Data-Driven SIRM-Connected FIS for Prediction of External Tendon Stress

See Hung Lau\textsuperscript{1}, Chee Khoon Ng\textsuperscript{1a} and Kai Meng Tay\textsuperscript{2b}

\textsuperscript{1}Department of Civil Engineering, Faculty of Engineering, Universiti Malaysia Sarawak, 94300 Kota Samarahan, Sarawak, Malaysia
\textsuperscript{2}Department of Electronic Engineering, Faculty of Engineering, Universiti Malaysia Sarawak, 94300 Kota Samarahan, Sarawak, Malaysia

Abstract. This paper presents a novel harmony search (HS)-based data-driven single input rule modules (SIRM)-connected fuzzy inference system (FIS) for the prediction of stress in externally prestressed tendon. The proposed method attempts to extract causal relationship of a system from an input-output pairs of data even without knowing the complete physical knowledge of the system. The monotonicity property is then exploited as an additional qualitative information to obtain a meaningful SIRM-connected FIS model. This method is then validated using results from test data from the literature. Several parameters, such as initial tendon depth to beam ratio; deviators spacing to the initial tendon depth ratio; and distance of a concentrated load from the nearest support to the effective beam span are considered. A computer simulation for estimating the bond reduction coefficient $\Omega_u$ is then reported. The contributions of this paper is two folds; (1) it contributes towards a new monotonicity-preserving data-driven FIS model in fuzzy modeling and (2) it provides a novel solution for estimating the $\Omega_u$ even without a complete physical knowledge of unbonded tendons.

Keywords: bond reduction coefficient; externally prestressed tendon stress; harmony search; monotonicity index; single input rule modules (SIRM)-connected fuzzy inference system (FIS)

1. Introduction

Externally prestressed beam is a structural concrete member where the prestressing tendons are placed on the outside of the concrete section and are attached by anchors and deviators at discrete locations (Naaman and Alkhairi 1991a, Ng 2003). The idea of prestressing tendon placement (or sometime known as externally prestressing technique) has been growing rapidly in rehabilitating and strengthening existing structure due to progressive aging and corrosion of steel reinforcement (Ariyawardena and Ghali 2002, Naaman and Alkhairi 1991a, Ng 2003). Comparing to the conventional prestressing technique (i.e., bonded tendon), externally prestressing technique has some advantages, such as simpler to construct, easier to inspect and maintain (Naaman and Alkhairi 1991a, Ng 2003). Regardless of its popularity, the structural behaviour of externally

\textsuperscript{1}Corresponding author, Ph.D., E-mail: lauseehung@gmail.com
\textsuperscript{a}Professor., E-mail: ckg@feng.unimas.my
\textsuperscript{b}Senior Lecturer, Dr., E-mail: kmtay@feng.unimas.my
prestressed beam is still not fully understood due to its complicity compared to the conventional prestressing technique (Tan and Tjandra, 2007). The study of externally prestressed beam is difficult because of the increase of stress in the external tendons which depends on the overall deformation of a beam member that varied over the entire length of the beam. This causes the eccentricity of the unbonded tendons to vary under loading i.e., second-order effects (Alkhairi and Naaman 1993, Ariyawardena and Ghali 2002, Mutsuyoshi et al. 1995, Naaman and Alkhairi 1991a, b, Ng and Tan 2006a) and it involved complicated numerical analysis (Alkhairi and Naaman, 1993, Mutsuyoshi et al. 1995, Rao and Mathew, 1996).

The common approach used for estimating the stress in externally prestressed tendon at ultimate, \( f_{psf} \), for an externally prestressed beam is to determine the stress increase caused by an external loading(s), \( \Delta f_{ps} \), beyond the effective prestress, \( f_{pe} \), i.e., \( f_{ps} = \Delta f_{ps} + f_{pe} \) (Alkhairi and Naaman 1993, Naaman and Alkhairi 1991a). Since the member analysis method involved complicated analysis, it can be further simplified by a “pseudo-section analysis” by considering the bond reduction coefficient, \( \Omega_u \) (Alkhairi and Naaman 1993, Mutsuyoshi et al. 1995, Naaman and Alkhairi 1991a, Ng 2003). A search in the literature reveals that efforts to predict an accurate \( \Omega_u \) (via analytical approaches and/or by experiment), either directly or indirectly, have been reported. A number of parameters have been identified to have contributed to \( \Omega_u \); e.g., concrete strength, \( f'c \), area of prestressed reinforcement, \( A_{ps} \), area of non-prestressed reinforcement, \( A_s \), span to depth ratio, \( L/d_{p0} \), the effective prestressed, \( f_{pe} \), ratio of initial tendon depth to beam depth, \( d_{p0}/h \), the ratio of deviators spacing to the initial tendon depth, \( S_d/d_{p0} \), ratio of the distance of a concentrated load from the nearest support to the effective beam span, \( L_s/L \) and so on (Harajli et al. 1999, Mutsuyoshi et al. 1995, Naaman and Alkhairi 1991b, Ng 2003). For details, refer to Figs. 1-2.

