



Faculty of Engineering

**EXPERIMENTAL STUDY ON THE HIGH STRENGTH CONCRETE  
WITH DIFFERENT FINE AGGREGATE**

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**Bachelor of Engineering with Honours  
(Civil Engineering)  
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DIFFERENT FINE AGGREGATE**

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DIFFERENT FINE AGGREGATE

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**To all my family members**

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## **ABSTRACT**

The results of an experimental study on the high strength concrete (HSC) with different fine aggregate is presented. The study mainly concern on the influence of the characteristic of shape and texture of the fine aggregate on mechanical properties of HSC. Two types of fine aggregate were used namely crushed granite sand and natural river sand. The HSC mixtures consist of high cement content of  $530 \text{ kg/m}^3$  and low water-cement ratio of 0.30 was selected. The presence of crushed rock dust in crushed rock sands that influence the mechanical properties of HSC was also considered in this study. HSC mixtures comprises of crushed granite sand with various percentage of crushed granite dust content such as 0, 5, 10 and 15% were prepared. The compressive strength of the HSC was determined by conducting a series of cube compression tests. The experimental results indicated that: (a) The performance of HSC in term of compressive strength was superior with crushed granite sand (angular and rough grains) rather than natural river sand (rounded and smooth grains). (b) amount of 10% of crushed granite dust content was found to improve the compressive strength of the HSC mixtures, thus, the 10% was considered as the optimum dust content because the compressive strength of the HSC decreased slightly as the content of crushed granite dust increased to higher amount of 15 %.

## **ABSTRAK**

Hasil ujian daripada kajian eksperimen mengenai konkrit berkekuatan tinggi dengan pelbagai jenis pasir dipersembahkan. Kajian ini memberi perhatian terhadap pengaruh ciri-ciri bentuk dan tekstur permukaan pasir terhadap ciri-ciri mekanikal konkrit berkekuatan tinggi. Dua jenis pasir iaitu pasir batu granite dan pasir sungai telah digunakan untuk kajian. Campuran konkrit berkekuatan tinggi yang mengandungi kandungan simen yang tinggi iaitu  $530 \text{ kg/m}^3$  dan nisbah air kepada simen yang rendah iaitu 0.30 telah dipilih. Kewujudan debu batu dalam pasir batu kasar yang mempengaruhi ciri-ciri mekanikal konkrit berkekuatan tinggi juga diambil perhatian dalam kajian ini. Campuran konkrit berkekuatan tinggi yang mengandungi pasir batu granite dengan pelbagai peratusan kandungan debu batu kasar granite sebanyak 0, 5, 10 dan 15 % telah disediakan. Kekuatan mampatan konkrit berkekuatan tinggi telah ditentukan dengan satu siri ujian mampatan kiub. Hasil eksperimen telah menunjukkan bahawa: (a) prestasi konkrit berkekuatan tinggi daripada segi kekuatan mampatan adalah lebih tinggi dengan pasir batu granite (butiran bersegi dan kasar) berbanding pasir sungai (butiran bulat dan halus). (b) 10 % kandungan debu batu granite telah meningkatkan kekuatan mampatan campuran konkrit berkekuatan tinggi, oleh itu, kandungan 10 % tersebut telah diambil kira sebagai kandungan optimum debu kerana kekuatan mampatan konkrit berkekuatan tinggi menurun secara perlahan apabila kandungan debu tersebut meningkat kepada 15 %.

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# **SECTION 1**

## **INTRODUCTION AND SCOPE OF STUDY**

### **1.1 INTRODUCTION**

The general purpose of this project was to investigate the mechanical properties of high strength concrete (HSC) with different types of fine aggregates. The types of fine aggregates used were natural river sand (NRS) and crushed granite sand (CGS). Both of the fine aggregates were chosen due to its differences in terms of their characteristics such as shape and texture. The cube compression tests were conducted in order to determine the mechanical properties such as compressive strength of HSC with different types of fine aggregate.

