A New Two-Stage Fuzzy Inference System-Based Approach to Prioritize Failures in Failure Mode and Effect Analysis

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***Abstract*—This paper presents a new Fuzzy Inference System (FIS)-based Risk Priority Number (RPN) model for the prioritization of failures in Failure Mode and Effect Analysis (FMEA). In FMEA, the monotonicity property of the RPN scores is important. To maintain the monotonicity property of an FIS-based RPN model, a *complete* and *monotonically-ordered* fuzzy rule base is necessary. However, it is impractical to gather all (potentially a large number of) fuzzy rules from FMEA users. In this paper, we introduce a new two-stage approach to reduce the number of fuzzy rules that needs to be gathered, and to satisfy the monotonicity property. In *stage-1*, a Genetic Algorithm (GA) is used to search for a small set of fuzzy rules to be gathered from FMEA users. In *stage-2*, the remaining fuzzy rules are deduced approximately by a monotonicity-preserving similarity reasoning scheme. The monotonicity property is exploited as additional qualitative information for constructing the FIS-based RPN model. To assess the effectiveness of the proposed approach, a real case study with information collected from a semiconductor manufacturing plant is conducted. The outcomes indicate that the proposed approach is effective in developing an FIS-based RPN model with only a small set of fuzzy rules, which is able to satisfy the monotonicity property for prioritization of failures in FMEA.**

***Index Terms*—Failure mode and effect analysis, fuzzy inference system, similarity reasoning, monotonicity property, fuzzy rule reduction.**

ABBREVIATIONS & ACRONYMS

AARS	Approximate Analogical Reasoning Schema
D	*Detection*
FCBGA	Flip Chip Ball Grid Array
FIS	Fuzzy Inference System
FMEA	Failure Mode and Effect Analysis
FRI	Fuzzy Rule Interpolation
FRPN	fuzzy RPN
GA	Genetic Algorithm
O	*Occurrence*
MF	membership function
NLP	Non-Linear Programming
RPN	Risk Priority Number
S	*Severity*
S-1 *FRs*	Stage 1 Fuzzy Rules
S-2 *FRs*	Stage 2 Fuzzy Rules
SQP	Sequential Quadratic Programming
SR	Similarity Reasoning

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NOTATIONS

$\mu_x^{n_x}(x)$	A membership function for $x \in [S, O, D]$, where $n_x = 1, 2, 3, \ldots, m_x$
$A_x^{n_x}$	A linguistic term for $x \in [S, O, D]$, where $n_x = 1, 2, 3, \ldots, m_x$
m_x	Number of membership function for $x \in [S, O, D]$
p_x	$p_x \in [1, 2, 3, \ldots, m_x - 1]$
x	An input for RPN model, $x \in [S, O, D]$

I. INTRODUCTION

FAILURE mode and effect analysis (FMEA) is a popular reliability analysis tool that is used to evaluate the risks associated with potential failure modes of a complex system or process [1]–[3]. In FMEA, the risk of a failure mode is determined by a Risk Priority Number (RPN) [1], i.e., $RPN = f(S, O, D)$ whereby three risk factors, i.e., *Severity* (S), *Occurrence* (O), and *Detection* (D), act as the inputs, and an RPN score is produced as the output. In this aspect, the fuzzy RPN model has been successfully applied to a variety of domains, which include maritime [3], fishing vessel [4], manufacturing [5], power generation [6], product development [7], and agriculture [8]. The focus of this paper is on the use of the Fuzzy Inference System (FIS) in FMEA, i.e., the FIS-based RPN model [9]. The advantages of using the FIS-based RPN model, as compared with the conventional RPN model, are well-established, *viz.*, (i) the FIS-based model allows modeling of nonlinear relationships between RPN and risk factors [9]; (ii) FIS is a solution for the attribute scales, which can be qualitative, instead of quantitative [9]; (iii) FIS is able to incorporate human knowledge, whereby information can be described with vague and imprecise linguistic statements [10]; and (iv) FIS avoids the scenario whereby two or more sets of S, O and D settings with different risk implications produce an identical RPN score [4]. Despite the popularity of the FIS-based RPN model, a few limitations pertaining to FIS have been pointed out. With a grid partition strategy, the number of fuzzy rules required increases in an exponential manner, i.e., the curse of dimensionality or the combinatorial rule explosion problem. Indeed, as stated in [11], the FIS-based RPN model requires a large number of rules, and it is a time-consuming, tedious process to acquire the rules from domain experts for building a fuzzy if-then rule base.

Recent research findings have shown that it is important to maintain the monotonicity property of the FIS-based RPN model [12]–[18]. If the monotonicity property is violated, the RPN scores produced can be invalid, and with contradictions [12]–[15]. Besides that, the monotonicity property acts as useful qualitative information for building the FIS-based RPN model [12]. In our previous investigations [12]–[15], a set of *sufficient conditions* for the FIS-based RPN model to fulfill the monotonicity property has been developed. The *sufficient conditions* are used as a set of governing equations for building the FIS-based RPN model. The *sufficient conditions* in this paper (i.e., Theorem 1) are conditions with mathematical support [17] to guarantee the fulfillment of the monotonicity property pertaining to the FIS-based RPN model, i.e., a three-input FIS model. In addition, our previous studies [12], [15] showed that non-fulfillment of Theorem 1 led to violation of the monotonicity property.

