EFFECTS OF MANIPULATIVES ON THE LEARNING OF ALGEBRA

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ABSTRACT

This study aimed to investigate the effects of using manipulatives on students' algebraic achievement, attitudes toward algebra, manipulatives preferences and conceptual understanding of algebra in Malaysian secondary schools. This research employed a non-equivalent quasi-experimental pretest-posttest design. A total of 207 form four arts stream students from three secondary schools in Kuching and Samarahan divisions participated in the study. The students learned quadratic expansion and factorization using either virtual manipulatives (treatment group) or physical manipulatives (control group) as the instructional method. The virtual manipulatives were developed using Dreamweaver and Java applet based on the guidelines from past studies such as Durmus and Karakirik (2006) and Moyer, Bolyard, and Spikell (2002). The data collection in each school was conducted over a two-week time period during regular school hours. For the first session involving quadratic expansion, group one worked with physical algebra tiles while group two worked with virtual algebra tiles. For the second session involving quadratic factorization, group one worked with virtual algebra tiles and group two worked with physical algebra tiles. Several research instruments were used to collect the required data in this study. The quadratic expansion and factorization pretests and posttests were used to measure the impact of physical and virtual manipulatives on students' algebraic achievement. Kolb's learning style inventory was used to establish the students' learning style preferences. The manipulative preferences questionnaires were used to identify students' manipulatives preferences. The attitude inventory was used to measure students' attitudes toward algebra. Lastly, the conceptual understanding task sheets were used to measure the impact on students' conceptual understanding of algebra. In addition, another 15 students were purposively selected for semi-structured interviews based on the conceptual task sheets. Statistical analyses revealed that students showed progress in algebraic achievement in both instructional methods with virtual manipulatives having more positive impact than physical manipulatives for students with concrete learning style. The analysis also showed that female students performed better than male students when physical manipulatives was used as the instructional method. When the groups were examined for attitudes toward algebra,
both instructional methods were equally effective in improving students’ attitudes toward algebra. However, the results also showed that male students exhibited more positive attitudes than female students in motivation and enjoyment toward learning algebra. Further result indicated no statistical significant differences in students’ preferences toward manipulatives. Finally, analysis showed that physical manipulative and virtual manipulatives performed equally well in promoting students’ conceptual understanding of algebra. The findings of this study indicated that the three stages of Bruner’s learning theory; enactive, iconic and symbolic could be implemented using virtual manipulatives as the instructional method to promote students’ conceptual understanding of algebra. The study showed that secondary school students also benefited from using virtual manipulatives. In general, these findings on virtual manipulatives concurred and supported past findings on positive impact of virtual manipulatives in the learning of mathematics such as studies by Reimer and Moyer (2005), Smith (2006), and Suh (2005). It was recommended that teachers should use more virtual manipulatives in classrooms to promote conceptual learning. Hence, the Ministry of Education should recommend policies that include the use of virtual manipulatives in the teaching of mathematics by providing support such as in-house training for teachers and develop more software on virtual manipulatives. Further research could be carried out to extend the findings of this study by allowing more time for treatment, using larger sample size other than form four arts classes, investigating wider mathematics contents besides algebra, and by comparing the effectiveness between two and more virtual manipulatives to further illuminate greater understanding of the effect of virtual manipulatives on students’ conceptual learning.
ABSTRAK

# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>i</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>ABSTRAK</td>
<td>iv</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>x</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xii</td>
</tr>
</tbody>
</table>

## CHAPTER 1: INTRODUCTION

1.0 Introduction
1.1 Background of the study
1.2 Statement of the problem
1.3 Research framework
1.4 Research objectives
1.5 Research questions
1.6 Significance of the study
  1.6.1 Contribution to literature and practice in Malaysia
1.7 Scope and limitations of the study
1.8 Definition of terms
1.9 Summary

