Why is teaching mathematics so difficult?

By Dr Hong Kian Sam

CAMPUS Line
EVERY FRIDAY

Thought 1

ASK a reasonably large random selection of people this question: *Using the letters B and G, express the following statement algebraically: 'In this class there are twice as many boys as there are girls'.* You will find that even among 'the educated', somewhat more than half will come up with the incorrect expression. Don't comment on what they say. Just write it down.

Now ask them to translate the equation they wrote as literally as possible. This not the same thing as repeating the original information. Most likely they will have written either B=2G or G=2B They will usually say: *Boys equal two girls* or *Boys are twice the girls* or *Boys are two times the girls* or the reverse of these.

Now what does this tell you? It tells you that the letters B and G are not clearly identified in their minds as *numbers*. If they were, we would hear *The number of boys is equal to two times the number of girls* or something similar. And herein lies one major problem. **The symbols used in algebraic expressions are not clearly perceived as numbers, but rather as objects.**

Thought 2

A corollary of Thought 1 is that because algebraic symbols are not perceived as numbers, the missing operators [2G is better written 2·G, etc.] aren't missed and the algebraic expression 2G takes on a meaning of "two boys". This is not a numerical concept, but a physical one. The notion that the B in 2B is a representation of a number as opposed to "boys" is lost. There is no operation of multiplication involved. Maybe it would be better to leave the operator in.

Thought 3

Now, repeat your experiment with different people, but this time use this question: *Using the letters T and S, express the following statement algebraically: 'In this school there are twice as many students as teachers'.*

There will still be a high proportion of wrong answers; but not as high as before. One probable reason is the possibility that 'checking the answer' now exists. In the first version we had no way of knowing whether there were more boys in the class than there were girls; and so no attempt was made to compare the suggested algebraic expression with reality. Here, however, it would be reasonable to assume that there were a greater number of students than teachers, and so some people, will take the trouble to check the expression against what they know logically must be the case.

Thought 4

Suppose we present someone with the inverse of the problems discussed above. *Translate the algebraic statement B=2G into a plain English sentence.* Would the respondent now come up with a lucid sentence in which it is objectively clear that he sees B and G as numbers? Probably not and a lot of people will react to it by saying: *So what. I know that B and G and T and S represent numbers rather than things. Just because I don't express this in an unambiguous way doesn't mean that I don't understand it.*

However, we should be aware that the careless use of language in normal communication frequently leads to misunderstandings. Mathematics is also a language and unfortunately the careless use of misleading, inconsistent and sometimes incorrect terminology in school mathematics is quite common; and that this is at least part of the reason why there are so many problems with learning mathematics. What, one must ask, is the advantage in being careless with terminology in a field which is supposed to be the most precise of all the school subjects?

Here are some more examples on this topic. For instance, there doesn't seem to be much emphasis placed on the distinction between the symbol indicating addition, operation of addition, the result of addition and the sign associated with non-negative numbers. *Plus*, and, *add*, *sum* and *positive* are sometimes used interchangeably, as in 2 plussed with 3 is 5 and 2 summed with 3 is 5 as opposed to 2 added to 3 is equal to 5

"Addition" is the name of an operation. "Plus" is the name of the symbol. "Add" is the directive to carry out the operation. "Sum" or "total" is what you get. "And", despite common usage, has nothing to do with addition whatever! "2 and 3 is 5" simply makes no sense until you come to Boolean operations at which time it is classified as a false statement. Similarly, students refer to negative numbers as "minus". What possible justification can there be for not using the terms "subtract", "difference", "minus" and "negative" in the correct context. We have often seen this sort of thing: 2 - 3 = - 1 [two minus three equals minus one].

All computer languages - except for BASIC which is long gone - make a distinction between the assignment operator "=" and the logic operator "=". In school mathematics, for whatever reason, this distinction is never made. In x = 3 and 2 + 3 = 5 the "=" sign is presumed to have precisely the same meaning.

Thought 5

In primary schools it is not uncommon to see an assortment of different people assigned to teach mathematics, even those clearly not prepared for it. There was a common conception particularly, among administrators, that mathematics is probably the easiest subject to teach at the primary level. Just about anyone could do it. They might not like it. But they can do it. However, the question that forms the title of this article suggests that this attitude is changing.

