What is Algebra and why learn it?

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MANY mathematics reference and textbooks suggest strategies for helping secondary school students with mathematics topics they most often have difficulties with. In these texts, usually students are assumed to be helped effectively by giving them *rules* for how to deal with various situations.

For example, many pages are devoted to rules and procedures by which students can *get the right answer* when asked to factor quadratic expressions such as: Factor: $2x^2 + 10x + 12$

Some students find this topic more frustrating than others. The simple examples, such as the one above, were no problem; but for some reason the texts have all sorts of exercises that are not simple.

Why are we doing this? Is factoring complex quadratic expressions a worthy objective in its own right, or does it serve some further purpose? If the purpose of the exercise is to solve quadratic equations, is factoring the most sensible way to go about that? Keep in mind that no real-world problems lead to quadratic equations with integer coefficients - which are the only kind that can be factored.

Thus **"What is the purpose of solving equations?"** And the next question is, **"Why learn algebra?"**

The answers to these questions should be clearly stated. They should be subjected to, and be able to withstand, critical analysis.

An often given answer to the last question runs something like this: **The purpose of learning algebra is to extend students' repertoire of techniques for solving appropriate and plausible *word problems*.**

Word problems exclude symbolic "problems" such as 'find *x*' or 'simplify', etc. outside the context of some plausible real world situation. These are activities involving abstract symbol manipulations.

Whether symbol manipulations have any inherent value in their own right is debatable.

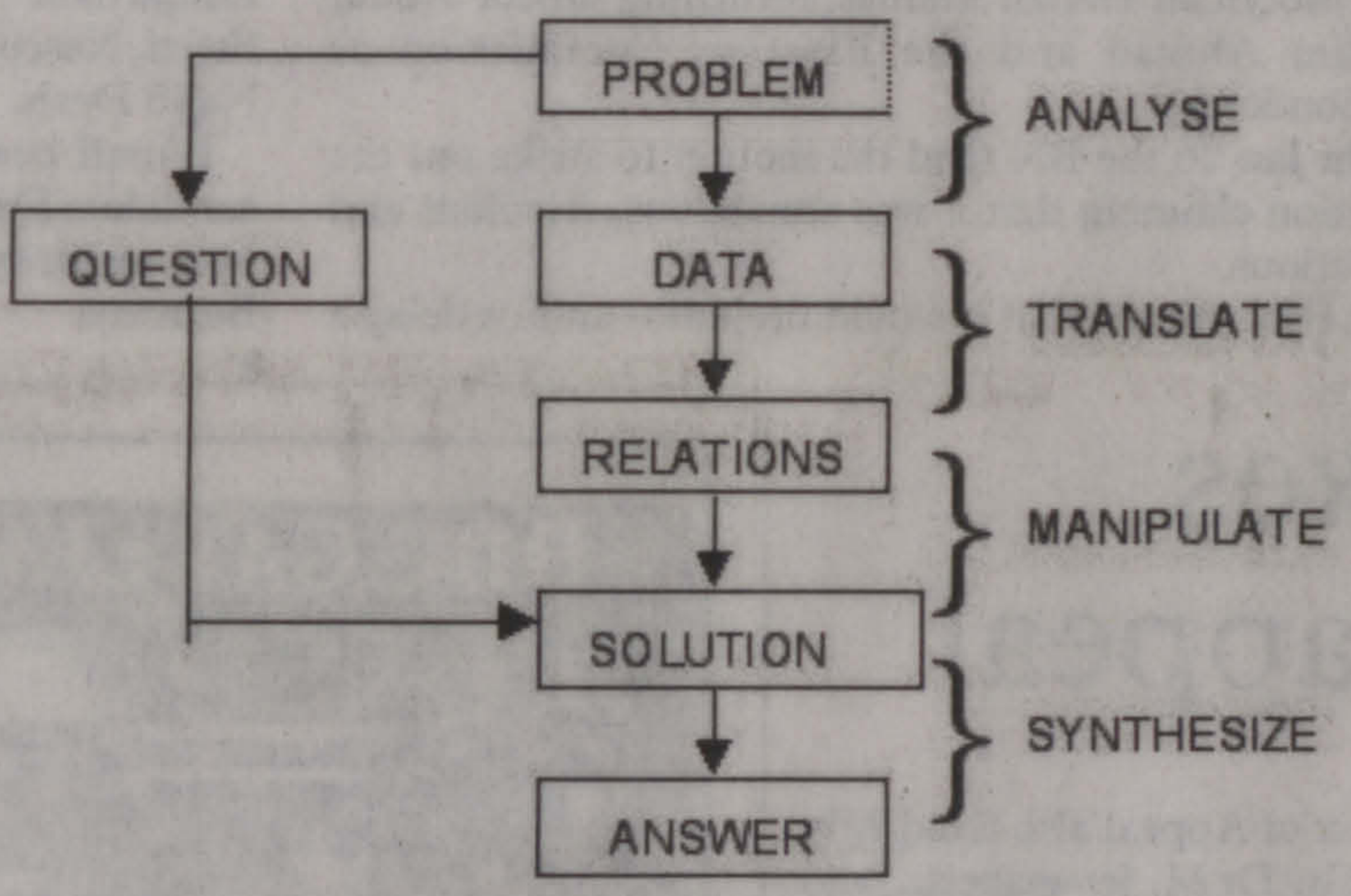
The standard algebra and calculus curricula in schools are examples of how we can lose track of where we are going and why we are going there.