## CHAPTER 2: LITERATURE REVIEW

2.0 Introduction
2.1 Conducting the literature review
2.2 Shortcomings in mathematics learning
2.3 Mathematics learning and conceptual understanding
  2.3.1 Learning theory and mathematical concept development
  2.3.2 Mathematics representations and conceptual understanding
2.4 Use of manipulatives in mathematics learning
  2.4.1 Mathematics manipulatives and conceptual understanding
  2.4.2 Description of the physical algebra tiles
  2.4.3 Previous research on the use of physical manipulatives in mathematics learning
2.5 Use of computer technology in mathematics learning
  2.5.1 Emergence of virtual manipulatives
  2.5.2 Previous research on the use of virtual manipulatives in Mathematics learning
CHAPTER 3: METHODOLOGY ......................................................... 61
3.0 Introduction.......................................................................... 61
3.1 Research design.................................................................... 61
3.2 Population and sample............................................................ 62
3.3 Description of the algebraic lessons........................................... 63
  3.3.1 General feature of the manipulative materials......................... 64
  3.3.2 Manipulating the algebra tiles ........................................... 71
  3.3.3 Application of PM and VM on algebraic lessons..................... 88
  3.3.4 Evaluation of VM.......................................................... 95
3.4 Research instruments............................................................. 96
  3.4.1 Tests and survey questionnaires.......................................... 97
  3.4.2 Interview questions ........................................................ 103
3.5 Pilot study ........................................................................... 106
3.6 Data collection procedures..................................................... 108
3.7 Data analysis ........................................................................ 111
3.8 Summary ............................................................................. 112

CHAPTER 4: RESULTS................................................................... 114
4.0 Introduction.......................................................................... 114
4.1 Reliability of the research instruments...................................... 114
4.2 Demographics and students details of the quadratic expansion and factorization sessions.................................................. 115
4.3 Students’ achievement in algebra............................................. 117
  4.3.1 Research Question 1: Would the use of VM, gender and Learning styles affect students’ achievement in learning algebra? ..................................................... 117
4.4 Students’ attitudes toward algebra........................................... 129
  4.4.1 Research Question 2: Would the use of VM, gender and
learning styles affect students' attitudes toward learning mathematics? ................................................................. 131
4.5 Students' preferences toward manipulatives in learning algebra ......... 140
4.5.1 Research Question 3: Would gender and learning styles affect students' manipulatives preferences toward learning algebra? ........................................................................................... 141
4.6 Students' conceptual understanding in learning algebra .................... 145
4.6.1 Research Question 4: Would the use of VM impact students' conceptual understanding in learning algebra? ....................................................... 145
4.6.2 Findings from quantitative analysis ...................................... 146
4.6.3 Findings from qualitative analysis ..................................... 147
4.7 Summary ......................................................................... 153

CHAPTER 5: DISCUSSIONS AND CONCLUSIONS ........................................ 156
5.0 Introduction ....................................................................... 156
5.1 Summary of the research .................................................... 156
5.2 Discussion of research findings ............................................. 157
5.2.1 Effects of VM on students' algebraic achievement ............... 157
5.2.2 Changes in students' attitudes toward algebra ................. 159
5.2.3 Students' preferences toward manipulatives ...................... 161
5.2.4 Effects of virtual manipulatives on students' conceptual understanding of algebra ...................................................... 161
5.3 Implications of the study ..................................................... 164
5.3.1 Implication for theories .................................................. 164
5.3.2 Implication for students' learning ..................................... 165
5.3.3 Implication for teachers' teaching .................................... 166
5.3.4 Implication for instructional design .................................. 166
5.3.5 Implication for other stakeholders (The Ministry and Department of Education, and Teachers' training institutes) ......................... 167
5.3.6 Implication for future research ....................................... 168
5.4 Conclusions ...................................................................... 169

REFERENCES ........................................................................... 171

APPENDICES
APPENDIX 1 Kolb Learning Style Inventory .................................. 186
APPENDIX 2 Attitudes toward Algebra Inventory ............................ 187
APPENDIX 3 Manipulatives Preferences Questionnaires .................... 189
APPENDIX 4 Quadratic Expansion Pretest ..................................... 190
LIST OF TABLES