Fig. 1 Strain and stress distributions at critical section of an externally prestressed beam at ultimate flexural limit state. (Ng 2003)
To predict $f_{ps}$ at ultimate flexural failure, numerical technique (Pisani, 2009), nonlinear analysis (Dall’Asta et al. 2007, Zona et al. 2009), rational analysis (Ozkul et al. 2008), and finite element analysis (Lou and Xiang 2006, Sivaleepunth et al. 2006, 2007) have been adopted to estimate $\Omega_u$ analytically. However, it was argued that the aforementioned analytical solutions maybe tedious due to the second-order effects (Harajli et al. 1999, Mutsuyoshi et al. 1995) and maybe inconsistent with actual test values (He and Liu 2010, Nataraja et al. 2006, Sivaleepunth et al. 2006). Many tests had been carried out to estimate $f_{ps}$, that is, indirectly estimating $\Omega_u$. Various statistical tools, e.g., linear regression (Naaman and Alkhairi 1991a) and correlation (Ng 2003), were used to approximate $\Omega_u$. These lines of study results in various approximated mathematical models which best describe a set of experimental data. However, it is realized that most of these equations may not cover all the parameters that have significant effects on $\Delta f_{ps}$ and tend to overestimate it.

Instead of using statistical tools in approximating $\Omega_u$, soft computing approach is an alternative solution to solve this approximation problem. The soft computing model introduced herein is a data-driven harmony search (HS) zero-order single input rule modules (SIRMs)-connected fuzzy inference system (FIS) hereafter abbreviated as HS-SIRMs connected FIS. FIS model is used because of its interpretability capability to express the behaviour of the system in a human understandable way (Jin, 2000). It is worth mentioning that the use of fuzzy set related techniques in civil engineering is new. It is a popular research direction in the predictions of compressive strength (Subaşı et al. 2012) and shear strength (Nasrollahzadeh and Basiri, 2014) of concrete. SIRMs-connected FIS is chosen because of its capability to overcome the issue related to the curse of dimensionality when the number of input increases (Yubazaki et al. 1997). To improve the validity of the resulting SIRMs-connected FIS model, additional qualitative
knowledge (i.e., monotonicity property) is imposed in the modelling process. Three non-dimensionless parameter (i.e., \( \frac{d_p}{h} \), \( \frac{S_d}{d_p} \), and \( \frac{L_s}{L} \)) are chosen. HS is then used to search for an SIRMs-connected FIS model which best fit the experimental data. To preserve the monotonicity property, monotonicity index (MI) from previous works (Lau, et al. 2013; Tay, et al. 2012) is used as the constraint of the HS search.

This study is significant because it contributes to a new data-driven SIRMs-connected FIS model with monotonicity preserving capability. A musical-inspired meta-heuristic optimizer (i.e., HS) is used to search for a set of parameters that best describe \( \Omega_u \). The proposed approach may further lead to time saving and cost reduction in externally prestressed beams analysis.

2.0 Background

In this section, existing prediction equations for externally prestressed tendon using simplified method (i.e., pseudo-section analysis) is described. This is followed by a review of a SIRMs-connected FIS, monotonicity index (MI), and HS.

2.1 Review of Prediction Equations

Several pseudo-section analysis based \( \Omega_u \) equations have been developed to evaluate the \( f_{ps} \) and flexural strength of externally prestressed beam (Alkhairi and Naaman 1993, Mutsuyoshi et al. 1995, Naaman and Alkhairi 1991b, Ng and Tan 2006a, Ng 2003).