The *sufficient conditions* indicate that a *complete* and *monotonically-ordered* fuzzy rule base is important to maintain the monotonicity property. However, it is difficult and impractical to establish a *complete* and *monotonically-ordered* fuzzy rule base in practice [5], owing to a potentially large number of fuzzy rules that need to be gathered from FMEA users. Therefore, an effective, systematic approach to reduce the number of required fuzzy rules is essential [5]. Nevertheless, reducing the number of fuzzy rules is risky, because this can lead to gaps in the input (S, O, and D) domains, resulting in invalid RPN scores, i.e., the "*tomato classification problem*" [20]. As a solution, different Similarity Reasoning (SR) schemes have been proposed to deduce the incomplete or missing fuzzy rules, e.g., the Approximate Analogical Reasoning Schema (AARS) [21], and Fuzzy Rule Interpolation (FRI) [20]. In our previous studies [15], [22], [23], a monotonicity-preserving SR scheme was proposed based on AARS. The approach manages to approximately deduce the missing fuzzy rules for building a monotonic FIS-based RPN model.

In this paper, we further extend our previous results [12]–[15] to tackle the pressing issues in practical implementation of the FIS-based RPN model, i.e., how to minimize the number of fuzzy rules required from FMEA users, and to ensure the resulting FIS-based RPN model satisfies the monotonicity property. The proposed solution comprises two stages: 1) a Genetic Algorithm (GA)-based fuzzy rule search procedure; and 2) a monotonicity-preserving AARS rule deduction procedure. In Stage 1, the minimum number of fuzzy rules that need to be gathered from FMEA users (i.e., the Stage 1 Fuzzy Rules, or *S-1 FRs*) is determined using a GA-based procedure. *S-1FRs* are then collected from FMEA users. In Stage 2, the remaining fuzzy rules (i.e., the Stage 2 Fuzzy Rules or *S-2 FRs*) are deduced, approximately, with a monotonicity-preserving AARS-based procedure [15], [22], [23]. *S-1 FRs* and *S-2 FRs* are aggregated to form a complete fuzzy rule base. A user-defined threshold is introduced to ensure that each *S-2 FR* has a minimum level of similarity measure with at least one *S-1 FR*. As such, each *S-2 FR* is guaranteed to be deducible with the AARS scheme from *S-1 FRs* [21]. In this paper, the monotonicity property is exploited as useful qualitative information to design the fuzzy membership functions (MFs), and to deduce the *S-2 FRs*, when the fuzzy rules solicited from experts are incomplete.

In our previous studies, we identified the importance of selecting and reducing fuzzy rules in the FIS-based RPN model [5]. The monotonicity property for tackling this task subject to a *complete* fuzzy rule base was described in [12]–[14]. In [15], [22], [23], a monotonicity-preserving SR model was devised. The monotonicity index was suggested in [24]. However, it is not clear how fuzzy rules from human experts can be minimized, and how the problem associated with an *incomplete* rule base can be handled so that the monotonicity property can be preserved for the FIS-based RPN model. As stated earlier, it is impractical to solicit a complete rule base from human experts [5], [11]. As such, the main contributions of this study are a new theorem to construct a monotonic FIS-based RPN model (which is motivated by the *sufficient conditions*), and a monotonicity-preserving approach comprising fuzzy rule selection and SR to handle the challenges associated with an incomplete rule base in FMEA applications. Besides that, a new monotonicity test is devised to evaluate the monotonicity property of the resulting model using a real case study.

This study is motivated by a number of important issues in fuzzy rule reduction and selection [25], [26], SR [20], [21], [27], and the monotonicity property [12]–[18], which have been highlighted in many recent publications. However, to the best of our knowledge, little attention has been given to the practical application of SR. One of the focal points of this study is the use of the FIS-based RPN model to prioritize failure modes, which constitutes a new application of fuzzy rule reduction and selection, as well as SR techniques. The proposed approach facilitates the practical implementation of fuzzy FMEA, i.e., the difficulty in fuzzy rule elicitation from human experts [5], [11], [28]. Besides that, the importance of the monotonicity property in fuzzy systems for assessment and decision making problems has been highlighted in [12]–[18]. The monotonicity property, as useful qualitative information for modeling, has also been stressed in [29]. In short, this study provides a solution to two key issues in the FIS-based RPN model, i.e., how to reduce the number of fuzzy rules from human experts, and how to handle an incomplete fuzzy rule base so that it satisfies the monotonicity property.

The organization of this paper is as follows. In Section II, the FIS-based RPN model, the monotonicity property, and some essential mathematical formulations are presented. In Section III, the proposed approach is described. Details of the GA-based search procedure are explained. The monotonicity-preserving AARS rule deduction procedure is presented too. The experimental results are analyzed and discussed in Section IV. Finally, concluding remarks and suggestions for further work are presented in Section V.

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II. Preliminaries

A. The FIS-Based RPN Model

The FIS-based RPN model has three inputs, i.e., S, O, D, and one output, i.e., the fuzzy RPN (FRPN) score. In general, each input (i.e., $x \in [S, O, D]$) is defined using a scale table in the range of $[\underline{x}, \overline{x}]$ (usually [1], [10]). Each partition is represented by a fuzzy MF (i.e., $\mu_x^{n_x}(x)$), and is associated with a linguistic term (i.e., $A_x^{n_x}$). The fuzzy MFs follow an ordered sequence, i.e., $\mu_x^{p_x}(x) \preccurlyeq \mu_x^{p_x+1}(x)$. The relationship between S, O, D, and the FRPN of a fuzzy rule is as follows.