Table 3.1: The design of the study................................................... 61
Table 3.2: Sample items for evaluation of virtual algebra tiles............ 95
Table 3.3: Evaluation of virtual algebra tiles........................................ 96
Table 3.4: Sample items on quadratic expansion and factorization tests.... 97
Table 3.5: Sample items from reviewer form on quadratic expansion tests.... 98
Table 3.6: Sample items from reviewer form on quadratic factorization tests................................................... 98
Table 3.7: Sample items by categories of attitudes............................... 100
Table 3.8: Sample items on manipulatives preferences......................... 100
Table 3.9: Sample items on students' learning style inventory................. 101
Table 3.10: The interview questions on conceptual understanding of algebra... 105
Table 3.11: Categories, rules of inclusions, and examples for coding the Students’ conceptual understanding on the quadratic expansion.... 106
Table 3.12: Reliability coefficients of the research instruments in pilot study... 107
Table 3.13: Summary of instruments used in data collection.................. 111
Table 4.1: Reliability coefficients of the research instruments................... 115
Table 4.2: Descriptive statistics: Quadratic expansion pre and post test mean scores................................................... 118
Table 4.3: Analysis of covariance: Achievement in quadratic expansion........................................................................ 121
Table 4.4: Simple effect analysis: Interaction between method and learning styles .............................................................. 122
Table 4.5: Estimated marginal means: Interaction between method and learning styles .............................................................. 122
Table 4.6: Descriptive statistics: Quadratic factorization pre and post test mean scores................................................... 123
Table 4.7: Analysis of covariance: Achievement in quadratic factorization........................................................................ 127
Table 4.8: Simple effect analysis: Interaction between method and gender........................................................................ 127
Table 4.9: Estimated marginal means: Interaction between method and gender........................................................................ 128
Table 4.10: Descriptive statistics: Pre and post attitudes and categories of attitudes mean scores................................................... 130
Table 4.11: Analysis of covariance: Attitudes toward algebra.................. 134
Table 4.12: Analysis of covariance: Motivation..................................... 138
Table 4.13: Analysis of covariance: Self-Confidence............................. 139
Table 4.14: Analysis of covariance: Value.......................................... 139
Table 4.15: Analysis of covariance: Enjoyment..................................... 140
Table 4.16: Descriptive statistics: Manipulatives preferences mean scores...... 141
Table 4.17: Analysis of variance: Manipulatives preferences (PM).............. 144
Table 4.18: Analysis of variance: Manipulatives preferences (VM).............. 145
Table 4.19: Descriptive statistics: Conceptual understanding scores.............. 146
Table 4.20: Independent t-test: Achievement in conceptual understanding...... 147
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>The research framework of the study</td>
<td>10</td>
</tr>
<tr>
<td>2.1</td>
<td>Interactions between learners’ internal and external representations</td>
<td>26</td>
</tr>
<tr>
<td>2.2</td>
<td>Five representation systems in mathematics learning</td>
<td>27</td>
</tr>
<tr>
<td>2.3</td>
<td>Different types of physical algebra tiles</td>
<td>30</td>
</tr>
<tr>
<td>2.4</td>
<td>The student set algebra tiles</td>
<td>31</td>
</tr>
<tr>
<td>2.5</td>
<td>The Lab Gear algebra tiles</td>
<td>32</td>
</tr>
<tr>
<td>2.6</td>
<td>The Algeblocks</td>
<td>33</td>
</tr>
<tr>
<td>2.7</td>