Reading most textbooks and curricula descriptors and observing what students actually spend most of their time at, it would seem that the focus in school mathematics is on symbol manipulations rather than application.

In his book *The Pleasure of Finding Things Out*, Richard Feynman describes how he perceived Algebra as a child:

...a set of rules which if you followed them without thinking you could get the answer ... if you didn't know what they were trying to do.

Here is a proposed model for the **algebraic process**:



To illustrate the above model, consider the following two examples, one from elementary algebra and one from introductory calculus:

Algebra (problem)	Calculus (problem)
Ali and Ahmad have 5 apples between them. Ali has one more apple than Ahmad has. How many apples does each have?	If a rock dropped from a cliff is seen to strike the ground below after 10 seconds, how high is the cliff?

Stage 1

Process (**analysis**): This word means to take things apart. To separate something into its components.

Result 1 (**question(s)**): What do we eventually want to know?

Algebra (question)	Calculus (question)
How many apples does Ali have? How many apples does Ahmad have?	How high is the cliff?

Result 2 (**data statements**): These are unambiguous ways of expressing the essential data needed to answer the question.

Algebra (data)	Calculus (data)
The number of apples Ali has plus the number of apples that Ahmad has add up to five. The number of apples that Ali has is one more than the number of apples that Ahmad has.	The stone is in free fall for 10 seconds.

Stage 2

Process (**translation**): The data statements are translated into *algebraic statements*. In algebra, we use letter symbols to represent numbers or values. In many cases we need additional relation statements. For example, that theacceleration due to gravity is 9.8 m/s/s.

Result (**relations**):

Algebra (relations)	Calculus (relations)
B + S = 5 B = 1 + S	height = *integrate* (g * t) from t = 0 to 10 g = 9.8 m/s/s

Stage 3

Process (**manipulation**): The details of the various processes and techniques are discussed elsewhere. Symbolic approaches were developed as a matter of necessity at a time when calculating devices were not available; and their end result is a numeric solution to the problem. Numerical approaches are impractical when done by hand; but readily available software does either. Some people feel that calculators/computers should not be used.

Result (**solution**): Numerical values for the variables

Algebra (solutions)	Calculus (solutions)
S = 2 B = 3	height = 490 m

Stage 4

Process (**synthesis**): The word synthesise means to put together or assemble from component pieces. The solutions and the original question are synthesised to produce a final result - the answer to the question(s).

Result (**answer to the problem**):

Algebra (answer)	Calculus (answer)
Ali has two apples and Ahmad has three apples	The cliff is 490 m high.

Conclusions

Analysis and translation require human intelligence and knowledge. They require the ability to read, familiarity with the English language and its idioms, and they require familiarity with, and selection of, relevant properties of the systems under investigation.

As such, analysis can serve as a vehicle for learning about relations between all sorts of real world objects and phenomena. This activity has value in its own right, probably much more real educational value than the symbol manipulation which is the actual emphasis.

Unless the student understands the problem in the sense that he can analyse it as described, then proceeding to the solution stage serves no plausible purpose.

There are several methodologies for obtaining solutions. The traditional algorithms were developed in an age when numerical methods were impractical. They have some severe limitations and are not demonstrably superior from the pedagogic perspective.

On the assumption that what has been stated in the preceding paragraphs makes sense, then we are permitted to ask:

Why is it that the majority of time and effort in school mathematics algebra and calculus is devoted to symbol manipulation as opposed to problem analysis?" and,

Why do we teach symbolic, and not numerical solutions methods in school mathematics?

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