R^{n_S,n_O,n_D} : *If Severity is* $A_S^{n_S}$ *and Occurrence is* $A_O^{n_O}$ *and Detect is* $A_D^{n_D}$*, then RPN is* B^{n_S,n_O,n_D}

where B^{n_S,n_O,n_D} is the fuzzy consequent in the RPN domain. To simplify the notation, a fuzzy rule is written as $R^{n_S,n_O,n_D} : A^{n_S,n_O,n_D} \rightarrow b^{n_S,n_O,n_D}$, where $A^{n_S,n_O,n_D} = A_S^{n_S} \bigwedge A_O^{n_O} \bigwedge A_D^{n_D}$, and b^{n_S,n_O,n_D} is the fuzzy singleton [10] for B^{n_S,n_O,n_D}. Based on the zero-order Sugeno FIS model [10], the FRPN score is obtained using (1), shown at the bottom of the page. The total number of fuzzy rules required for the FIS-based RPN model with a complete rule base is $m_S \times m_O \times m_D$.

B. The Monotonicity Property

A sequence, $\bar{s}$, denotes a subset of $[S, O, D]$ with two elements, whereby x is excluded, i.e., $\bar{s} \subset [S, O, D]; x \notin \bar{s}$. The monotonicity property of the FIS-based RPN model is formally established as follows.

Definition 1: The FIS-based RPN model is said to fulfill the monotonicity property if the $FRPN$ score increases or remains unchanged as x increases, i.e., $FRPN(\bar{s}, x_2) \geq FRPN(\bar{s}, x_1), \forall x_2 > x_1$.

A theorem for the FIS-based RPN model is established as follows.

Theorem 1: The FIS-based RPN model (1) is said to fulfill the monotonicity property if the following two conditions are satisfied.

a) Condition 1: $(d\mu_x^{p_x+1}(x)/dx)/\mu_x^{p_x+1}(x) \geq (d\mu_x^{p_x}(x)/dx)/\mu_x^{p_x}(x)$.

At the rule antecedent, $(d\mu_x^{p_x+1}(x)/dx)/\mu_x^{p_x+1}(x) \geq (d\mu_x^{p_x}(x)/dx)/\mu_x^{p_x}(x)$. Note that $(d\mu(x)/dx)/\mu(x)$ is the ratio between the rate of change in the fuzzy MF degree and the MF itself. In this paper, the Gaussian MF $G(x : c, \sigma) = e^{-[x-c]^2/2\sigma^2}$, is used. The derivative of a Gaussian MF with respect to x is $G^{'}(x) = -((x-c)/\sigma^2)G(x)$. As such, $(d\mu(x)/dx)/\mu(x)$ for a Gaussian MF, i.e., $(G^{'}(x)/G(x))$, is a linear function, i.e., $E(x) = G^{'}(x)/G(x) = -(1/\sigma^2)x + (c/\sigma^2)$ An example of the usefulness of *Condition 1* is demonstrated in Figs. 2 and 3.

b) Condition 2: $b^{p_x+1,p_{\bar{s}}} \geq b^{p_x,p_{\bar{s}}}$.

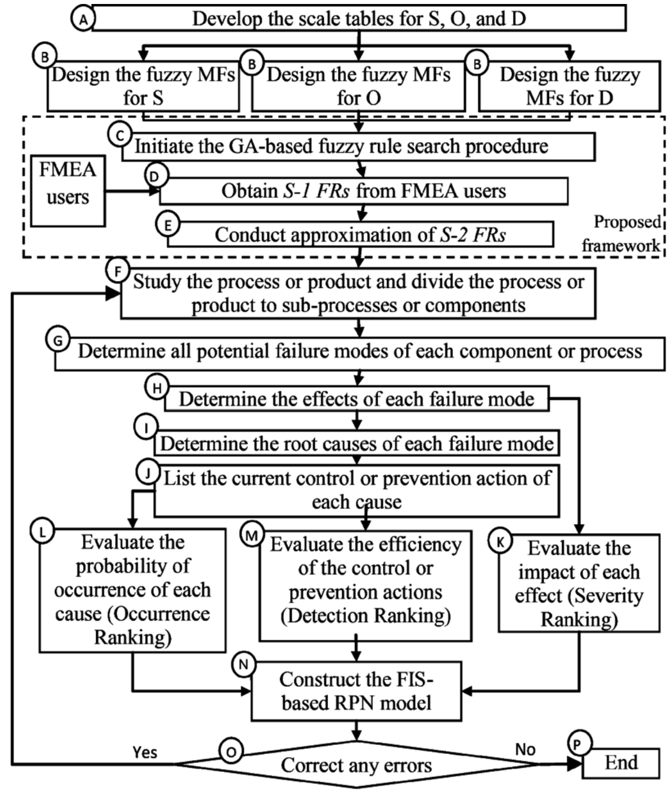

Fig. 1. The proposed approach for building a monotonicity preserving FIS-based RPN model.

Condition 2 implies that the fuzzy rule base should be complete.

An extension of Condition 1 is expressed as Corollary 1, as follows.

Corollary 1: Let $\mu_x^{n_x}(x)$ be a Gaussian MF, i.e., $\mu_x^{n_x}(x)(x : c_x^{n_x}, \sigma_x^{n_x}) = e^{-[x-c_x^{n_x}]^2/2\sigma_x^{n_x}}$, where $c_x^{n_x}$, and $\sigma_x^{n_x}$ respectively denote the centre, and width of the Gaussian MF.