<td>Three different types of virtual algebra tiles</td>
<td>38</td>
</tr>
<tr>
<td>2.8</td>
<td>Scaffolding of the snapping feature of virtual algebra tiles</td>
<td>46</td>
</tr>
<tr>
<td>2.9</td>
<td>Kolb's Experiential Learning Style</td>
<td>51</td>
</tr>
<tr>
<td>3.1</td>
<td>The homemade physical algebra tiles</td>
<td>64</td>
</tr>
<tr>
<td>3.2</td>
<td>The L-shaped frame of the physical algebra tiles</td>
<td>65</td>
</tr>
<tr>
<td>3.3</td>
<td>The zero pairs algebra tiles</td>
<td>66</td>
</tr>
<tr>
<td>3.4</td>
<td>The welcoming screen of the virtual manipulatives used in the study</td>
<td>67</td>
</tr>
<tr>
<td>3.5</td>
<td>The main menu for virtual algebra tiles</td>
<td>67</td>
</tr>
<tr>
<td>3.6</td>
<td>The main menu for algebra tiles used in quadratic expansion session</td>
<td>68</td>
</tr>
<tr>
<td>3.7</td>
<td>The instruction menu for algebra tiles used in quadratic expansion session</td>
<td>68</td>
</tr>
<tr>
<td>3.8</td>
<td>The content in the instruction menu for quadratic expansion session</td>
<td>69</td>
</tr>
<tr>
<td>3.9</td>
<td>The exercise menu for algebra tiles used in quadratic expansion session</td>
<td>69</td>
</tr>
<tr>
<td>3.10</td>
<td>The content in the exercise menu for quadratic expansion session</td>
<td>70</td>
</tr>
<tr>
<td>3.11</td>
<td>The build-in functions in the virtual algebra tiles</td>
<td>70</td>
</tr>
<tr>
<td>3.12</td>
<td>Example of the item in conceptual understanding post task sheet</td>
<td>102</td>
</tr>
<tr>
<td>3.13</td>
<td>The procedures involve in data collection</td>
<td>108</td>
</tr>
<tr>
<td>3.14</td>
<td>Data analysis procedure based on the research questions</td>
<td>112</td>
</tr>
<tr>
<td>4.1</td>
<td>Students categorized according to gender, learning style, and instructional method in quadratic expansion session</td>
<td>115</td>
</tr>
<tr>
<td>4.2</td>
<td>Students categorized into respective cells in quadratic expansion session</td>
<td>115</td>
</tr>
<tr>
<td>4.3</td>
<td>Students categorized according to gender, learning style, and instructional method in quadratic factorization session</td>
<td>116</td>
</tr>
<tr>
<td>4.4</td>
<td>Students categorized into respective cells in quadratic factorization session</td>
<td>116</td>
</tr>
</tbody>
</table>
Figure 4.5: Box plot: Quadratic expansion pre and posttest scores with outliers ................................................................. 118
Figure 4.6: Box plot: Quadratic expansion pre test scores with outliers removed .................................................................. 119
Figure 4.7: Interaction plot: Interaction between learning style and method ........................................................................ 123
Figure 4.8: Box plot: Quadratic factorization pre test and posttest scores with outliers ..................................................... 124
Figure 4.9: Box plot: Quadratic factorization pre test scores with outliers removed .......................................................... 125
Figure 4.10: Interaction plot: Interaction between gender and method ................................................................................ 129
Figure 4.11: Box plot: Pre and post attitudes scores with outliers ...................................................................................... 132
Figure 4.12: Box plot: Pre and post attitudes scores with outliers removed ................................................................. 132
Figure 4.13: Box plot: Pre and post categories of attitudes scores with outliers ................................................................. 135
Figure 4.14: Box plot: Pre and post categories of attitudes scores with outliers removed ..................................................... 136
Figure 4.15: Box plot: Physical and virtual manipulatives preferences scores with outliers ..................................................... 142
Figure 4.16: Box plot: Physical and virtual manipulatives preferences scores with outliers removed ................................. 143
CHAPTER ONE
INTRODUCTION