The main interest of the study was to investigate the effect of crushed granite dust content on the hardened properties of HSC. Therefore, four different mixtures of HSC that consists of crushed granite sand with various percentage of dust content were produced in order to determine the effect on the hardened

properties of HSC. The percentages of dust content of 0, 5, 10, 15 by weight of fine aggregate were chosen in this experimental investigation.

## **1.2 BACKGROUND**

HSC has been used in construction all over the world. It has been applied in many fields of construction and used in the completion of structures such as core wall and column in high buildings, long span bridges and some other structures that require higher compressive strength of concrete. HSC can be classified by its mechanical properties, mainly compressive strength which is higher compared to normal strength concrete. The mixtures of HSC usually composed of high cement content, low water-cement ratio and presence of several admixtures. In daily engineering practice, aggregates that obtained from various sources are used to produce concrete. Thus, the differences of characteristics of aggregate used in concrete mixture might influence the properties of concrete.

For instance, Donza et al. (2002) found that properties of HSC made from different type of fine aggregate was influenced by the different characteristics of the fine aggregates used. Natural sand and crushed rock sand made of granite, limestone and dolomite with different shape, texture and mineralogical source influenced the mechanical properties such as compressive and splitting tensile strength. Consequently, the mechanical strength of HSC produced from the crushed sand was similar or better than HSC with natural sand.

The difference of mechanical properties with various types of sand were reported by Kim et al. (1997). The particular amount of crushed rock dust included in crushed rock sand was proved to improve the properties of concrete based on the correlation made between mechanical properties such as compressive strength, splitting tensile strength and fracture energy of concrete. Accordingly, the mechanical properties of concrete with natural sand and crushed rock sand were different due to the inclusion of crushed rock dust. Moreover, Celik and Marar (1996) also reported the inclusion of crushed rock dust in crushed rock sand improved the mechanical properties such as compressive and flexural strength. The series of laboratory tests conducted concluded that the mechanical properties as well as fresh properties were influenced by the particular amount of crushed rock dust included in crushed rock sand.

### **1.3 AIMS AND OBJECTIVES**

The aims of the present study were to investigate the influence of fine aggregate type and dust content on the mechanical properties of HSC. In accordance with the aims of the study, the main objectives were as follows:

- 1) To conduct a series of cube compression tests of HSC with NRS and CGS.

- 2) To conduct a series of cube compression tests in order to determine the compressive strength of HSC that consists of CGS with the percentages of 0, 5, 10 and 15 of dust content by weight of fine aggregate.

#### **1.4 OUTLINE OF PROJECT REPORT**

Section 2 provides information and studies which are related to this project. The contents of this section also represents the source references used to conduct this project.

Section 3 describes the programme and methodology of the laboratory works such as cube compression test, materials selection and the preparation of specimens.

Section 4 illustrates the results gathered from the experimental works and analysis of the data obtained from the test results.

Finally, section 5 contains a conclusions based on the discussion of results in the preceeding section, and recommendations for future works.

## **SECTION 2**

### **LITERATURE REVIEW**

#### **2.1 GENERAL**

The purpose of this section is to present the information gathered which are related to this project. The first part presents the information on crushed fine aggregates and its effect on some properties of concrete.

The second part of this section presents the researches conducted and information related to HSC. The information of HSC based on researches done includes the mix proportion of HSC, materials and properties of HSC, and application of HSC.

## **2.2 AGGREGATES**

### **2.2.1 Crushed fine aggregate**

In practice, natural aggregates from natural deposits are usually taken and used in the production of concrete. Natural river sand is existing fine aggregate that commonly used for concrete mixture. Typically, crushed rock coarse aggregates produced by crushing a large parent mass of rock are used as coarse aggregate. During the process of rock crushing in quarry, the fine aggregates of crushed rock are likely occur instead of coarse aggregate. The crushed rock fine aggregate which has the particle size similar to sand particles could also being used for the preparation of concrete mixture. Several studies have been undergone on normal concrete and HSC incorporated with crushed rock fine aggregate such as research by Donza et al. (2002), Kim et al. (1997), and Celik and Marar (1996).