1.1) The ratio $(d\mu_x^{p_x}(x)/dx)/\mu_x^{p_x}(x)$ of a Gaussian MF returns a linear function, i.e.,

$$E_x^{p_x}(x) = -\left(1/\sigma_x^{p_x^2}\right)x_i + \left(c_x^{p_x}/\sigma_x^{p_x^2}\right). \qquad (2)$$

1.2) Condition 1 of Theorem 1 is satisfied if $(E_x^{p_x+1}(\underline{x}) \geq E_x^{p_x}(\underline{x}))$ and $(E_x^{p_x+1}(\overline{x}) \geq E_x^{p_x}(\overline{x}))$ are true.

III. The Proposed Approach

Fig. 1 depicts an overview of the proposed approach. To clarify the proposed approach (Steps A through P), information and data from a semiconductor manufacturing plant are used as an example.

$$FRPN(S, O, D) = \frac{\sum_{n_S=1}^{m_S} \sum_{n_O=1}^{m_O} \sum_{n_D=1}^{m_D} \left(\mu_S^{n_S}(S) \times \mu_O^{n_O}(O) \times \mu_D^{n_D}(D) \times b^{n_S,n_O,n_D}\right)}{\sum_{n_S=1}^{m_S} \sum_{n_O=1}^{m_O} \sum_{n_D=1}^{m_D} \left(\mu_S^{n_S}(S) \times \mu_O^{n_O}(O) \times \mu_D^{n_D}(D)\right)} \qquad (1)$$

TABLE I
THE SCALE TABLE FOR SEVERITY

Rank	Linguistic Terms	Criteria
10	Very High (Liability)	Failure will affect safety or compliance to law.
9~8	High (Reliability or reputation)	Customer impact. Major reliability excursions.
7~6	Moderate (Quality or convenience)	Impacts customer yield. Wrong package, par, or marking.
5~2	Low (Special Handling)	Yield hit, Cosmetic.
1	None (Unnoticed)	Unnoticed.

TABLE II
THE SCALE TABLE FOR OCCURRENCE

Rank	Linguistic Terms	Criteria
10~9	Very high	Many/shift, many/day
8~7	High	Many/week, few/week
6~4	Moderate	Once/week, several/month
3	Low	Once/month
2	Very low	Once/quarter
1	Remote	Once ever

TABLE III
THE SCALE TABLE FOR DETECTION

Rank	Linguistic Terms	Criteria
10	Extremely low	No control available
9	Very low	Controls probably will not detect
8~7	Low	Controls may not detect excursion until reach next functional area
6~5	Moderate	Controls are able to detect within the same functional area
4~3	High	Controls are able to detect within the same machine or module
2~1	Very High	Controls will detect excursions before next lot is produced

A. Develop the Scale Tables for S, O, and D

Tables I, II, and III show the evaluation criteria used in the semiconductor manufacturing plant for S, O, and D, respectively. In this implementation, $m_S = 5, m_O = 6, m_D = 6, \bar{x} = 10$, and $\underline{x} = 1$.

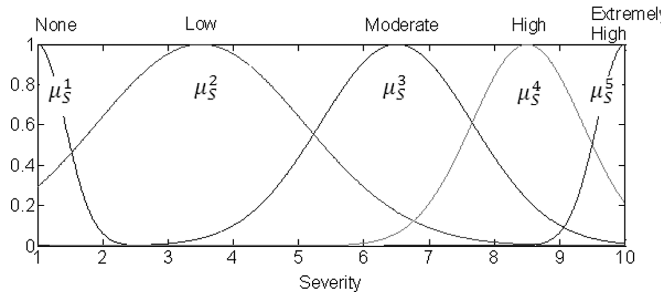

Fig. 2. Fuzzy MFs for Severity.

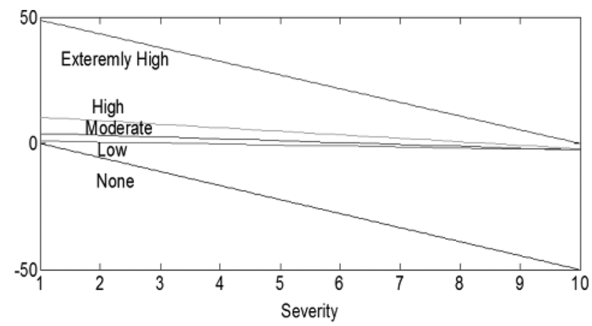

Fig. 3. Projection of fuzzy MFs for Severity using *Condition 1*.

B. Design the Fuzzy MFs for S, O, and D

Condition 1 of Theorem 1 is adopted as the governing equation to design the fuzzy MFs of S, O, and D. As an example, Fig. 2 illustrates the Gaussian MFs of S. These MFs are designed in such a way that *Corollary 1.2* is satisfied. Fig. 3 shows the projection of the Gaussian MFs with (2). It can be easily verified that *Corollary 1.2* is satisfied, i.e., $E_S^{ps+1}(1) \geq E_S^{ps}(1)$ and $E_S^{ps+1}(10) \geq E_S^{ps}(10)$ are always true, for $p_S = 1, 2, 3, 4$.