1.0 Introduction

This chapter provides an overview and introduction to the research. It describes the background of the study, statement of the problem, research framework, research objectives and questions, significance of the study, scope and limitations of the study, and definitions of important research terms. It concludes with a summary of the chapter.

1.1 Background of the study

Mathematics, despite its utility and importance is perceived by most students as difficult, boring, not very practical, abstract, and its learning as requiring a special ability that is not always within everyone’s reach (Ignacio, 2006). Research increasingly indicates that one of the reasons for failure in mathematics is due to student’s inability to apply concepts learned in formal contexts and in many cases this is caused by the abstraction and decontextualization of learning (Brown, Collins, & Duguid, 1989). Furthermore, many pupils generate negative attitudes toward mathematics in the course of their academic life. For them, learning the subject is not a source of satisfaction, but rather one of frustration, discouragement, and anxiety. Many of them, even some of the most able, find mathematics to be just a tiresome chore (Ignacio, 2006).

In 2003, the Ministry of Education, Malaysia started the extensive use of ICT in schools with the implementation of English in the teaching and learning of mathematics and sciences in all the government schools. Teachers were given personal notebook computers, LCD projectors and teaching and learning course wares. In order to promote the learning of mathematics, the new mathematics curriculum in the Malaysian’s schools was revised to give flexibility for teachers to implement an enjoyable, meaningful, useful and challenging teaching and learning environment
(Pusat Perkembangan Kurikulum, 2004). With that, teachers have been urged to adopt and to diversify their mode of teaching especially so in departing from teacher-centred teaching to student-centred learning. It is hoped that these changes will result in improving the learning process in the mathematics and science classrooms. However, this envisioned instructional changes do not seem to take off the ground and many still see the learning of mathematics to be problematic (Yahya Hassan, 2005). Yahya Hassan (2005) even describes school as a place that produces students who are only good at examinations but fail to solve a mathematics problem not from the textbook.

Nonetheless, the teaching of mathematics in Malaysia has undergone many changes in recent years (Pusat Perkembangan Kurikulum, 2004). As was reported by Stacey, Chick, and Kendal (2004), the growth of universal secondary education in developed countries during the 20th century, which then continued all over the world, has impacted on the learning of mathematics. In Malaysia, for example, the increase in access to education has resulted in a drastic increase of students entering secondary form 4 to almost double, from an enrolment of 215,736 students in 1994 to 394,305 students in 2006 (Education Planning Research and Development, 2006). This increase raises two important issues in learning mathematics.

The first issue is the relevance of learning mathematics. If mathematics like algebra is interpreted just as symbolic manipulation, then it has little relevance to everyday life. For instance, presently the topic on factorization of quadratic expressions in form four algebra are being taught in classroom by using symbolic rather than concrete manipulation. Two common and popular algorithmic methods used in solving the quadratic expressions are cross method and inspection method (Cheang, Moidunny, Khaw, & Yong, 2005). These approaches emphasize solely on imparting algorithmic procedures and skills of factorization to students enabling them to solve the given questions. This has resulted in the tendency of learning by rote without developing any understanding of what they are actually doing (Effandi Zakaria & Zanaton Iksan, 2007). In addition, in doing so, students are not able to see the relevant of mathematics in everyday life.
On the other hand, while the two stated approaches only emphasize on the manipulation of algebraic expressions that portray algebra as a meaningless activity involving manipulation of letters, many of the new curriculum approaches have tried to change this presentation of algebra (Stacey et al., 2004). One of the approaches is linking algebra to real situations so that the algebraic letters have clear referents and meanings. For example, the algebraic function, in this case the quadratic expression may be geometrically presented as the concept of the area of a rectangle and the two factors obtained from the factorization of this expression are represented by the concept of the length and the width of that rectangle respectively. Thus, learning by conceptual understanding are encouraged by educationists where they generally believe that mathematics should be learnt through conceptual understanding like representation in real situation (Stacey & MacGregor, 1999). Therefore, the challenge for educators is to reconceptualize the learning of mathematics as a subject that does have relevance to students in everyday life.

