

# Estimating Non-Conformance Using the Modified Tolerance Region Method and the Target Distance Method

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## Abstract

In many cases, the quality of a manufactured product is determined by more than one characteristic and often, these quality characteristics are correlated. A number of methods for dealing with quality evaluation of multivariate processes have been proposed in the literature. However, some of these studies do not consider correlation among quality characteristics. In this paper, two new approaches for estimating the proportion of non-conformance for correlated multivariate quality characteristics with nominal specifications are proposed: (i) the modified tolerance region approach and (ii) the target distance approach. In the first approach, the  $p$  number of correlated variables are analysed based on the projected shadow of the  $p$ -dimensional hyper ellipsoid so that the ability to visualise the tolerance region and the process region for  $p > 2$  is preserved. In the second approach, the correlated variables are combined and a new variable called the target distance is introduced. The proportion of non-conformance results estimated using both methods were used to compute the multivariate capability index and the total expected quality cost. This study also suggest modification to the  $NMC_p$  index as proposed in Pan and Lee (2010) such that the process capability for  $p > 2$  can be measured correctly. The application of both approaches is demonstrated using two examples and it is shown that both methods i.e. the modified tolerance region and the target distance methods are capable of estimating the capability of multivariate processes.

**Keywords:** Multivariate quality control; Correlated characteristics; Tolerance region; Proportion of non-conformance; Mahalanobis Distance

## 1. Introduction

The common goals for any profit-based manufacturing companies are to have financial controls and to gain substantial share in the global market. One of the important fiscal components in many manufacturing companies is the quality costs. These costs can be categorised into four elements i.e. prevention costs, appraisal costs, internal failure costs and external failure costs [1]. Prevention costs are costs associated to activities required in avoiding occurrence of poor quality at design and production stage. Appraisal costs are costs due to monitoring and inspection of incoming materials, products, consumables and equipment. When non-conformance of products is detected within the vicinity of the manufacturing site, activities such as rework, scrap, retest and machine unscheduled downtime may be required and these incur internal failure costs. External failure costs are costs associated to the negative financial impact that occurs after the end product is procured by the customer such as warranty claims, product recall and loss of market share. External failure costs are difficult to quantify, however the opportunity to identify and mitigate these external costs can provide significant benefit to the manufacturer.

[2] defined quality from the customers' perspective and highlighted that loss incurs on the society when a quality characteristic deviates away from its target value. This definition of quality relates the quality loss to the internal and external failure costs, thus advocates that these costs can be estimated using the quality loss function (LF). When a product quality is defined by more than one

key quality characteristics, applying the univariate LF for individual quality characteristic to estimate total quality loss may results in erroneous conclusion, especially when correlation exists. Motivated by this need, this paper intends to answer the following research question: "How can we measure the probability of non-conformance for products with correlated multivariate quality characteristics so that the costs of quality failure can be estimated effectively?"

Therefore, the objectives of this paper are: (i) to review and analyse related models as proposed by past researches, (ii) to present two new models for estimating the proportion of non-conformance ( $P_{NC}$ ) for multivariate data, as developed in this study, and (iii) to estimate the internal and external failure costs based on the estimated  $P_{NC}$ . The following parts of this paper are organised as follows: Section 2 reviews some of the multivariate LF and multivariate process capability index (mPCI) published in the literature related to this study. Section 3 presents both the modified tolerance region (MTR) and the target distance (TD) methods developed in this study, Section 4 discusses the application and results obtained using both methods, and Section 5 concludes this paper.

## 2. Literature Review

### 2.1. Multivariate Loss Functions

Earlier studies on multivariate LF neglects the factor of correlation among the quality characteristics and mostly suggest that the total