Naaman and Alkhairi (1991) proposed that

\[
f_{ps} = f_{pe} + \Omega_u E_{ps} \varepsilon_{cu} \left( \frac{d_p}{c} - 1 \right) \leq f_{py}
\]

where

\[
\Omega_u = \begin{cases} 
2.6 & \text{(for 1 point load)} \\
\frac{L}{d_{ps0}} & \text{(for third point loading or uniform)} \\
\frac{5.4}{d_{ps0}} & 
\end{cases}
\]

in which \( E_{ps} \) is modules of elasticity of tendon; \( \varepsilon_{cu} \) is concrete strain in top tendon at ultimate; \( d_{p0} \) is initial depth of the external tendon; \( c \) is depth of neutral axis at critical section at ultimate; \( L \) is total span length; \( f_{pe} \) and \( f_{py} \) is effective prestress and yield strength of prestressing tendons respectively.
Mutsuyoshi et al. (1995) then modified $\Omega_u$ in Naaman’s Equation (Naaman and Alkhairi 1991a) based on a numerical analysis and introduced a depth reduction factor, $R_d$, to estimate the tendon depth at ultimate. The tendon stress is given as:

$$f_{ps} = f_{pe} + \Omega_u E_{ps} \epsilon_{cal} \left( \frac{R_d d_{p0}}{c} - 1 \right) \leq f_{py}$$  \hspace{1cm} (3)$$

with the depth reduction coefficient given by

$$R_d = 1.0 - 0.022 \left( \frac{L}{d_{p0}} - 5 \right) \left( \frac{S_d}{L} - 0.2 \right) + 0.0186 \left( \frac{L}{d_{p0}} \right) \left( \frac{A_y f_y}{bd_s f_c} \right)$$  \hspace{1cm} (4)$$

and bond reduction coefficient given by

$$\Omega_u = \begin{vmatrix}
4.36 & -0.084 \left( \frac{S_d}{L} \right) \\
1.47 + 10.3 \left( \frac{L_d}{L} \right) & -0.29 \left( \frac{L_d}{L} \right) \left( \frac{S_d}{L} \right)
\end{vmatrix}$$  \hspace{1cm} (5)$$

where $S_d$ is deviator spacing; $b$ is beam width of compression zone; $L_d$ is distance between loading points; $A_y$, $d_s$, $f_y$ is area, depth and yield strength of tension reinforcement respectively; and $f_c$ are cylinder compressive strength of the concrete.

Aravinthan et al. (1997) then improved the equation proposed by Mutsuyoshi et al. (1995) based on the investigation on simply-supported externally prestressed beams. The proposed equation considered several factors that influenced the second-order effect such as: distance between deviators-to-span ratio, $S_d/L$, span-to-effective depth ratio, $L/d_{p0}$, bonded-to-total tendon area ratio $A_{ps,\text{int}}/A_{ps,\text{tot}}$.

The $\Omega_u$ is proposed as:

$$\Omega_u = \begin{vmatrix}
0.21 \frac{L}{d_{p0}} + 0.04 \left( \frac{A_{ps,\text{int}}}{A_{ps,\text{tot}}} \right) + 0.04 \text{ for one point load} \\
2.31 \frac{L}{d_{p0}} + 0.21 \left( \frac{A_{ps,\text{int}}}{A_{ps,\text{tot}}} \right) + 0.06 \text{ for three point loading}
\end{vmatrix}$$  \hspace{1cm} (6)$$

with depth reduction factor given as;
\[ R_d = \begin{cases} 
1.14 - 0.005 \left( \frac{L}{d_{p0}} \right) - 0.19 \left( \frac{S_d}{L} \right) & \leq 1.0 \text{ for onepoint load} \\
1.25 - 0.010 \left( \frac{L}{d_{p0}} \right) - 0.38 \left( \frac{S_d}{L} \right) & \leq 1.0 \text{ for thirdpoint loading} 
\end{cases} \] (7)

From a series of theoretical and experimental investigations, Ng (2003) showed that the span to depth ratio, \( L/d_{p0} \), has insignificant effect on \( \Delta f_{ps} \). A new dimensionless parameter, \( S_d/d_{p0} \), is introduced to cater for the second-order effects for longer span beam. Ng (2003) proposed a modified equation for \( \Omega_u \) using the correlation of average strains in the external tendons obtained through the rational analysis based on strain compatibility and force equilibrium on externally prestressed beam:

\[ \Omega_u = 0.895 - 1.365 \left( \frac{L_s}{L} \right) \left( \frac{d_{p0}}{h} \right) - k_s \] (8)

with coefficient accounting for second-order effect given as:

\[ k_s = \begin{cases} 
0.0096 \left( \frac{S_d}{d_{p0}} \right) & \text{for } \frac{S_d}{d_{p0}} \leq 15 \\
0.144 & \text{for } \frac{S_d}{d_{p0}} > 15 
\end{cases} \] (9)

where \( h \) is beam height and \( L_s \) is shear span.