Practically, coarse aggregate and fine aggregate produced from crushed rock can be differentiate by sieve analysis. “According to ASTM C125 (Concrete and Concrete Aggregates), fine aggregate is defined as aggregate passing a 3/8 in (9.5 mm) sieve and almost entirely passing a No. 4 (4.75 mm) sieve and predominantly retained on the No. 200 (75  $\mu$ m) sieve or that portion of an aggregate passing the No.4 (4.75 mm) sieve and retained on the No. 200 (75  $\mu$ m) sieve. Coarse aggregate is defined as aggregate predominantly retained on the No. 4 (4.75 mm) sieve or that portion of an aggregate retained on the No. 4 (4.75 mm) sieve” (Derucher et al. ,1998).

Neville and Brooks (1990) mentioned that fine aggregate and coarse aggregate are separated according to a size of 5 mm BS sieve (3/16 in.) or No. 4 ASTM sieve (4.75 mm), which is the main division to obtain the two different type of aggregate in term of size. The separation of fine and coarse aggregate is always used to produce good quality concrete. Table 2.1 shows the BS and ASTM sieve sizes that typically used for grading of aggregate.

Instead of the appearance of coarse and fine aggregate in quarry, quarry rock or stone dust is formed during the process of rock crushing. The by-product generally accounting less than 1% of the production of aggregate (Ho et al., 2002). Moreover, crushed rock sand is slightly different from natural sand in term of very fine particle or crushed stone dust which are the particles passing 75  $\mu$ m BS test sieve (BS 882: Part 2, 1983: quoted from Celik and Marar, 1996). Moreover, BS 882 proposed the limit of dust content in crushed sands to 16 %, but lower limit is proposed by ASTM C33 with maximum dust content of 7 % (Donza et al., 2002).

According to Neville (1995), the particle shape of aggregate is generally classified by its roundness. The term roundness referred to the measurement of the relative sharpness or angularity of the edges and corners of a particle. The major factors of roundness are the strength and abrasion resistance of the parent rock, and the amount of wear subjected to the particles. Furthermore, the roundness of crushed aggregate is affected by the reduction ratio and type of crusher, instead of

the nature of parent rock. The classification of roundness according to BS 812: Part 1: 1975 is presented in Table 2.2.

Mindess and Young (1981) pointed out that crushed stone is different from natural aggregate such as natural sand and gravel in term of surface texture. The surface texture of natural aggregate is generally smooth due to the effect of weathering and water. In the other hand, crushed stone has rougher surface than natural aggregate. The classification of surface texture by BS 812: Part 1: 1975 is shown in Table 2.3.

In term specific gravity, Neville (1995) mentioned that specific gravity for most of natural aggregate are in the range of 2.6 to 2.7. Table 2.4 shows the range of specific gravity of different group of rock.

### **2.2.2 Effect of crushed rock fine aggregate on properties of concrete**

Consequently, the characteristics of crushed rock fine aggregate as mentioned above have influences on the properties of fresh and hardened concrete. In the case of workability, Mindess and Young (1981) stated that aggregates with rounded shape and smooth surface texture such as natural sand and gravel are more advantageous than angular shaped and rough surfaced crushed aggregates. The angular shape leads to the interference of movement between aggregate particles. In addition, more paste are required to coat all particles due to higher surface to volume ratio of angular shaped aggregates.

Moreover, the rough surface texture also effects the workability of fesh concrete as more paste is required to lubricate the adjacent particles of aggregate and permit them to move easily.

