Adjusted and Controllability Properties of Memristor Rucklidge System

1st Hongyan Zang
1：*Collaborative Innovation Center of Memristive Computing Application*
Qilu Institute of Technology
Jinan 250200, P.R. China
2：*Faculty of Engineering*
Universiti Malaysia Sarawak
Kota Samarahan, Sarawak. Malaysia
hyzang@qlit.edu.cn

2nd Lili Huang
College of Intelligent Manufacturing and Control Engineering
Qilu Institute of Technology
Jinan 250200, P.R. China
llhuang@qlit.edu.cn

3rd Shourong Zhang
Shandong Vanform High Energy Physics Technology Co.Ltd.
Jinan 250100, P.R. China.
zhangshourong@van.form.com

4thAnnie Anak Joseph
Department of Electrical and Electronics
Universiti Malaysia Sarawak
Kota Samarahan, Sarawak. Malaysia
jannie@unimas.my

5th Haiyan Fu
College of Computer and Information Engineering
Qilu Institute of Technology
Jinan 250200, P.R. China
57802020@qq.com

6th Tengfei Lei
Collaborative Innovation Center of Memristive Computing Application (CICMCA)
Qilu Institute of Technology
Jinan 250200, P.R. China
leitengfei2017@qlit.edu.cn

2023 IEEE International Conference on Memristive Computing and Applications (ICMCA) | 979-8-3503-1623-0/23/$31.00 ©2023 IEEE | DOI: 10.1109/ICMCA59770.2023.10481231

***Abstract*—There are many models of memristors, but there are no reports on exponential memristors. In this paper, a three-dimension memristor Rucklidge system with exponential memristor is proposed by introducing exponential function and constructing exponential memristor. The dynamic behavior of the system is analyzed by means of Lyapunov exponent and bifurcation diagram. The complex attractor coexistence and amplitude modulation characteristics of the system are found. The internal variable of memristor is introduced into the second two-dimensional equation as a feedback term, and a conditional symmetric system is constructed for the three-dimensional memristor system, which can realize the polarity control of output signal. By discussing the dynamic characteristics of exponential memristor chaotic system and the control mechanism of amplitude modulation, bias and conditional symmetry, the complex dynamic characteristics of exponential function memristor are revealed, which lays a foundation for its application in the field of secure communication.**

***Keywords*—memristor, conditional symmetry, coexistence attractors，amplitude modulation**

I. Introduction

As an electronic component with memory properties, memristors play a very important role in the development of artificial intelligence technologies such as image encryption and synaptic simulation. The introduction of memristors into chaotic systems can affect system dynamics and even lead to extreme multistability [1]-[5]. Numerous studies of memristor models and typical chaotic systems have been published, and memristors have been widely studied in the fields of secure communication [6,7], cryptography [8], [9], image encryption [10], [11], memory [12], [13], speech encryption [14], recognition and sequencing applications [15].

The results show that the adjustment of the system itself can produce different chaotic signals without providing peripheral Settings to interfere with the adjustment. Similar self-modulation methods include frequency control [16], medium amplitude control and conditional symmetry control [17]-[19], and super multi-stable attractors [20]-[22], and so on. Conditional symmetry overcomes the structural limitations of the system and finds possible polar opposite-coexisting attractors in asymmetric systems. Many systems have symmetries in science and engineering [23]-[25]. A chaotic system with two cubic nonlinear terms is proposed, which has fully symmetrically coexisting bifurcation behavior and has the same Lyapunov exponent [26]. A 4-dimensional conditional symmetric system memristor chaotic system is proposed, which also has rich characteristics of amplitude modulation and frequency modulation [27].

However, most of these memristor systems are four-dimensional or higher, which leaves a lot of room for the study of three-dimensional memory chaotic systems. A three-dimensional memristor chaotic system with conditional symmetry is constructed by embedding a secondary magnetic-controlled memristor in varible-boostable chaotic system [28]. The offset boosting and conditional symmetry of the system are discussed emphatically. It can be seen that three-dimensional memristor chaotic system also has the characteristics of high dimensional system. It is a kind of dimensionality reduction processing to combine the feedback term introduced into the system with the internal variables of the memristor. Since the introduction of the memristor system will maintain a high complexity, it also provides ideas for dimensionality reduction processing of the high-level system.