The preceding description have identified several significant non-dimensionless parameters (i.e., \( d_{p0}/h \), \( S_d/d_{p0} \), and \( L_s/L \)) which then served as the basis for prediction of external tendon stress, \( f_{ps} \), for externally prestressed beams using HS-SIRMs connected FIS model, which indirectly approximates the bond reduction coefficient, \( \Omega_u \).

2.2 A general formulation of the Zero-order SIRMs-Connected FIS

A relatively new fuzzy inference model, SIRMs-connected FIS model is proposed for multi-input fuzzy system with \( n \)-input (Yubazaki et al. 1997). Consider a zero-order SIRMs-connected FIS model with \( n \)-input (i.e., \( y = f(\bar{x};\theta) \)), where \( \bar{x} = (x_1, x_2, ..., x_n) \) and \( \theta = (w_1, w_2, ..., w_n; A_1, A_2, ..., A_m; c_1, c_2, ..., c_n) \). It consists of \( n \) fuzzy rule modules as in Fig. 3. Note that \( SIRM-i \) represents the \( i \)-th rule module, where \( x_i \) is the sole variable in the antecedent, where \( i = 1, 2, ..., n \). \( R_i^j \) is the \( j \)-th rule in \( SIRM-i \), where \( j = 1, 2, ..., m_i \), while \( c_i^j \) is a variable output value in the consequent part. A fuzzy rule \( R_i^j \) can be viewed as
a mapping from \( A_i^h \) to \( c_i^h \).

\[
\begin{align*}
SIRM - 1: \{ & R_i^h : \text{if } x_1 = A_i^h \text{ then } y_i^h = c_i^h \} \\
& \vdots \\
SIRM - i: \{ & R_i^h : \text{if } x_i = A_i^h \text{ then } y_i^h = c_i^h \} \\
& \vdots \\
SIRM - n: \{ & R_n^h : \text{if } x_n = A_n^h \text{ then } y_n^h = c_n^h \} \\
\end{align*}
\]

Fig. 3 Fuzzy rules for a zero-order SIRMs-connected FIS model

The output of \( SIRM - i \), i.e., \( y_i(x_i) \) is obtained using Eq. (10). The membership function (MF) for \( A_i^h \) is denoted as \( \mu_i^h(x_i) \). The final inference result of SIRMs-connected FIS is obtained by a weighted sum of rule modules, as in Eq. (11). In which \( w_i \) reflects the relative importance of the \( SIRM - i \) which is defined according to the contribution of the input item to the system performance.

\[
y_i(x_i) = \frac{\sum_{h=1}^{m} [\mu_i^h(x_i) \cdot y_i^h]}{\sum_{h=1}^{m} [\mu_i^h(x_i)]}
\]

\[\text{(10)}\]

\[
y = \sum_{i=1}^{n} w_i \cdot y_i(x_i)
\]

\[\text{(11)}\]

### 2.3 A Monotonicity Index (MI) for Zero-order SIRMs-Connected FIS

Let \( f(\overline{x}) \) denote as \( n \)-input zero-order SIRMs-connected FIS model, where \( \overline{x}=(x_1, x_2, \ldots, x_n) \in X_1 \times X_2 \times \ldots \times X_n \). The \( i \)-th input in \( \overline{x} \) is represented by \( x_i \) is excluded from \( \overline{x} \), i.e., \( \overline{x} \subset \overline{x} \); \( x_i \notin \overline{x} \). The definition for monotonicity of \( f(\overline{x}) \) can be formally written as:

**Definition 1** An SIRMs-connected FIS model is said to fulfill the monotonicity increasing or decreasing property between its output, \( y \), and its input, \( x_i \), when \( y \) monotonically increases or decreases respectively, as \( x_i \) increases, i.e., \( f(\overline{x}, x_i) \geq f(\overline{x}, \tilde{x}) \) or \( f(\overline{x}, x_i) \leq f(\overline{x}, \tilde{x}) \), respectively, where \( x_i > \tilde{x} \in X_i \).

The proposed procedure for MI is summarized as follows:
(i) Determine the upper and lower limits of the universe of discourse for \( x_i \), and denote as \( \bar{x}_i \) and \( \underline{x}_i \) respectively.

(ii) Divide \( x_i \) domain to \( n_i \) divisions. Determine the grid size of \( x_i \), \( s_i = (\bar{x}_i - \underline{x}_i)/n_i \).