C. Initiate the GA-Based Fuzzy Rule Search Procedure

Given a complete rule base with $m_S \times m_O \times m_D$ rules, a set of *S-1 FRs* is searched in such a way that the *S-2 FRs* can be approximated with the AARS-based rule deduction procedure. Consider *S-1 FRs*, i.e., $R_{stage-1}^{n_{stage-1}} : A_{stage-1}^{n_{stage-1}} \rightarrow b_{stage-1}^{n_{stage-1}}$, where $n_{stage-1} = 1 \ldots, m_{stage-1}$, and *S-2 FRs*, i.e., $R_{stage-2}^{n_{stage-2}} : A_{stage-2}^{n_{stage-2}} \rightarrow b_{stage-2}^{n_{stage-2}}$, where $n_{stage-2} = 1 \ldots, m_{stage-2}.m_{stage-1} + m_{stage-2} = m_S \times m_O \times m_D$ is always true. Note that $b_{stage-1}^{n_{stage-1}}$ need to be gathered from FMEA users, while $b_{stage-2}^{n_{stage-2}}$ are unknown and need to be approximated. The minimum degree of similarity measure (i.e., supreme) between the union of the antecedents of *S-1 FRs* and the antecedent of each *S-2 FR* is as shown in (3) at the bottom of the next page.

In this study, the GA is used to search for a minimum set of *S-1 FRs* in such a way that each *S-2 FR* has a minimal level of similarity measure (indicated by a user-defined threshold) with at least one *S-1 FR*. Each fuzzy rule is represented as a binary chromosome, $S_i = 0, 1$, where 0 represents *S-2 FR*, and 1 represents *S-1 FR*, where $i = 1, \ldots, m_S \times m_O \times m_D$. The proposed *S-1 FRs* selection procedure is generalized as a constrained optimization problem as shown in (4) at the bottom of

GA_fuzzy_rule_selection ($t_{max}, n_{individual}, P_{crossover}, P_{mutation}, \lambda, w$)

$t = 1$
Initiate population $P(t)$ with $n_{individual}$ individual
While $t \leq t_{max}$
Compute objective value for each individual of $P(t)$ with (5)
Fitness and Ranking
Select $P_{gap}(t)$ from $P(t)$and Crossover$P_{gap}(t)$ with crossover rate $P_{crossover}$
Mutate $P_{gap}(t)$ with mutation rate$P_{mutation}$
Create new generation, $P(t+1)$
$t = t + 1$
End While
Compute objective value for each individual of P(t)with (5)
Identify the individual(s) with lowest objective value, i.e., P_{Best}
Return (P_{Best})

Fig. 4. The proposed GA-based procedure for fuzzy rule selection.

the page, where $\lambda, 1 \geq \lambda \geq 0$ is the user-defined threshold. The GA objective function is formulated as shown in (5) at the bottom of the page, where w is the weight, $w = m_S \times m_O \times m_D$. Such setting ensures that the GA-generated *S-1 FRs* always satisfy inequality (4). If inequality (4) is not satisfied, the GA-generated *S-1 FRs* are not sufficient to allow *S-2 FRs* to be deduced, pertaining to a minimal level of similarity with at least one *S-1 FR*. A summary of the GA-based procedure is shown in Fig. 4. It comprises several user-defined parameters, i.e., the number of iteration (t_{max}), number of individuals ($n_{individual}$), crossover rate ($P_{crossover}$), mutation rate ($P_{mutation}$), λ, and w. The output is the best individual with the lowest objective value, i.e., P_{Best}.

D. Obtain S-1 FRs From FMEA Users

The output of the FIS-based RPN model varies from 1 to 1000. It is represented by n_{FRPN} MFs. In this study, a discussion with the engineers from the manufacturing plant led to the setting of $n_{FRPN} = 5$, with linguistic terms of *Low*, *Low Medium*, *Medium*, *High Medium*, and *High*. This approach was deemed sufficient as more linguistic terms would result in more effort in judging all the failure modes and determining their relative importance. Note that b for these linguistic terms comprises 1, 250.75, 500.5, 750.25, and 1000, respectively. Based on the output of the GA-based procedure, *S-1 FRs* are gathered from FMEA users, with Condition 2 of Theorem 1 imposed.

E. Conduct Approximation of S-2 FRs

In accordance with [20], [21], [27], SR is useful for deducing the conclusion of an observation. In this study, the AARS [21] procedure is adopted to deduce $b_{stage-2}^{n'_{stage-2}}$. See (6) at the bottom of the next page.

Specifically, an optimization-based AARS procedure [15], [22], [23] is used. The deduced fuzzy rules from (6) are optimized to ensure that Condition 2 of Theorem 1 is satisfied. The proposed procedure serves as an initiative to re-label the *non-monotonically-ordered* conclusions [30]. Equation (7) is introduced to indicate the difference between the deduced (i.e., $b_{stage-2}^{n'_{stage-2}}$) and optimized ($b_{stage-2}^{n_{stage-2}}$) conclusions, i.e.,

$$diff = \sqrt[2]{\sum_{n=1}^{m_{stage-2}} \left(b_{stage-2}^{n_{stage-2}}{}' - b_{stage-2}^{n_{stage-2}}\right)^2}. \quad (7)$$

One of the Non-Linear Programming (NLP) methods, i.e., Sequential Quadratic Programming (SQP) [31], is adopted, which is an effective technique to solve constrained non-linear optimization problems. Note that the GA is not used in this stage because the presence of too many constraints reduces the feasible region, and complicates the search process [32]. The optimization problem is formulated as follows.