In 1992, Professor Rucklidge proposed the Rucklidge system in his research on fluid mechanics. This system is like as Lorenz system. It is a three-dimensional autonomous chaotic system. Lai Qiang [29] introduced the piece linear term with absolute value into Rucklidge system, and constructed a minimalist three-dimensional chaotic system with amplitude modulation and multiple coexistence. The absolute piecewise linear term used in this system is similar to the memristor expression in the quadratic active magnetron memristor model proposed by Bao [30]. Therefore, this term can add memory function to the system.

The study of nonlinear vibration and chaotic phenomena in fluid dynamics is of great significance for understanding the motion，and has certain reference value many complex processes in fluid dynamics and improving various fluid dynamics control methods. We study the Rucklidge system in this context. In order to enrich the dynamic behavior of the system, improve the un predictability of the system and obtain better chaos effect, the memristor is introduced into the

Natural Science Foundation of Shandong Province under Grant (ZR2022MA073).
Research Program of Qilu Institute of Technology (QIT22NN007).

Rucklidge system. In this study, an exponential memristor is constructed by introducing an exponential function, then a three-dimensional Rucklidge system with exponential memristor is proposed. In order to introduce the memristor without increasing the dimension of the system, the derivatives of the internal variables of the memristor are considered here using the third dimension of the Rucklidge system directly. In this new model, we find complex attractor coexistence and amplitude modulation properties. The effects of the initial values of the system parameters and state variables on the system are analyzed in detail by means of the system phase trajectory diagram, bifurcation diagram and Lyapunov exponent.

II. Model of Memristor Chaotic Systems

A. System Mathematical Model

The Rucklidge system was proposed by Rucklidge while studying a class of fluid dynamics problems in 1992. It is related to thermodynamics and is a three-dimensional autonomous chaotic system similar to the Lorenz system.

$$\begin{cases} \dot{x} = -ax + by - yz \\ \dot{y} = x \\ \dot{z} = y^2 - z \end{cases} \tag{1}$$

B. Memristor Model

Chaotic systems and memristor chaotic systems with exponential terms have been studied, but there is no exponential memristor. In order to embed the memristor into the three-dimensional system without increasing the dimension, the exponential memristor model is constructed by taking the third dimensional equation of Rucklidge system as the internal variable of the memristor. Mathematical expression of the memristor model can be described as follows:

$$\begin{cases} i = W(\varphi)\ u \\ W(\varphi) = e^{m\varphi} \\ \dot{\varphi} = u^2 - \varphi \end{cases} \tag{2}$$

The exponential memristor is introduced into the first dimension of Rucklidge system as a feedback term, and a new three-dimensional memristor chaotic system is obtained. The mathematical model of Memristor Rucklidge system is expressed as:

$$\begin{cases} \dot{x} = -ax + by - cye^{mz} \\ \dot{y} = x \\ \dot{z} = y^2 - z \end{cases} \tag{3}$$

where *x*, *y*, and *z* are three dynamical variables and *a*, *b*, *c*, *m* are all positive parameters. In system (3). The parameters of the system are $a = 2$, $b = 10$, $c = 1$, $m = 3$, and initial condition (IC) is (1, 0, 2). Numerical simulation software Matlab is used to numerical simulation, and the memristor Hysteresis characteristic curve and the phase trajectories of each plane are shown in Figure 1. The attractor of system (3) is symmetric attractor.