(iii) Compare each pair of \( y_i(x_i + s_i \times n_i) \) and \( y_i(x_i + s_i \times (n_i + 1)) \) with a function denote as \( \text{monotone}(y_i(x_i + s_i \times n_i)) \). Eq. (12) or Eq. (13) is adopted for a monotonic increasing or decreasing relationship respectively.

\[
\text{monotone}(y_i(x_i + s_i \times n_i)) = \begin{cases} 
1 & \text{if } y_i(x_i + s_i \times n_i) \leq y_i(x_i + s_i \times (n_i + 1)) \\
0 & \text{if } y_i(x_i + s_i \times n_i) > y_i(x_i + s_i \times (n_i + 1))
\end{cases}
\]  

\[
\text{monotone}(y_i(x_i + s_i \times n_i)) = \begin{cases} 
1 & \text{if } y_i(x_i + s_i \times n_i) \geq y_i(x_i + s_i \times (n_i + 1)) \\
0 & \text{if } y_i(x_i + s_i \times n_i) < y_i(x_i + s_i \times (n_i + 1))
\end{cases}
\] 

(iv) Obtain the MI between \( y_i \) and \( x_i \) for an SIRM-connected FIS model using Eq. (14)

\[
MI_i = \frac{\sum_{n_i=1}^{n_i=n_i-1} \text{monotone}(y_i(x_i + s_i \times n_i))}{\sum_{n_i=1}^{n_i=n_i-1} (i)}
\] 

2.4 Harmony Search (HS) Algorithm

The SIRM-connected FIS model is then optimized using a music-inspired meta-heuristic optimizer (i.e., HS). The HS is conceptualized using the musical process of searching for a perfect state of harmony. HS is chosen because it does not require initial values for the decision variables. Besides, it uses a stochastic random search based on the memory considering rate (HMCR) and the pitch adjusting rate (PAR) so that derivative information is unnecessary (Geem et al. 2008, Geem et al. 2001). Consider an optimization problem with \( m \) variables (i.e., \( \vec{z} = (z_1, z_2, ..., z_m) \)). The aim is to search for a set of \( \vec{z} \) in such that \( g(\vec{z}) \) is optimized. Fig. 4 summarizes the optimization procedure for HS.
Begin
Define objective function \( g(z) , \quad z = (z_1, z_2, ..., z_m)^T \) and
Define harmony memory size (HMS), harmony memory considering rate (HMCR),
pitch adjusting rate (PAR), and termination criterion (maximum number of search)
Generate Harmony Memory (HM) with random harmonies
while \( t < \text{max number of iterations} \)
while \( (i \leq \text{number of variables}) \)
if \( (\text{rand} < \text{PAR}) \), choose a value from HM for the variable \( i \)
if \( (\text{rand} < \text{PAR}) \), adjust the value by adding certain amount
else Choose a random value
end if
end while
Accept the new harmony (solution) if better
end while
Find the current best solution
end

Fig. 4 Pseudo code for HS algorithm (Geem et al. 2001)

3.0 Proposed Framework

In this section, the HS-SIRM connected FIS model is expressed as a constraint optimization problem. In this study, the non-dimensional parameters considered are: \( d_{p0}/h \), \( S_d/d_{p0} \), and \( L_s/L \).

3.1 The Zero-order SIRM-connected FIS for Estimating \( \Omega_u \)

A zero-order SIRM-connected FIS model with three inputs (i.e., \( \Omega_u = f(\bar{z}; \theta) \), where \( \bar{z} = (d_{p0}/h, S_d/d_{p0}, L_s/L) \) and \( \theta \) is the parameters describing the model is considered. It consists of three fuzzy rule modules as in Fig. 5, i.e., \( i = 1, 2, 3 \). \( SIRM - d_{p0}/h \) represents the \( d_{p0}/h \) rule module, where \( A_i^{(l)} h \) is the sole variable in the antecedent. \( R_i^{(l)} \) is the \( j_i \)-th rule in \( SIRM - d_{p0}/h \), where \( j_i = 1, 2, ..., m_i \) and \( c_i^{(l)} \) is a numerical output in the consequent or fuzzy singleton. Thus, a fuzzy rule \( R_i^{(l)} \) can also be viewed as a mapping from \( A_i^{(l)} h \) to \( c_i^{(l)} \), i.e., \( R_i^{(l)} : A_i^{(l)} h \rightarrow c_i^{(l)} \). The same applies to modules \( SIRM - S_d/d_{p0} \) and \( SIRM - L_s/L \).
The zero-order SIRMs-connected FIS model is written as Eq. (15).