$$Minimize\ diff = \sqrt[2]{\sum_{n=1}^{m_{stage-2}} \left(b_{stage-2}^{n_{stage-2}}{}' - b_{stage-2}^{n_{stage-2}}\right)^2}$$

$$\bigcap_{n_{stage-2}=1}^{m_{stage-2}} \left(A_{stage-2}^{n_{stage-2}}, \bigcup_{n_{stage-1}=1}^{m_{stage-1}} A_{stage-1}^{n_{stage-1}}\right) = min_{n_{stage-2}=1}^{m_{stage-2}}\left(A_{stage-2}^{n_{stage-2}}, max_{n_{stage-1}=1}^{m_{stage-1}}\left(A_{stage-1}^{n_{stage-1}}\right)\right) \quad (3)$$

$$\begin{aligned} &\text{Minimize} && \sum_{i=1}^{m_S \times m_O \times m_D} S_i, \\ &\text{subject to} && \bigcap_{n_{stage-2}=1}^{m_{stage-2}} \left(A_{stage-2}^{n_{stage-2}}, \bigcup_{n_{stage-1}=1}^{m_{stage-1}} A_{stage-1}^{n_{stage-1}}\right) \geq \lambda \end{aligned} \quad (4)$$

$$Objective\ function\ (\lambda, w) = w \times \left(\begin{cases} 0 & if\ Inequality\ (4)\ is\ fulfilled \\ 1 & if\ Inequality\ (4)\ is\ not\ fulfilled \end{cases}\right) + \sum_{i=1}^{m_S \times m_O \times m_D} S_i \quad (5)$$

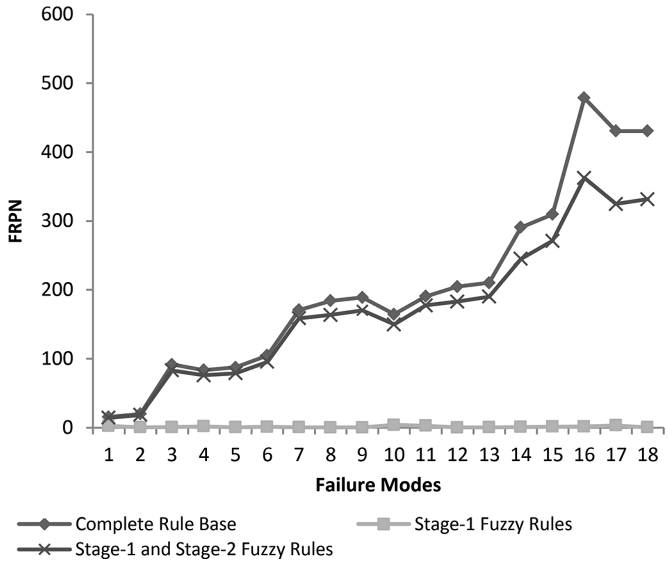

Fig. 5. Correlation analysis of FRPN values obtained while $\lambda = 0.05$.

subject to $b^{p_x+1,p_{\bar{s}}} \geq b^{p_x,p_{\bar{s}}}$, i.e., Condition 2 of Theorem 1.

F. Study the Process or Product, and Divide the Process or Product Into Sub-Processes or Sub-Components

The intention, purpose, goal, and objective of a product or process are studied. These can be achieved by scrutinizing the interaction among the components or processes, which is followed by a careful task analysis by FMEA users.

G. Determine All Potential Failure Modes of Each Component or Process

All potential failures of the component or process which include problems, concerns, and opportunities for improvement are identified by FMEA users.

H. Determine the Effects of Each Failure Mode

Immediate consequences for each potential failure are identified by FMEA users.

I. Determine the Root Causes of Each Failure Mode

The potential root cause of each failure mode is identified by FMEA users.

J. List the Current Control or Prevention Action of Each Cause

The first level method or procedure to detect or prevent failures of the product or process is conducted by FMEA users.

K. Evaluate the Impact of Each Effect (Severity Ranking)

The severity score is executed and evaluated by FMEA users.

L. Evaluate the Probability of Occurrence of Each Cause (Occurrence Ranking)

The occurrence score is executed and evaluated by FMEA users.

M. Evaluate the Efficiency of the Control or Prevention Actions (Detection Ranking)

The detection score is executed and evaluated by FMEA users.

N. Construct the FIS-Based RPN Model

S-1 FRs and *S-2 FRs* are aggregated to form a complete fuzzy rule base. With the designed MFs for S, O, and D, and the aggregated fuzzy rule base, the final FIS-based RPN model (1) is constructed.

O. Correct Any Errors

Return to Step F if there is any correction or refinement to be made.

P. End

IV. A Case Study

To evaluate the effectiveness of the proposed FIS-based RPN model, we conducted a series of experiments with real data and information collected from Flip Chip Ball Grid Array (FCBGA) products in a semiconductor manufacturing plant. Specifically, we examined the wafer mounting process with the proposed approach. We used computer equipped with Intel (R) Core (TM) i5-2300 CPU @2.80 GHz 2.79 GHz., and 3.35 GB of RAM. We analyzed several aspects of the proposed approach. The details follow.