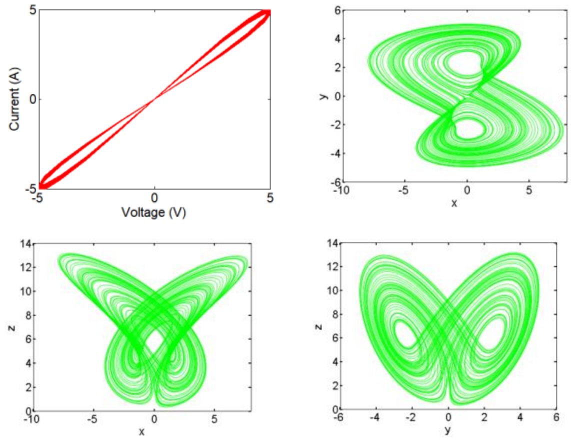

Fig. 1. memristor Hysteresis characteristic curve and attractor of memristor chaotic system

In order to study the influence of internal variables of memristor on the system, the internal variables of memristor are introduced into the second dimension. A new asymmetric memristor chaotic system (4) is obtained. A variable with a polarity reversal in the system may involve an offset enhancement of conditional symmetry. Here the absolute value function simply replaces the introduced variable f(*z*), and the system can be conditionally symmetric:

$$\begin{cases} \dot{x} = -ax + by - cye^{mz} \\ \dot{y} = x + f(z) \\ \dot{z} = y^2 - z \end{cases} \tag{4}$$

Choosing f (*z*) = |*z*| -*d* or f (*z*) = *z* presents two symmetric attractors. The conditional symmetry here is not in the strict sense of conditional symmetry, and the bias of *z* does not manifest, but presents inversion symmetry of (*x*, *y*, *z*) →(−*x*, −*y*, *z*).

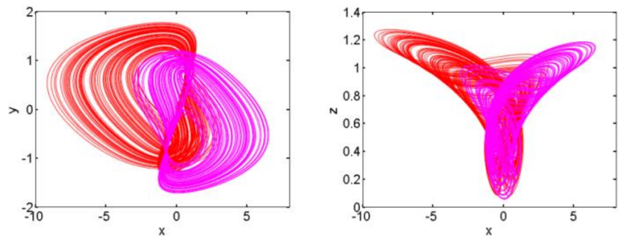

Fig. 2. Conditional symmetric coexisting attractors of system(4) and the initial conditions IC =(1 , 0 ,-2)(red) and IC =(1, 0, 2) (pink) .(a) *x-y*, (b) *x-z*

C. Dynamical Analysis

Analysis the dissipation of system (3), $\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -1 - a < 0$, Indicates that the system is dissipative. Let, $\dot{x} = \dot{y} = \dot{z} = 0$, the system have a mathematical solution, the equilibrium point is E (x_0, y_0, z_0) = (0, 0, 0).

The Jacobian matrix at the equilibrium point E of a three-dimensional memristor chaotic system is solved by linear transformation as follows:

$$J = \begin{cases} -a & b - c\exp(m z_0) & -mc\, y_0 \exp(m z_0) \\ 1 & 0 & 1 \\ 0 & 2 y_0 & -1 \end{cases} \tag{5}$$

The eigenvalue satisfying of the characteristic equation λ I−J = 0 : λ1 = − 1.0 + 19.8098 * i; λ2 = − 1.0− 19.8098*i;

λ3 = −1. The equilibrium point is stable. It can be said that this system is a system with a hidden attractor. The following conclusions can be obtained through Wolf algorithm, that Lyapunov exponents (Shorthand for LyE) are: LyE1 = 0.224374, LyE2 = −0.000594, LyE3 = −3.223748. The value of the above LyE was obtained when the time step was set as 1S and the time length was set as 5000S.

With fixed parameters $b = 10$, $c = 1$, $m = 3$, when parameter changes within the range of $a \in [1, 2.2]$, the bifurcation diagram and the corresponding Lyapunov exponent spectrum of system (3) is presented in Fig.3.

With the increase of a, the system enters the chaotic state through period-doubling bifurcation, and a narrow period window appears in the interval $a \in (1.63, 1.67)$ and $a \in (1.86, 1.88)$, the maximum value shows an exponential decreasing trend from a =1.58. The bifurcation diagram and the Lyapunov exponent as shown in Fig.3(a-b).