\[
\Omega_u = f\left( \frac{d_{p_{0}}}{h}, S_d/d_{p_{0}}, \text{and } L_s/L \right)
= w_1 \times \Omega_{u_1} + w_2 \times \Omega_{u_2} + w_3 \times \Omega_{u_3}
= \frac{\sum_{j=1}^{m} [\mu_1^h(d_{p_{0}}/h)c_1^h] + \sum_{j=1}^{m} [\mu_2^j(S_d/d_{p_{0}})c_2^j] + \sum_{j=1}^{m} [\mu_3^j(L_s/L)c_3^j]}{\sum_{j=1}^{m} [\mu_1^h(d_{p_{0}}/h)] + \sum_{j=1}^{m} [\mu_2^j(S_d/d_{p_{0}})] + \sum_{j=1}^{m} [\mu_3^j(L_s/L)]}
\]  

\[\text{Eq. (15)}\]

3.2 Monotonicity Index (MI)

The monotonicity relationship between the inputs and output of the zero-order SIRMs-connected FIS model can be observed from experiments. It is generally agreed that when \( d_{p_{0}}/h \) increases, \( \Omega_u \) increases. Besides, when \( S_d/d_{p_{0}} \) and \( L_s/L \) increase, \( \Omega_u \) decrease (Ng, 2003). An input \( x_i \in \left[ d_{p_{0}}/h, S_d/d_{p_{0}}, L_s/L \right] \) is considered, the proposed procedure is summarized as follows:

(i) Determine the upper and lower limits of the universe of discourse for \( x_i \), and denote as \( \bar{x}_i \) and \( x_i \), respectively.

(ii) Divide \( x_i \) domain to \( n_i \) divisions. Determine the grid size of \( x_i \), \( s_i = (\bar{x}_i - x_i)/n_i \).

(iii) Compare each pair of \( \Omega_{u_1} \left( x_i + s_i \times n_i \right) \) and \( \Omega_{u_2} \left( x_i + s_i \times (n_i + 1) \right) \) with a function denote as \( \text{monotone} \left( \Omega_{u_1} \left( x_i + s_i \times n_i \right) \right) \). Eq. (16) or Eq. (17) is adopted for a monotonic increasing or decreasing relationship respectively.

\[
\text{monotone} \left( \Omega_{u_1} \left( x_i + s_i \times n_i \right) \right) = \begin{cases} 1 & \text{if } \Omega_{u_1} \left( x_i + s_i \times n_i \right) \leq \Omega_{u_2} \left( x_i + s_i \times (n_i + 1) \right) \\ 0 & \text{if } \Omega_{u_1} \left( x_i + s_i \times n_i \right) > \Omega_{u_2} \left( x_i + s_i \times (n_i + 1) \right) \end{cases}
\]  

\[\text{Eq. (16)}\]

\[
\text{monotone} \left( \Omega_{u_2} \left( x_i + s_i \times n_i \right) \right) = \begin{cases} 1 & \text{if } \Omega_{u_1} \left( x_i + s_i \times n_i \right) \geq \Omega_{u_2} \left( x_i + s_i \times (n_i + 1) \right) \\ 0 & \text{if } \Omega_{u_1} \left( x_i + s_i \times n_i \right) < \Omega_{u_2} \left( x_i + s_i \times (n_i + 1) \right) \end{cases}
\]  

\[\text{Eq. (17)}\]
(iv) Obtain the MI between $\Omega_i$ and $x_i$ for an SIRMs connected FIS model using Eq. (18)

$$MI_i = \frac{\sum_{i=1}^{n_i} \text{monotone}(\Omega_i (x_i + s_i \times n_i))}{\sum_{i=1}^{n_i} (1)}$$

In this paper, $MI_i$ is preprocessed with Eq. (19)

$$MI_i = \begin{cases} 0, & MI_i = 1 \\ 1, & MI_i \neq 1 \end{cases}$$

3.2 A Monotonicity Preserving HS-SIRMs Connected FIS Model for Estimating $\Omega_u$

A HS-SIRMs connected FIS model, i.e., $\Omega_u = f(\bar{x}; \theta)$, is considered. A system identification problem attempts to determine a set of $\theta$, in such a way that $f(\bar{x}; \theta)$ best represents a system when it is observed by $j$ desired input-output pairs of data, i.e., $\{(\bar{x}_k, \Omega_u)\}$, where $\bar{x}_k = (d_{p,0}/h, \{S_{d}/d_{p,0}\}, (L_x/L_x)^k)$. $k = 1, 2, ..., j$. Fig. 6 shows the schematic diagram of parameter identification for an HS-SIRMs connected FIS.
A data set composed of \( j \) desired input-output pairs \((\bar{x}_k, \Omega_{u_k})\), where \( k = 1,2,3,\ldots,j \), is used to construct an HS-SIRMs connected FIS model. The inputs (i.e., \( d_{p\theta_0}/h \), \( S_d/d_{p\theta_0} \), and \( L_x/L \)) is applied to both the system and the HS-SIRMs connected FIS model, while the square of the difference between the target system output (i.e., \( \Omega_u \)) and the model output (i.e., \( \hat{\Omega}_u \)), is \( (\Omega_u - \hat{\Omega}_u)^2 \). The total of \( (\Omega_u - \hat{\Omega}_u)^2 \) for the \( j \) set of data is used to give an indication of how near the FIS model with the target system.

