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# On the Monotonicity Property of the TSK Fuzzy Inference System: The *Necessity of the Sufficient Conditions* and the *Monotonicity Test*

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**Abstract** The *sufficient conditions* for satisfying the monotonicity property of the Takagi–Sugeno–Kang (TSK) fuzzy inference system (FIS) have shown to be useful in many different applications. However, the related *sufficient and necessary condition*s are still unknown. As such, even when the sufficient conditions are violated, the TSK FIS model may still able to satisfy the monotonicity property. Therefore, the *monotonicity test* is used as an approximated method to determine the validness of the monotonicity property. To the best of our knowledge, the use of the monotonicity test in FIS is new. In this paper, we focus on single-input zero-order TSK FIS with Gaussian fuzzy membership functions. An algorithm to test the monotonicity property, either accepting or rejecting an TSK FIS model of being monotone, is devised and analyzed. The relationship between TSK FIS and its capability of satisfying the monotonicity property, along with the sufficient conditions and the outcome of the monotonicity test, is established through a Monte Carlo simulation. The Monte Carlo simulation is a useful *necessity test* for the sufficient conditions. We define a *necessity measure of the sufficient conditions* (*NMSC*) as the probability that a randomly generated monotone TSK FIS model (evaluated based on the monotonicity test algorithm) satisfies the sufficient conditions. We empirically show that the NMSC score reduces with increasing number of fuzzy rules. In addition, an application of the TSK FIS model to failure modes and effects analysis is demonstrated. As compared with the sufficient conditions, a better FIS-based model with a lower error measure can be obtained using the monotonicity test. The outcome indicates the effectiveness of the monotonicity test for designing low-dimensional TSK FIS models with large numbers of fuzzy rules.

**Keywords** Takagi–Sugeno–Kang fuzzy inference system · Monotonicity property · Monotonicity test · Sufficient conditions · Failure mode and effects analysis · Fuzzy occurrence model

## 1 Introduction

A fuzzy inference system (FIS) has been shown to be a universal approximator, i.e., the ability to approximate an arbitrary continuous function within any given degree of accuracy [1–5]. The use of FIS to approximate monotone functions has been reported, and the importance of the monotonicity property pertaining to an FIS model has been highlighted in a number of recent publications. These include the development of mathematical conditions with respect to the monotonicity property of single-input rule modules connected FIS (SIRM-FIS) [6, 7], Mamdani FIS [8], Takagi–Sugeno–Kang (TSK) FIS [9, 10], and interval type-2 FIS [11]. As an example, a set of *sufficient conditions* for TSK FIS to satisfy the monotonicity property was derived in [9]. The sufficient conditions were demonstrated to be useful governing equations in real-world applications

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