When b changes from 2 to 12 and other parameters remain unchanged, the bifurcation diagram of state variable x and the corresponding Lyapunov exponent spectrum are respectively shown in Fig.3(c-d). When $b\in[3.7, 8.2]$ and $b \in [9.07, 12]$, the system is chaotic and the maximum Lyapunov exponent is positive.

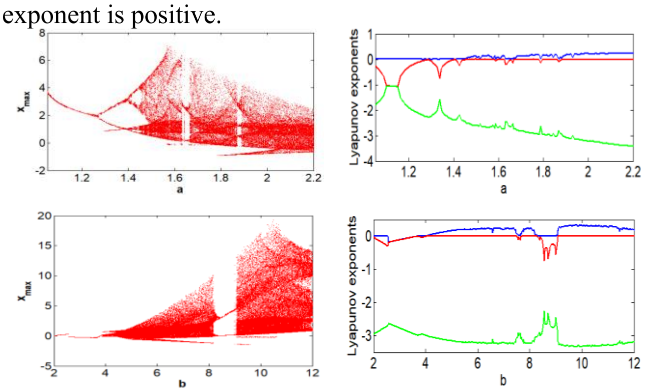

Fig. 3. Bifurcation diagram and Lyapunov exponent of system (3) with parameter a and b.

Similarly, other parameters are fixed, and the bifurcation parameter is selected as c. With the change of c, the system enters a chaotic state at 0.2, and presents a narrow periodic window in the interval $c \in (0.40, 0.42)$ and $c \in (0.75, 0.79)$. The bifurcation diagram and Lyapunov index of c are shown in Fig.4.

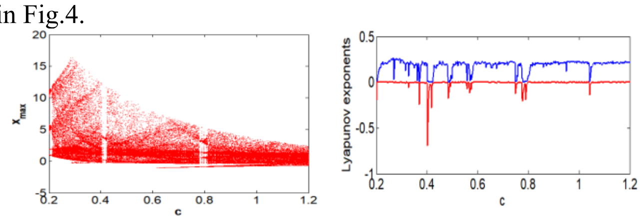

Fig. 4. Bifurcation diagram and Lyapunov exponent of system (3) with parameter c.

III. Amplitude Modulation and Offset Boosting

A. *Amplitude Modulation Based on Parameter m*

Set m as the bifurcation parameter, the bifurcation diagram and Lyapunov exponent as shown in Fig.5.

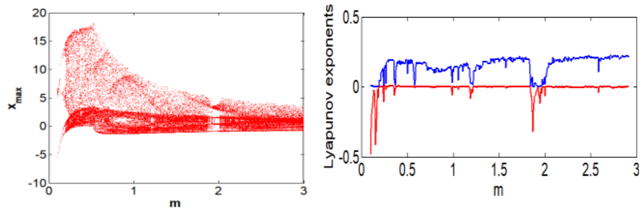

Fig. 5. Lyapunov exponent spectrum and bifurcation diagram under diff erent values of the parameter m . (a) Lyapunov exponent, (b) Bifurcation digram

The parameter m in the exponential memristor is a special parameter. As a power coefficient, it has obvious influence on the whole exponential function. We know that the range of the exponential function must be greater than 0. We believe that the memristor feedback term we introduce here must be a positive feedback term. This will make our system more prone to oscillations.

Parameter m affects the amplitude of the output signal, in addition to the influence of m on the chaos and periodic after the system enters chaotic state. With the increase of m, the amplitude decreases exponentially, mainly due to the influence of exponential memristor. In particular, the phase trajectory diagram of the system when m takes different values as shown in Fig.6. When m > 1, the phase trajectory not only presents amplitude modulation, but also shows a symmetric transformation. The attractor is symmetric, At m = 1.5, and at m = 3 the attractor is inverse to the attractor at m<=1.