The constrained optimization problem is formulated as Eq. (20):

\[
\text{Minimize } \text{Error}(\theta) = \sum_{k=1}^{j \in \Omega_u} (\Omega_u - f(x_k; \theta))^2
\]

Subjected to \( MI_i = 1 \), in which \( i \in [1, 2, 3] \)

Thereafter, the objective function to be minimized by HS is as shown in Eq. (21).

\[
\text{Objective function} = \text{Error}(\theta) + w \times \sum_{i=1}^{n} MI_i
\]

where \( w \) is a weightage constant.

### 4.0 Model Development

#### 4.1 Data Collection

A total of 27 beams \( (j = 27) \) from experimental investigations were used to examine the applicability of the proposed model in estimating \( \Delta f_{ps} \) and \( f_{ps} \) of the externally prestressed beams and hence \( \Omega_u \). The flexural strength of the beams were analysed based on strain compatibility and force equilibrium on beams reported in Table 1 and Figs. 1-2. These beams were: (i) Series T, ST, and SR beams with straight tendons, tested under third-point loading in (Ng 2003); (ii) Series M beams with an effective span of 5200 mm and draped tendons, tested under two symmetrical loads spaced at 900 mm apart by (Mutsuyoshi et al. 1995) and (iii) Series Y with an effective span of 4000 mm and straight tendons, tested under two symmetrical loads spaced at 600 mm apart (Yaginuma 1995). Data used in this paper is obtained from (Mutsuyoshi et al. 1995, Ng 2003), as summarized in Table 1.

<table>
<thead>
<tr>
<th>Beam No</th>
<th>( S_d/d_{p\theta_0} )</th>
<th>( d_{p\theta_0}/h )</th>
<th>( L_x/L )</th>
<th>( \Delta f_{ps} )</th>
<th>( f_{ps} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-1</td>
<td>7.2000</td>
<td>0.7692</td>
<td>0.4135</td>
<td>357.4000</td>
<td>1347.6000</td>
</tr>
<tr>
<td>M-2</td>
<td>12.0000</td>
<td>0.7692</td>
<td>0.4135</td>
<td>341.5000</td>
<td>1331.7000</td>
</tr>
</tbody>
</table>
### Table 2 Parameter setting used for the simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harmony memory size (HMS)</td>
<td>30</td>
</tr>
<tr>
<td>Harmony consideration rate (HMCR)</td>
<td>0.90</td>
</tr>
<tr>
<td>Pitch adjusting rate (PAR)</td>
<td>0.20</td>
</tr>
<tr>
<td>Number of iterations</td>
<td>10,000</td>
</tr>
</tbody>
</table>

#### 4.2 Simulation

In the simulation, Gaussian membership function (MF) is used. It is further assumed that there are five Gaussian MFs for each of the non-dimensional parameter. The parameters setting for HS-SIRM connected FIS model is depicted in Table 2.
5. Results and Discussions

5.1 Comparison with Previous Equations

Fig. 7 shows the plot for $\Delta f_{ps}^{(\text{predicted})}$ versus $\Delta f_{ps}^{(\text{experimental})}$ and $f_{ps}^{(\text{predicted})}$ versus $f_{ps}^{(\text{experimental})}$ in externally prestressed tendons between the existing prediction equations (Mutsuyoshi et al. 1995, Naaman and Alkhairi 1991a, Ng 2003) and the proposed model. It is observed that most of the existing prediction equations can reasonably predict $f_{ps}$, but they tend to overestimate $\Delta f_{ps}$, except for Ng (2003) which underestimates $\Delta f_{ps}$, and showed the scattering phenomenon far from the perfect line. Besides, it is observed that the existing prediction equations (Mutsuyoshi et al. 1995, Naaman and Alkhairi 1991b, Ng 2003) tend to give inconsistent and unconservative results in predicting $\Delta f_{ps}$ and $f_{ps}$.