The second issue is related to the effectiveness of mathematics instructions. Recent years have seen rapid improvements in technology, and especially computers and information communication and technology (ICT). The advancement in technology enables the extensive use of computer and ICT to improve classroom mathematics instructions (Moyer, Bolyard, & Spikell, 2002). For example, the curriculum reform of the teaching and learning of mathematics by the Ministry of Education, Malaysia, since 2003 resulted in the extensive use of ICT in classroom. The growing technology tools provide an avenue and opportunity to assist teachers in enhancing their teaching and in improving the mathematics learning experiences of students (Yahya Hasan, 2005).

In order to promote the use of ICT in classroom mathematics instruction, the Ministry of Education, Malaysia has been developing and providing teaching and learning course wares in the form of multimedia CD-ROM since 2003 to all the schools and also bought the license to use Geometer's Sketchpad, a learning software, in secondary schools in 2004 (Teoh & Fong, 2005). In addition, in house training for teachers to get use to the new technology has been going on regularly (Chong, Horani & Jacob, 2005).
In order to make sure that the reform is successfully implemented, the Ministry of Education has been keeping track and monitoring on the effectiveness of using ICT in mathematics instruction (Bahagian Teknologi Pendidikan, 2005). Following the implementation of the new reform, a few studies have been carried out to gain the feedback on the effectiveness of the mathematics education reform. For example, a massive study on the effectiveness of the teaching and learning materials on mathematics and science developed by the Educational Technology Division involving 7,956 learners and 130 teachers in 130 primary and secondary schools throughout the nation indicated that the instructional ICT materials was effective in improving the learners’ mastering of mathematics and science concepts (Bahagian Teknologi Pendidikan, 2005).

However, the level of effectiveness of mathematics teaching remains questionable (Lim & Hwa, 2006). Three factors are largely responsible for the effectiveness in mathematics instruction. Firstly, teacher-centred approach still dominates classroom instruction (Effandi Zakaria & Zanaton Iksan, 2007; Koh & Hong, 2007). Likewise, well before the implementation of ICT in teaching mathematics, a study by Mohd. Majid Konting (1997) on effective mathematics teachers classroom practices in Malaysia showed that the effective mathematics teachers were inclined to use traditional teacher-centered teaching strategies and to dominate classroom interactions without much group work and evidence of student centeredness. Secondly, in spite of the advancement in ICT, mathematics teachers were slow in adopting innovative methods in facilitating the use of technology in mathematics instruction (Mokhtar Nawawi, Ahmad Fauzi Ayub, Wan Zah Wan Ali, Aida Suraya Yunus, & Rohani Ahmad Tarmizi, 2005). Thirdly, the influence of the examination-oriented culture. The over emphasize on performance based learning that force the teachers to use ‘teach to test’ procedure indirectly may reduce the effectiveness of mathematics instruction (Lim & Hwa, 2006; Yahya Hasan, 2005). Teachers are more concerned with rushing and finishing the syllabus to help the children score in the examination rather than focusing on understanding of the algebra concepts.
With the latest curriculum reform, teachers are encouraged to engage in different instructional approaches like conceptual learning rather than just concentrating on rote learning (Pusat Perkembangan Kurikulum, 2004).

Manipulatives refer to objects that can be touched and moved by students to introduce or reinforce a mathematical concept (Hartshorn & Boren, 1990). Educators in the West have recommended the use of manipulatives in classroom teaching as one of the mathematics instructional approach in teaching algebra concepts (National Council of Teachers of Mathematics, 1989). In doing so, the learner will be able to construct a concept or discover an algebra relation through the proper use of manipulatives with an adequate task (Durmus & Karakirik, 2006). In addition, through manipulatives, the learner can also relate the real world to the abstract algebra world. For example, the use of manipulative materials as concrete models such as algebra tiles in expansion and factorization of the quadratic expressions relate the concept of area, length and width of a rectangle, representing the real world with the algebraic symbols in the algebra world. Besides conceptual understanding, other improvements claimed by proponents of manipulatives include increase in problem solving skills, enhancement of students' attitudes toward mathematics and increase in motivation (Durmus & Karakirik, 2006).

