Faculty of Computer Science and Information Technology

SCHWARZ DOMAIN DECOMPOSITION METHOD

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ABSTRACT

Schwarz domain decomposition methods have been invented by Hermann Amandus Schwarz in 1869. Basically, the main idea of this method is to be a parallel solver and solve the parallel computation. With the advent of the parallel computing, we are going to have a very complex data to be process within a time. Schwarz methods is the best way to illustrate how it can enhance the performance in solving the computation of parallel problem where the computational domain is decomposed into subdomains. We also indicate that the boundary-value problem is formulated and solved independently for each subdomain by computes the parallel computation. In addition, with parallel computing, the computational time also significantly reduced, which will be helpful to enhance the performance of the data computing. The Schwarz domain decomposition method provide efficient and robust scheme for solving large systems of linear algebraic equations. With these methods, we can reduce the amount of memory used to complete the parallel computing. Parallel computing is important since it helps to make the functionality of the domain decomposition become more resourceful in solving the computation iteratively.

ABSTRAK

Kaedah penguraian domain Schwarz telah diciptakan oleh Hermann Amnduss Schwarz pada tahun 1869. Umumnya, pendapat utama kaedah ini adalah untuk menjadi penyelesai selari dan menyelesaikan pengkomputeran selari. Dengan kemunculan pengkomputeran selari, kita akan mempunyai maklumat yang sangat kompleks untuk diproses dalam masa yang sama. Kaedah Schwarz adalah cara yang paling sesuai untuk menerangkan bagaimana ia dapat meningkatkan prestasi dalam menyelesaikan pengiraan secara selari dengan memisahkan bidang yang besar menjadi bidang-bidang yang kecil. Kami juga menyatakan bahawa masalah keadaan batas akan dirumuskan dan diselesaikan sendiri untuk setiap bidang-bidang dengan mengira pengiraan selari tersebut. Tambahan pula, dengan pengiraan selari, masa pengiraan penting juga disingkatkan, yang sangat membantu meningkatkan prestasi pengiraan maklumat. Kaedah penguraian domain Schwarz menyediakan skema yang cekap dan efisien untuk menyelesaikan sistem persamaan algebra yang besar. Dengan kaedah ini, kita dapat menguranrkan jumlah memori yang digunakan untuk melengkapi pengkomputeran selari. Pengkomputeran selari adalah penting untuk memastikan fungsi penguraian domain menjadi lebih berkesan dalam menyelesaikan pengiraan secara berterusan.

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Chapter 1: Introduction

1.1 Background of study

With the advent of parallel computers, domain decomposition algorithms become increasingly popular in scientist community because they define a good framework to derive efficient solvers for the resulting linear systems by using the mathematical properties of Partial Differential Equations (PDEs). Domain decomposition method are one of the solutions of linear systems that was use information regarding discretization or partial differential equations, where it is a family of methods to solve problems of linear equation on parallel machines in the context of simulation. The value of domain decompositions methods is part of a need for parallel algorithms for professional use (et. al Victoria, 2015). The well-posed of the Poisson problem in complex geometries and the convergence of the algorithm had been proved by H. Schwarz.

With the advent of the digital computers, the domain decomposition method acquired practical interest, as an iterative linear solver. Subsequently, the parallel computers become available and with a small modification of the algorithm of nonoverlapping subdomains that make the method suited with these architectures of parallel computing. The convergence can be proved by using the maximum principle. This project will use the first order of differential equation through the derivation of Laplace's equation.

This project will solve on how the domain decomposition can be solved by using the parallel method from the derivation of Poisson's equation, $-\Delta\mathbf{u} = \mathbf{f}$ and Laplace's equation to which it reduces $\Delta\mathbf{u} = \mathbf{0}$, with the boundary conditions which Dirichlet boundary condition. Dirichlet boundary conditions are condition where the value of the dependent variable is specified on the boundary.