A. Design of the Fuzzy MFs

Formulating an appropriate and systematic methodology for designing the fuzzy MFs has been highlighted as a challenging problem [19]. It is difficult to design the fuzzy MFs that ensure the resulting FIS-based RPN model satisfies the monotonicity property. Our proposed approach reduces the trial and error effort in obtaining a set of fuzzy MFs and fuzzy rules that satisfy the monotonicity property. It is a relatively simple, practical solution for designing the fuzzy MFs with fulfilment of the monotonicity property. As an example, for the S fuzzy MFs (as depicted in Fig. 2), they are able to fulfil the monotonicity property once Condition 2 is satisfied.

B. Selection of Fuzzy Rules

1) Fuzzy Rule Reduction: We conducted a series of experiments with the GA-based fuzzy rule selection procedure with $t_{max} = 50, n_{individual} = 40, P_{crossover} = 0.7, P_{mutation} = 0.05$, and $w = m_S \times m_O \times m_D$. We analyzed the computation complexity (i.e., c_λ in seconds), and the number of *S-1 FRs*, n_λ,

$$b_{stage-2}^{n'_{stage-2}} = \sum\nolimits_{n_{stage-1}=1}^{m_{stage-1}} \left(sup\left(A_{stage-1}^{n_{stage-1}}, A_{stage-2}^{n_{stage-2}}\right) \times b_{stage-1}^{n_{stage-1}}\right) / \sum\nolimits_{n_{stage-1}=1}^{m_{stage-1}} \left(sup\left(A_{stage-1}^{n_{stage-1}}, A_{stage-2}^{n_{stage-2}}\right)\right) \quad (6)$$

TABLE IV
THE EXPERIMENTAL RESULTS FOR VARIOUS SETTINGS OF λ

λ	n_λ	Percentage of fuzzy rules reduction	c_λ (second)
0.05	30	83.33	8518
0.10	31	82.78	11808
0.15	38	78.89	9087
0.20	39	78.33	9645

for a variety of λ settings. Equation (8) at the bottom of the page shows the percentage of fuzzy rule reduction.

Table IV shows a summary of the experimental results. Note that a conventional fuzzy FMEA model [4] would require 180 fuzzy rules, and it was time-consuming and impractical to gather such information. With the proposed approach, the number of fuzzy rules reduced drastically, ranging from 78.33% to 83.33%. As an example, with $\lambda = 0.05$, only 30 fuzzy rules were required, and the remaining 150 fuzzy rules were approximated by using the AARS-based procedure.

2) Variation of λ: The GA-based fuzzy rule selection procedure ensured that each *S-2 FR* has a minimum degree of similarity measure (indicated by λ) with at least one *S-1 FR*. With a higher λ setting, more *S-1 FRs* were required. As shown in Table IV, n_λ increased with an increasing value of λ. With different λ settings, different minimal sets of *S-1 FRs* that could meet the pre-determined minimum degree of similarity measure were obtained. Note that the number and location of *S-1 FRs* were not pre-determined, but were encapsulated in the GA objective function. Instead of determining the important fuzzy rules [5], the proposed approach focused on determining a minimal set of *S-1 FRs* in such a way that *S-2 FRs* could be approximated by the AARS-based procedure.

3) Computation Complexity: An important issue in the practical implementation of the GA is its computational complexity. The computational durations were between 2.37 and 3.28 hours, with a low-cost consumer-oriented computer. We deemed this duration to be an acceptable interval as FMEA initiatives normally require days or even weeks [33]. Besides that, it is a time-consuming, tedious process [5], [11], [28] to acquire fuzzy rules from domain experts in building a complete fuzzy rule base. As the GA was used as an approximate optimization tool, a near optimal solution was obtained. It was expected that, with a higher $t_{\max}$, a better set of *S-1FRs* (i.e., fewer number of fuzzy rules) could be obtained.

C. Analysis of the Risk Evaluation Results

1) Ranking Results: Table V summarizes the experimental results for the wafer mounting process with two λ settings (i.e., 0.05, and 0.15). A total of 18 failure modes are listed in the column labelled Failure Mode. Columns S, O, and D show the three risk factors associated with each failure mode. The failure risk evaluation outcomes with (i) a complete rule base (180 rules) provided by FMEA users (domain experts), (ii) with *S-1 FRs* only, and (iii) with aggregated *S-1 FRs* and *S-2 FRs*, are listed in the columns labelled Complete rule base, *S-1 FRs*, and *S-1 FRs* and *S-2 FRs*, respectively.

TABLE V
FAILURE RISK EVALUATION, RANKING AND PRIORITIZATION FOR THE WAFER MOUNTING PROCESS

Failure Mode	S	O	D	Fuzzy RPN score: Complete rule base	λ=0.05		λ=0.15	
					S-1FRs	*S-1FRs* and *S-2FRs*	*S-1FRs*	*S-1FRs* and *S-2FRs*
1	3	1	1	15	2	14	3	164
2	3	2	1	20	0	18	4	167
3	2	3	2	92	1	83	0	190
4	3	1	2	84	2	76	2	193
5	3	2	2	87	0	79	3	196
6	3	3	2	105	1	95	0	210
7	2	4	2	171	0	159	0	259
8	2	2	3	184	0	164	0	224
9	2	3	3	189	0	170	0	245
10	3	4	1	165	4	150	0	233
11	3	4	2	191	3	177	0	282
12	3	2	3	205	0	183	1	247
13	3	3	3	210	0	190	0	267
14	3	2	4	291	1	245	5	306
15	4	3	4	310	1	271	7	341
16	2	2	10	479	2	363	2	592
17	3	2	5	430	3	324	17	434
18	3	3	5	431	0	331	3	441

As an example, Failure Mode 1 (broken wafer), which led to yield loss, was given an S score of 3. This failure could happen because of the drawing out arm failure. As it rarely happened, it was given an O score of 1. To eliminate the problem, software enhancement was suggested as the corrective action. Owing to the effectiveness of the action to eliminate the root cause, a D score of 1 was given. With the complete fuzzy rules, an FRPN score of 15 was obtained. With $\lambda = 0.05$, the FRPN scores of 2, and 14 were obtained using *S-1 FRs* only, and *S-1 FRs* and *S-2 FRs*, respectively.

