportation	1		1	6	Quality control in installation and transportation	Quality supervisor
F_3	Sensor mount. seal	Damaged external thread	Cannot install wire nut	3	Damage during shipment to customer	2		1	6	Quality control in shipment	Logistic supervisor
F_4	Hose connection	Crack / break burst. Bad seal poor hose quality	Leak	7	Over pressure	7	Burst, validation pressure cycle	1	49	Test incuded in prototype and production validation testing	Reliability engineer
			Failed mount	4	Vibration	8	Vibration w/road tapes	3	96	Obtain vibration road tape	Quality supervisor
			Hose leak	6	Over pressure	5	Burst, validation pressure cycle with clamps	2	60	Obtain clamps and clamping specification	Quality supervisor
F_5	Heat transfer structure	Stress crack	Leak. Loss of heat transfer	7	Wicking. Material strength	6	Thermal cycle	1	42	Included in product specification	Quality supervisor

F_6	Heat transfer structure	Corrossion	Leak. Loss of heat transfer	7	Coolant quality. Contamination. Environment – int/ext	6	Service simulation coolant evaluation	5	210	Supplier coolant to be evaluated	Reliability engineer
F_7	Heat transfer structure	Steam fail	Leak. Lost of heat transfer	4	Environment – int/ext	1	Service simulation	1	4	Included in product specification	Quality supervisor

Appendix 4: Chemical Volume Control System FMEA from (Guimarães & Lapa, 2004) (page 206)

Component	F_i	Failure Mode	Failure Causes	Failure Effect	Failure Detection Methods	O	S	D	RPN	RPN Ranking	Fuzzy RPN	Fuzzy RPN Ranking
Let-down Flow Control Valves	F_1	Fails open	Mechanical Binding	Loss of redundancy. Unable to terminate let-down flow.	Remote valve position indication. Downstream flow and temperature indicator.	3	5	5	75	22	3	4
	F_2	Fails close	Loss of air or electrical power. Spurious signal	Loss of redundancy. Loss of normal let-down flow path through regenerative heat exchanger.	Remote valve position indication. Let-down flow and pressure indicators.	5	8	8	320	2	5	2
Regenerative Heat Exchanger	F_3	Plugged tubes	Corrosion product build-up. Born build-up. Foreign material in RCS.	Reduced let-down flow.	Flow indicator	4	5	3	60	25	2	5
	F_4	Insufficient heat transfer	Scale build-up on tubes.	Temperature of let-down flow may exceed design limits, resulting in possible damage to down-stream components,	Regenerative heat exchange outlet temperature indicators.	4	5	3	60	25	2	5

	F_5	External leakage	Casing crack. Vent valve seat leakage,	Reduced let-down floe. Primary coolant release.	Excessive makeup flow rate. Containment radiation monitors	3	4	5	60	25	5	2
	F_6	Tube leakage	Corrosion. Manufacturing.	No effect		3	4	5	60	25	5	2
Orifice Isolation Valves	F_7	Fails open	Mechanical binding	Loss of redundancy. Loss of normal let-down flow part.	Remote valve position indicator. Let-down flow and pressure indicators.	3	8	7	168	8	5	2
	F_8	Fails closed	Loss of air or electrical power. Spurious signal	Blockage of flow to VCT.	Remote valve position indicator. Let-down flow and pressure indicators.	5	10	10	500	1	9	1

List of Publication

Book Chapter

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