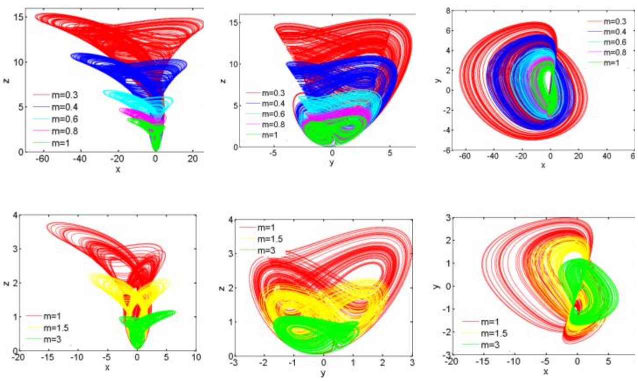

Fig. 6. Projections of the system under different values of the parameter m

B. *Amplitude Modulation Based on Scale Transformation*

In order to further realize the amplitude modulation of other signals, a scale modulation parameter k is introduced here. The system (3) is scaled to $x \to kx$, $y \to yk$, $z \to zk$. The mean value of the system under different scales of initial value IC = (1, 0, 2) and $k \in (0, -4]$ as shown in Fig. 7.

The amplitude of the mean value of the signal x and z presents an inverse exponential relationship with the scale parameter k. The mean (x) is negative and mena (z) is positive, which is related to the quadrant where the phase trajectory is located, which can also be confirmed by the sequential waveforms. The attractors on the x and z axes are asymmetrical. And the mean of y changes very little, because the phase orbitals are two-sided on the Y-axis, and the mean is basically 0.

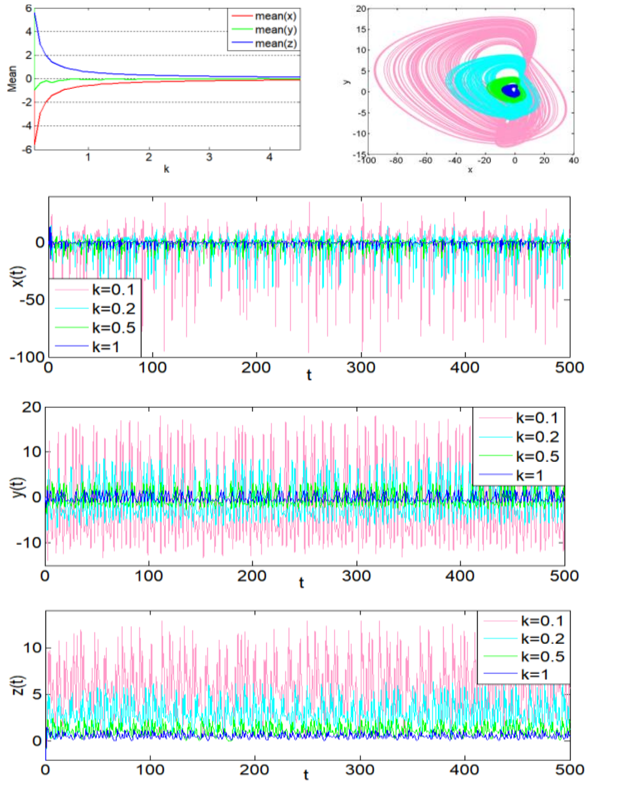

Fig. 7. The sequential waveformswaveforms and average values of x(t), y(t), z(t) with parameter k

C. *Offset Boosting of the Conditional Symmetric System*

If a certain state variable can control the polarity of the output signal in the system by changing the parameters, it can adjust the polarity of the output signal by adding a voltage source.

DC bias can adjust the mean value and polarity of the output signal. In this system, x variable is simplest to control the signal bias by changing the parameters. Here the parameter n is used as the DC bias of the variable x. Fig.8 shows the position of the attractor at $n = -10$, $n = -5$, $n = 0$, and $n = 5$ to explain the effect of DC bias on the mean and polarity of the system.