On the contrary, the particle shape and texture of crushed aggregate have more beneficial effects on mechanical properties of hardened concrete. Better bonding or adhesion between the aggregate particles and cement matrix in concrete microstructure are due to the rough surface texture of aggregate. Furthermore, the angular shape also improving the mechanical properties as the characteristic results in larger surface area, the larger contact area is available and thus greater bond can be achieved (Neville and Brooks, 1990).

More detail information on the infulences of the fine aggregate characteristic including the content of crushed stone dust are presented based on the following studies.

Donza et al. (2002) have conducted a study on HSC with different fine aggregate. In the study, two mixtures containing  $530 \text{ kg/m}^3$  cement and water-cement ratio of 0.30 were prepared by crushed granite sand (GS) and river sand (RS). Accordingly, the mixture of crushed granite sand concrete required higher dosage of superplasticizer to obtain high slump due to the effect of angular shape and rough surface texture of granite sand particles. On the other hand, the granite sand concrete has the higher strength than river sand concrete as presented in Figure 2.1. They stated that rough surface texture of granite sand particles that

contributed to the strong paste-fine aggregate interface could be related to the greater compressive strength. Moreover, six other mixtures with low water-cement ratio and cement content of 450 and 485 kg/m<sup>3</sup> were prepared by granite (GS), limestone (LS) and dolomite (DL) crushed sand with similar grading.. The gradation and physical characteristics of the three different crushed sands are presented in Table 2.5. As a result, granite sand concrete had shown the highest compressive strength. They stated that the shape and texture of granite sand particles which were different from the other two crushed sand particles have contributed to the higher compressive strength. Mix proportions and slump test results obtained in the study are presented in Table 2.6.

Kim et. al. (1997) have presented a study on fracture characteristics of crushed rock sand concrete. One of the mixture proportion in the study was prepared with cement content of 628 kg/m<sup>3</sup> and water-binder ratio of 0.20. Crushed limestone sand, crushed granite sand, and river sand were selected for the mixture. As a result, river sand concrete and crushed granite sand concrete have shown the highest compressive strength ( $f_c$ ) and tensile splitting strength ( $f_{sp}$ ), respectively. However, the fracture energy ( $G_f$ ) of river sand concrete was slightly lower than the other two crushed sand concretes as presented in Table 2.7. They stated that the improvement of cohesion between cement paste and aggregate in crushed sand concrete was contributed by crushed stone dust included in the fine aggregates. Moreover, the analysis of stone dust content on crushed limestone sand concrete was also conducted in the study. A mixture containing 331 kg/m<sup>3</sup> cement and water-cement ratio of 0.6 was prepared by crushed limestone sand

with adjusted content of crushed stone dust (0, 3, 6 % by weight of fine aggregate). Accordingly, the strength of concrete with 3% stone dust was better than concrete with 0% stone dust. Furthermore, the strength of concrete with 6% stone dust was slightly lower than concrete with 3% stone dust. Details of test results of compressive and tensile strength are shown in Figure 2.2 and Figure 2.3, respectively.

The other study on the effect of crushed stone dust on properties of concrete was also conducted by Celik and Marar (1996). Seven different mixtures were determined using crushed limestone fine aggregate with various percentages of stone dust of 0, 5, 10, 15, 20, 25, and 30 by weight of fine aggregate. Test results indicated that the increased content of crushed stone dust up to 10% of fine aggregate weight had contributed to the improvement of concrete strength but increasing the dust content more than 10% had decreased the strength gradually. Furthermore, the slump of fresh concrete decreased as the content of crushed stone dust increased. They stated that the development of compressive strength was due to the dust particles that filling in the voids between cement paste and aggregate particles. However the strength had decreased as a result of the excessive amount of stone dust as cement paste was insufficient to coat all the aggregate particles. Additionally, the fineness of fine aggregate increased since the percentage of dust increased. The increment resulted in the larger specific surface and more amount of water is required to wet the increased amount surfaces of particles. Table 2.8 shows the slump test results and the compressive strength test results obtained in the study are presented in Figure 2.4.