On the other hand, from the students' and parents' perspective, mathematics is seen not so much as a subject which is difficult to teach... but rather, one that is difficult to learn. In informal conversation with people, mention that you used to teach mathematics, the come-back is invariably "I was never very good at math", or words to that effect.

If one were to do a survey among students for the purpose of identifying their attitudes towards different school subjects, it would likely be found that mathematics had a relatively low 'satisfaction quotient'. There is also a perception that a disproportionate number of students who do well enough in other subject areas have difficulties with mathematics. Many students look on mathematics as a cross to bear rather than something, which is interesting or useful. At the same time everyone *knows* that mathematics is a 'very important subject'.

In the realities of the classroom rather than what ought to be, school mathematics is not really about understanding. It is about skills. A skill is defined in terms of being able to get the right answer to a particular type of 'problem'. What is involved is some sort of manipulation of algebraic symbols such as simplifying complex fractions. But being able to do something does not mean the same thing as understanding what one is doing. Furthermore, there are many different levels of understanding any one thing.

Though the term is frequently used in education circles and often by parents and students as well, there is no agreement among educators precisely what is meant by understanding. However, it is generally recognised that teaching for understanding, however it is defined, is much more difficult than teaching skills. Thus in school mathematics classes most activity is directed at skill building through drill practice exercises.

Thought 6

Another problem with school mathematics is that students are frequently unclear about how or what they are doing 'fits into the scheme of things'. There seems to be little opportunity to stand back and look at the overall view of what we are trying to do, what we have done, where we are going and why we are doing it. They cannot see the forest because there are so many trees.

School mathematics seems to consist of an endless series of topics amongst which there seems to be no obvious connection. It's easy enough for the mathematics teacher to rationalise here by stating that it is necessary to have this or that as a background before you can do, or even 'understand' such and such and so on. It can all be justified. But a student rarely sees the broader picture - and so the whole thing becomes a series of isolated rote type activities.

And where, when all is said and done, does it lead? What is the justification offered the student. "So that you can use mathematics to solve problems." What kind of problems? Let's look at the examples at the end of the chapters. For example: *If Ali has one more orange than Ahmad has and Ali and Ahmad have five oranges between them, how many oranges does Ali have?*

Is this what mathematics is about? How many problems in school mathematics are convincingly important enough for anyone to care about? Not many. And the reason is that the 'word problems' are selected to practise the 'theory' that has just been covered. Another approach would be to start with the problems - the justification for doing math - and then developing the math needed to deal with the problem. In other words, perhaps we should reverse the sequence of what we are doing. What do you think?

Thought 7

The word "equations" is another problematic word. Here is an example of an equation: 2 + 3 = 5. Here is another x = 7. This is *not* an equation: x + y = 7. This is a *relation*. One difference is that there are very few problems that result in equations - only puzzles.

Thought 8

The computer is arguably the most significant mathematics related invention in many centuries. It has made possible solution procedures to mathematical problems that are not only effective, but also much faster and easier than traditional approaches. In fact, much of what is covered in traditional school mathematics is rendered obsolete by computer based techniques. Here we are three decades into the personal computer revolution. Has the school mathematics curriculum acknowledged computers yet?

Thought 9

School mathematics is taught backwards. Consider the typical scenario. Introduction of the concept. Practice on the concept. At the end of the chapter some "word problems" the solution process to which involves applying the concept. From the general to particular.

Yet we all *know* that learning always takes place the reverse of this. From the particular to the general. Concepts cannot be 'taught' on schedule. Concept formation is a generalisation process. Generalisation from specific examples.

Conclusion

Student: *What do we have to learn this stuff for?*

Teacher: *So you will pass the course so that you will be able to do more of it next year*

Student: *What do we have to learn this stuff for?*

Teacher: *Unfortunately you haven't taken enough math to understand the answer. You will have to wait a couple of years. Then you will understand it.*

Student: *What do we have to learn this stuff for?*

Teacher: *You need it in your daily life*

Have you got a better answer? Is it true?

Dr Hong Kian Sam, is a lecturer in educational researc3h, ICT and mathematics education at the Faculty of Cognitive Sciences and Human Development (FSKPM), Universiti Malaysia Sarawak (Unimas). His research interests include learning processes in mathematics and statistics, ICT-supported learning, and evaluation of research instruments.