As expected, good results were obtained using the complete rule base. But, it was difficult to ensure a complete set of fuzzy rules to be gathered in the first place. With *S-1 FRs* only, the tomato classification problem occurred, as some of the FRPN scores were almost zero. As an example, the FRPN score of 2 was obtained for failure mode 1 with $\lambda = 0.05$, owing to the phenomenon of gaps, where some fuzzy rules were missing. Therefore, the inferred results were invalid. With the aggregated *S-1 FRs* and *S-2 FRs*, the tomato classification problem could be solved.

2) Correlation Analysis of the FRPN Scores: A correlation analysis [3] for the FRPN scores deduced from the FIS-based RPN models is summarized in Fig. 5. The FRPN scores for all

$$Percentage\ of\ fuzzy\ rule\ reduction = \frac{m_S \times m_O \times m_D - n_\lambda}{m_S \times m_O \times m_D} \times 100\% \tag{8}$$

$$monotone(x) = \begin{cases} 1 & FRPN(\bar{s}, x+1) \geq FRPN(\bar{s}, x) \\ 0 & else \end{cases} \tag{9}$$

$$Monotone\ Index\ (x) = \sum_{s_2=1}^{s_2=10} \sum_{s_1=1}^{s_1=10} \sum_{x=1}^{x=9} (monotone(x)) \tag{10}$$

18 failure modes are compared. The FIS-based RPN model with aggregated *S-1 FRs* and *S-2 FRs* was able to produce a set of FRPN scores that closely matched those from the complete rule base, as compared with *S-1 FRs* only. Without *S-2 FRs*, most of the deduced FRPN scores from *S-1 FRs* were close to zero. The correlation analysis ascertained the usefulness of the AARS procedure in producing *S-2 FRs* as a solution to the tomato classification problem.

3) Monotonicity Test: The monotonicity test [24] was adopted to evaluate the degree of fulfilment of the monotonicity property for x. Note that $\bar{s} = [s_1, s_2]$ denotes a subset of $[S, O, D]$, where x is excluded. The proposed test attempts to give an indication whether the FIS-based RPN model can be practically implemented. Equation (9) is devised to compare two comparable sets of risk factors with respect to the monotonicity property of the FIS-based RPN model. If the test result is unity, the monotonicity property is satisfied; otherwise, a score of 0 is returned. Equation (10) produces a Monotonicity Index (MI), where $0 \leq MI \leq 900$, for x by aggregating all the possible combinations of s_1 and s_2. The higher the MI is, the better the degree of fulfilment of the monotonicity property. If $MI = 9, x$ is said to satisfy the monotonicity property. See (9) and (10) at the top of the page.

Table VI summarizes the monotonicity test results for $x = S, O, D$. Columns x and λ indicate the risk factor with different λ settings. Columns $Monotone\ Index\ (x)$ show MI for x with (i) the complete rule base, (ii) *S-1 FRs* only, and (iii) aggregated *S-1 FRs* and *S-2 FRs*. It can be observed that, with the complete fuzzy rule base, MI $=$ 900, therefore satisfying the monotonicity property. With *S-1 FRs* only, all MI values are far lower than 900, indicating an inability to satisfy the monotonicity property. However, with *S-1 FRs* and *S-2 FRs*, MI $=$ 900 for all λ settings, therefore satisfying the monotonicity property again.

TABLE VI
THE MONOTONICITY TEST

x	λ	$Monotone\ Index\ (x)$		
		Complete rule base	*S-1FRs*	*S-1FRs and S-2 FRs*
S	0.05	900	602	900
	0.15		541	900
O	0.05	900	552	900
	0.15		496	900
D	0.05	900	424	900
	0.15		546	900

V. Summary

In this paper, we proposed a new approach comprising a GA-based fuzzy rule selection procedure and an AARS-based rule deduction procedure for the FIS-based RPN model. We addressed two important limitations of the FIS-based RPN model: obtaining a complete rule base, and satisfying the monotonicity property. We conducted a case study using data and information obtained from a semiconductor manufacturing plant. The results positively demonstrate the effectiveness of the proposed approach in constructing an FIS-based RPN model that aggregates the required fuzzy rules to preserve the monotonicity property.

For future work, we intend to develop a suitable technique to re-label non-monotonic *S-1 FRs* [23]. Instead of imposing a monotonicity constraint for *S-1 FRs*, the non-monotonic fuzzy rules can be identified and re-labeled by using the re-labeling technique. Other approaches such as evidential reasoning [34], [35] for re-labeling fuzzy rules and ensuring a monotonic fuzzy rule base can be examined. In this regards, evidential information in terms of a belief function associated with the fuzzy rule base, as used in [36], needs to be established. In addition, practical implementation of the proposed approach to other FMEA applications can be conducted. All these ideas constitute the direction for further work.

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