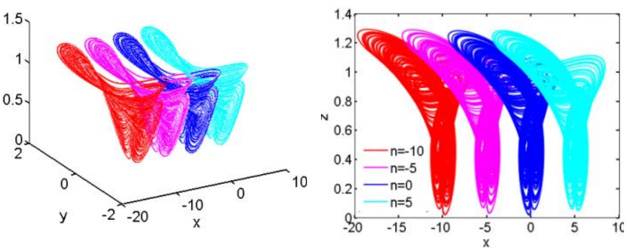

Fig. 8. The influence of offset parameter n on the phase trajectory diagram.

IV. Conclusion and Discussions

In this paper, an exponential magnetron memristor is constructed using an exponential function and embedded into a three-dimensional Rucklidge system. A chaotic system with stable equilibrium point is formed by taking the internal variable of the memristor as a feedback term. The dynamic characteristics of the exponential memristor chaotic system and the control mechanism of amplitude modulation, bias and conditional symmetry are explored. When n is used as the DC bias parameter of the output signal x, can you adjust it to get different polar output of x. Especially in the aspect of amplitude modulation, it shows the influence of exponential function on the whole characteristics of the system. Therefore, the exponential function memristor can make the output amplitude exponentially adjustable and increase the complexity of the output signal.

References

[1] Bao, B. C., Li, Q. D., Wang, N., & Xu, Q. (2016). Multistability in chua's circuit with two stable node-foci. Chaos. 2016;26(4):043111.

[2] Jaros P, Perlikowski P, Kapitaniak T. Synchronization and multistability in the ring of modified Rössler oscillators. Eur Phys J-Spec Top. 2015;224(8):1541–52.

[3] Njitacke ZT, Kengne J, Fotsin HB, Nguomkam AN, Tchiotsop D. Coexistence of multiple attractors and crisis route to chaos in a novel memristive diode bridge-based jerk circuit. Chaos Solitons Fractals. 2016; 91:180–97.

[4] Sprott JC, Jafari S, Khalaf AJM, Kapitaniak T.Megastability: coexistence of a countable infinity of nested attractors in a periodically-forced oscillator with spatially-periodic damping. Eur Phys J-Spec Top. 2017;226(9):1979–85.

[5] Li C, HuW, Sprott JC,Wang X. Multistability in symmetric chaotic systems. Eur Phys J-Spec Top. 2015;224(8):1493–506.

[6] Sahin, M. E., Taskiran, Z. G. C., Hasan Güler, & Hamamc, S. E.Simulation and implementation of memristive chaotic system and its application for communication systems. Sensors and Actuators A: Physical, vol.290, pp.107-118, 2019.

[7] Al-Yasir, Yasir I. A. Adaptive Control Synchronization of a Novel Memristive Chaotic System for Secure Communication Applications. Inventions. 2019; 4(2):30.

[8] Shi, X., Duan, S., Wang, L., Huang, T., & Li, C. A novel memristive electronic synapse-based Hermite chaotic neural network with application in cryptography. Neurocomputing 166.oct.20(2015):487-495.

[9] Kuka, C. S., Hu, Y., Xu, Q., & Alkahtani, M. An Innovative Near-Field Communication Security Based on the Chaos Generated by Memristive Circuits Adopted as Symmetrical Key. IEEE Access, vol.8, pp.167975-167984, 2020.

[10] Sun, J., Li, C., Lu, T., Akgul, A., & Min, F. A memristive chaotic system with hypermultistability and its application in image encryption. IEEE Access, vol. pp, no.99, 2020.

[11] Yang, Y., Wang, L., Duan, S., & Luo, L. Dynamical analysis and image encryption application of a novel memristive hyperchaotic system. Optics & Laser Technology,2021,133(1).

[12] Yang, J., Wang, L., Wang, Y., & Guo, T. A novel memristive Hopfield neuaral network with application in associative memory. Neurocomputing, 2016, 227(mar.1), 142-148.

[13] B, Leimin Wang A, and H. Z. A. B. A new emotion model of associative memory neural network based on memristor.Neurocomputing, 2020,410, 83-92.