With the advancement of ICT, a more powerful form of manipulatives called virtual manipulatives (VM) is attracting the attention of educationists. A VM is defined as "an interactive, web-based visual representation of a dynamic object that presents opportunities for constructing algebra knowledge" (Moyer et al., 2002, p. 373). Generally, VM enables as much engagement as physical manipulatives do and in addition, VM provide interactive environments where students could pose and solve their own problems to form connections between algebra concepts and operations, and get immediate feedback about their actions that lead them to reflect on their conceptualization (Durmus & Karakirik, 2006).

With other benefits such as easy availability, (Moyer et al., 2002), providing students with instantaneous, corrective feedback (Clements & McMillen, 1996; Durmus & Karakirik, 2006; Reimer & Moyer, 2005; Suh & Moyer, 2005), increasing motivation
and attention in students (Clements & McMillen, 1996; Reimer & Moyer, 2005); and promoting inquiry-based learning and problem solving (Clements & McMillen, 1996; Durmus & Karakirik, 2006), VM is emerging as an important mathematics instructional tool. Furthermore, as technology continues to evolve, VM may eventually overtake physical manipulatives and become an important and appropriate algebra instructional tool for the next generation (Moyer et al., 2002). Hence, advances in technology, accompanied by changes in instructional approaches offer a chance for improving the students’ mathematics learning outcomes.

1.2 Statement of the problem

The integrated curriculum for secondary schools states that the main aim for secondary school mathematics learning is to develop individuals who are able to think mathematically and to apply mathematics knowledge effectively and responsibly in solving problems and making decision. This will enable the individual to face challenges in everyday life that arises due to the advancement of science and technology (Pusat Perkembangan Kurikulum, 2004).

The learning of algebra in Malaysian schools for example started in the primary curriculum, but only in secondary level do students begin formal algebra, which is signified by the use of letters to denote unknown or variable quantities. This gradual introduction reflects the special role of algebra as a gateway to higher mathematics. As such algebra is the language of higher mathematics and is also a set of methods to solve problems encountered in everyday life.

The importance of algebra is reflected in the contents found in secondary school form four mathematics textbook that shows five out of eleven chapters are related to algebra (Cheang et al., 2005). This means that the students must understand as well as be capable in solving the algebraic problems before proceeding to form five where another five out of ten chapters are related to algebra (Yeow, Kalthom, Chen, & Gan, 2006).
However, the difficulties of learning mathematics especially algebra have always been highlighted by mathematics educationists. For example, an analysis on the Malaysian secondary schools students' mathematics performance by the Malaysian Examination Syndicate showed that a large number of them are still unable to master algebra concepts and skills (Lembaga Peperiksaan Malaysia, 1993). Some studies on the learning of algebra also noted students' incompetencies in algebra are caused by misconceptions and weak basic algebra operation skills (Chan, 1999; Saripah Latipah Syed Jaapar, 2000; Khamsan Omar, 1999). Two studies on the factorization of quadratic expressions further highlighted the common errors in factorization such as being unable to find suitable factor pairs, wrongly placing positive and negative sign on factor pairs and unable to factorize an expression completely (Azrul Fahmi Ismail & Marlina Ali, 2007; Rosli Dahlan, 2000). Both studies also suggested the importance of clear understanding of the concept of factorization in order to avoid these misconceptions and errors in factoring quadratic expressions.

Even though learning of algebra is difficult, physical manipulatives like algebra tiles can play an important role in developing the students' understanding of the concept of algebra. A study by Sharp (1995) on secondary schools' students on the use of algebra tiles found that by bridging the students' understanding of the concrete environment with the symbolic representations of algebra, the students successfully created the meaning of algebra ideas. Findings from a study by Thornton (1995) on algebra tiles and learning styles among secondary students supported the effectiveness of using manipulative materials in generating positive attitudes and greater understanding of algebraic concepts.