Table 3 further shows the results of the correlation between the experimental results (Alkhairi and Naaman 1993, Mutsuyoshi et al. 1995, Ng 2003) and the proposed model with and without considering monotonicity property for beams listed in Table 1 and the predicted values. The HS-SIRMs connected FIS model is first tested with the data from Table 1 without considering MI as a constraint. The study shows a relatively good coefficient correlation of 0.8468 and 0.9538 respectively for $\Delta f_{ps}$ and $f_{ps}$ in the external tendons at ultimate, with relatively less variability compared to the other proposed equations (Naaman and Alkhairi, 1991b; Mutsuyoshi et al. 1995; Ng 2003).

Table 3 Correlation of test results and theoretical predictions for $\Delta f_{ps}$ and $f_{ps}$

<table>
<thead>
<tr>
<th>Methodology</th>
<th>Stress increase, $\Delta f_{ps}$</th>
<th>Ultimate stress, $f_{ps}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Correlation Coefficient</td>
<td>Variability within 95% confidence</td>
</tr>
<tr>
<td>Proposed Without MI</td>
<td>0.8468</td>
<td>0.0874</td>
</tr>
<tr>
<td>Proposed With MI</td>
<td>0.7815</td>
<td>0.1097</td>
</tr>
<tr>
<td>Naaman (Naaman and Alkhairi, 1991b)</td>
<td>0.7356</td>
<td>0.1572</td>
</tr>
<tr>
<td>Mutsuyoshi (Mutsuyoshi et al. 1995)</td>
<td>0.7228</td>
<td>0.1610</td>
</tr>
<tr>
<td>Ng (Ng 2003)</td>
<td>0.8573</td>
<td>0.0971</td>
</tr>
</tbody>
</table>
Fig. 7 Comparison of experimental results with predicted values
5.2 Comparison between Model with and without MI

An evaluation of HS-SIRMs connected FIS model with and without MI as a constraint was also carried out in this study. It is observed that although HS-SIRMs connected FIS model without MI yields better correlations and less variability compared to other researchers as in Table 3, the monotonicity property of the model is not preserved as shown in Fig. 8. To improve the validity of the model, MI is considered as a constraint to the HS-SIRMs connected FIS model. It is identified that the model yields slight increase in correlation coefficient value and variability but the monotonicity property of the model is fulfilled as depicted in Fig. 8.

Fig. 8 shows in detail the surface plot for the HS-SIRMs connected FIS model with and without MI as a constraint. By considering $S_d/d_{ps0}$ to be constant (i.e., $S_d/d_{ps0} = 12$), it is observed that when the ratio of $d_{ps0}/h$ increases, the observed output increases monotonically; when the ratio of $L_s/L$ increases, the observed output decreases monotonically. Furthermore, when the ratio of $d_{ps0}/h$ is kept to be constant (i.e., $d_{ps0}/h = 0.7$), the increased in the ratio of $S_d/d_{ps0}$ and $L_s/L$ caused the observed output to decrease monotonically. Eventually, when the ratio of $L_s/L$ is kept constant (i.e., $L_s/L = 0.4$), the increase in the ratio of $S_d/d_{ps0}$ causes the observed output to decrease, while the increase in the ratio of $d_{ps0}/h$ causes the observed output to increase monotonically.

![Surface plot for $S_d/d_{ps0}=12$](image)

(a) Surface plot for $S_d/d_{ps0}=12$
Surface plot for $7.00 = h \frac{ps}{d}$

Surface plot for $4.0 = L \frac{LLs}{s}$

Fig. 8 Surface plots for the effects of MI to HS-SIRMs connected FIS model

6.0 Conclusions

In this paper, HS-SIRMs connected FIS model is proposed in a data-driven FIS model. The objective function is formulated as a constrained optimization problem. A HS optimization procedure is then used to search for a set of variables that obey the monotonicity property as the sufficient conditions. Experiments are conducted with data obtained from the developed by other researchers to study $\Delta f_{ps}$ and $f_{ps}$ in externally prestressed beams. The results show that the proposed approach is useful to generate a HS-SIRMs connected FIS model with HS optimization procedure for the predicting $\Omega_u$ and hence $\Delta f_{ps}$ and $f_{ps}$ with acceptable computation complexity and error.

The parametric study was carried out for determining $\Delta f_{ps}$ and $f_{ps}$ in externally prestressed beams. The influential non-dimension parameters include (i) $d_{p0} / h$; (ii) $S_d / d_{p0}$; and (iii) $L_s / L$ were examined by HS-SIRMs connected FIS model analysis. The proposed model overall provides a better correlation for predicting $\Delta f_{ps}$ and $f_{ps}$ compared with other existing prediction equations while preserving the monotonicity property of the model.
References