[14] aidyanathan, S., Azar, A. T., Akgul, A., Lien, C. H., & Cavusoglu, U. A memristor-based system with hidden hyperchaotic attractors, its circuit design, synchronisation via integral sliding mode control and an application to voice encryption. International Journal of Automation and Control,2019, 13.6:644.

[15] Sun, J., Xiao, X., Yang, Q., Liu, P., & Wang, Y. Memristor-based Hopfield network circuit for recognition and sequencing application. AEU: Archiv fur Elektronik und Ubertragungstechnik: Electronic and Communication, 2021(134-):134.

[16] Zhang, X., Li, C., Chen, Y., Iu, H. H. C., & Lei, T. A memristive chaotic oscillator with controllable amplitude and frequency. Chaos Solitons & Fractals, 2020, 139(1).

[17] Zhang X, Li C, Min F. Broken Symmetry in a Memristive Chaotic Oscillator. IEEE Access, 2020, PP (99):1-1.

[18] Hua, M., Yang, S., Xu, Q., Chen, M., & Bao, B. "Forward and reverse asymmetric memristor-based jerk circuits. AEU - International Journal of Electronics and Communications,2020,123:153294.

[19] Zhang, S., Zheng, J. H., Wang, X., & Zeng, Z. A novel no-equilibrium HR neuron model with hidden homogeneous extreme multistability. Chaos Solitons & Fractals ,2021145.4:110761.

[20] Dolvis L G, Jacques K. A unique chaotic snap system with a smoothly adjustable symmetry and nonlinearity: Chaos, offset-boosting, antimonotonicity, and coexisting multiple attractors. Chaos Solitons & Fractals, 2018, 113:275-293.

[21] Zhang, S., Li, C., Zheng, J., Wang X., Zeng Z., Chen, G. Generating Any Number of Diversified Hidden Attractors via Memristor Coupling. IEEE Transactions on Circuits and Systems I: Regular Papers, 2021, Vol 68, No.12, pp.4945-4956.

[22] Gu J, Li C, Chen Y. A Conditional Symmetric Memristive System With Infinitely Many Chaotic Attractors. IEEE Access, 2020, 8:12394-12401.

[23] Dai, W., Xu, X. Song, X., Li, G. Audio Encryption Algorithm Based on Chen Memristor Chaotic System. Symmetry, 2022, 14(1): 17.

[24] Victor Kamdoum Tamba, Janarthanan Ramadoss, Viet-Thanh Pham, elt. Symmetric Oscillator: Special Features, Realization, and Combination Synchronization, Symmetry 2021, 13(11), 2142.

[25] Rajagopal, K., Jafari, A., He, S., Parastesh, F., Jafari, S., & Hussain, I. Simplest symmetric chaotic flows: the strange case of asymmetry in Master Stability Function. The European Physical Journal Special Topics, 2021, 230(7):1999-2010.

[26] Li, X., Zheng, C., Wang, X., Cao, Y., & Xu, G. Symmetric coexisting attractors and extreme multistability in chaotic system. Modern Physics Letters B, 2021,35(32):1-17.

[27] Zang, H., Huang, L., Lei, T., Wang, Y, & Wang, Z. Conditional Symmetry of a Memristive System with Amplitude and Frequency Modulation Control. Mathematical Problems in Engineering: Theory, Methods and Applications, 2021,2021(Pt.32):4924323.1-11.

[28] Wang, R., Li, C., Kong, S., Jiang, Y., & Lei, T. A 3D memristive chaotic system with conditional symmetry. Chaos, Solitons & Fractals 2022,158:111992-.

[29] RUCKLIDGE A M. Chaos in models of doublec onvection. Fluid Mech,1992,237:209-229

[30] Bao, B. C., Xu, J. P., Zhou, G. H., Ma, Z. H., & Zou, L. Chaotic memristive circuit: equivalent circuit realization and dynamical analysis. Chinese Physics B, 2011,20(12):120502