With the advancement in ICT, dynamic and interactive teaching and learning materials can be easily created using computer software. One of the materials that is increasingly attracting educators' attention is the web-based, visual representations of dynamic objects that allow users to construct algebraic knowledge called virtual manipulatives created using Java Applet. The advantages of virtual manipulatives include being easier to use, to reproduce and to distribute over the internet. For example, in the case of using algebra tiles as the manipulatives; the virtual algebra tiles have all the
advantages of physical algebra tiles. In addition the virtual tiles are easily accessible by the students at school and even at home if they have computers with internet access. Furthermore, virtual algebra tiles are also easier to manage and space saving compared to physical tiles. In terms of students' achievements, several literature have found that students using computer-based manipulatives alone or in combination with physical manipulatives demonstrated higher achievement than students using either only physical or no manipulatives (Kieren & Hillel, 1990; Moyer, Niezgoda, & Stanley, 2005; Smith, 1995; Suh, 2005).

Students' learning of mathematics can be affected by many factors. One factor that may affect the learning of mathematics is the students' learning styles. Learning styles refer to the individual differences between students and how they prefer to learn. Each student's experience of learning is not the same and can be categorized in several different ways. Kolb's learning style theory categorizes individual learning preference into four types of learning styles: diverger, assimilator, converger, and accommodator (Kolb & Kolb, 2005). The four learning styles can further be grouped into two experiential groups of concrete learners and abstract learners. An understanding of the ways students learn will help teachers to make accommodations and modify instructions to meet the needs of their students. This will produce a better and more effective instruction so that students' achievement in mathematics can be improved. Another factor that may play a significant role in learning is gender. Research on the relationship between gender and learning reported that male students and female students learn differently and they have a preferred learning style of their own (Bevan, 2001).

The use of manipulatives in teaching mathematics in Malaysia classroom has been proposed in the mathematics curriculum specification (Pusat Perkembangan Kurikulum, 2004). According to the curriculum specification (Pusat Perkembangan Kurikulum, 2004), teachers are encouraged to use multiple approaches and examples, determine teaching and learning strategies most suitable for their students and provide appropriate teaching and learning materials. Several studies on effectiveness of using manipulatives in the teaching and learning of form one mathematics in Malaysian
classroom reported manipulatives improve mathematics achievement and attitudes toward mathematics (Justinah Sualin, 2008; Law, 2006; Ng, 2006). In addition, the curriculum specification (Pusat Perkembangan Kurikulum, 2004) also recommended the use of technology in the teaching and learning of mathematics in Malaysian classroom.

Although the virtual manipulatives have not been significantly used in the Malaysian classroom instruction, the teaching and learning of geometry using Geometer’s Sketchpad (GSP) software have been explicitly indicated in the new Malaysian secondary school syllabus implemented in 2003 (Kamariah Abu Bakar, Rohani Ahmad Tarmizi, Ahmad Fauzi Mohd Ayub, & Aida Suraya Md Yunus, 2009). In the syllabus, teachers have been recommended to utilize the GSP softwares in enhancing students’ learning of geometry. Research studies on the use of GSP software in the Malaysian classroom reported positive results where GSP software has been shown to increase students’ achievement in geometry (Norhayati Mt. Ali, 2003) and resulted in significant change in attitudes toward geometry among students (Ahmad Fauzi Mohd Ayub, Rohani Ahmad Tarmizi, Kamariah Abu Bakar, & Aida Suraya Md Yunus, 2008).

Thus, this research compared the effectiveness of mathematics instruction between physical manipulatives and virtual manipulatives in secondary school students’ learning of quadratic expansion and factorization. The physical algebra tiles in this study were the modified version of concrete algebra tiles used by Lab Gear (Wah & Picciotto, 1994) whereas the virtual algebra tiles in this study were the modified version of virtual algebra tiles developed by NLVM (Utah State University, 2008). The manipulated variable in this study was the instructional methods, while gender and learning styles were the independent variables. The dependent variables were students’ algebraic achievement, students’ attitudes toward algebra, students’ manipulatives preferences and students’ conceptual understanding of algebra.