1.1 Statement of Research

Schwarz methods are well-known as the most top parallel algorithm solvers. Schwarz proposed the idea where the domain will be split into subdomains. It has been invented by Hermann Amandus Schwarz in 1869. There are some methods are being used to come out with the solution in solving the problems including alternating Schwarz method, parallel Schwarz method, additive Schwarz method and multiplicative Schwarz method. Alternating Schwarz method had been invented by H. Schwarz as the computational tools. The main classical methods for continuous level of Schwarz methods had been introduced by Jacques-Louis Lions for parallel computing. The influence of the number of subdomains in the convergence of the additive Schwarz algorithm will be seen through the solutions in solving the problem of domain decomposition. In this project, the research will focus on the study of the domain decomposition that will limited into two subdomain decompositions only. Extending the analysis to general decomposition would be valuable to have such a complete overview of the behaviour of Schwarz algorithm.

1.2 Research Objectives

The goals of this research are to evaluate and examine how the Laplace's equations can solve the Schwarz domain decomposition by using parallel method. Other objectives, include:

i. Identify the problems occur in the classical Schwarz methods,

ii. Visualization of the result of Laplace equation simulation

1.3 Scope of study

The scope of this project is to solve the domain decomposition problem with parallel machine by using MATLAB.

1.4 Significant of project

Based on the outcome of the project, understand on the domain decomposition problem and how the suitable method can solve the problem, then give an idea on how it will work. Furthermore, this project will be able to determine on how the application of parallel computing had been practically applied in daily life for example, in the analogy of the vacuum pump.

1.5 Project Schedule

The project schedule for Final Year Project is attached at the Appendix A.

1.6 Expected Outcome

With the parallel Schwarz method, we will be able to solve the Schwarz domain decomposition algorithm by using Laplace’s equation.

Chapter 2: Literature Review

2.1 Introduction

In this chapter, literature review is on the focus. Literature review is an important phase in the study of materials and sources which helps to support and encourage the success of this research. Domain decomposition methods are parallel, potentially fast, robust algorithms for the solution of linear equations. This technique had been known for a long various of time. However, the use has become more popular with the advent of parallel computing. A survey regarding domain decomposition methods can be found in (Chan and Mathew, 1994). In the following, we present the concepts involved in domain decomposition. For advance information, the specialized texts from Toselli and Widlund (2005) and Tarek (2008) would explain more regarding the theory and implementation of domain decomposition method. Domain decomposition algorithms may be applied to a variety partial differential equation. To simplify the presentation, we restrict the focus to linear, second order of Partial Differential Equations.

2.2 Parallel Computing for Aeroacoustic Problems

According to (Resiga and Atassi, 2014), domain decomposition method is being used to solve the time-harmonic aeroacoustics problem. Accurate resolution of the wave form with minimum dispersion and dissipation need the computational aeroacoustics schemes. The large-scale systems of equation will be solved with the direct solvers, and the computation time becomes very big. Therefore, the iterative solvers will converge in wave form like and possible to be not converge at all. The computational domain will be partitioned into subdomains and automatically generated the boundary value problem that will be solve independently. The transmission conditions are imposed that will ensure the uniqueness of the solution.

Transmission conditions will be updated for each iteration then will covered the global solution. As the problem have been solved concurrently with subdomains in parallel computers, it gives advantages in type of saving the amount of memory been used for solving large systems of linear equations and with the parallel computing, the computational time significantly reduced.

The boundary value problem for the computational domain had been defined through the Prandtl-Glauert space. After discretizing the problem occur, one end up solving large system of the linear equations. Because of the large number of mesh points are required to accurately preserve the wave form and to minimize the dispersion, the CPU time and the memory requirements become a problem towards implementing aeroacoustics computation. To solve the problem, we need to solve several small and independent problems instead of single large problem. This is where the idea where of domain decomposition methods had been applied with the additional advantage of solving the subproblems concurrently, on parallel computers.

2.3 Multi-Domain Methods

2.3.1 Schwarz alternating method

Schwarz alternating method is one of the well-known methods for the decomposition of the solution in connection with the Poisson equation. The Poisson equation with the Dirichlet boundary conditions on each subdomain boundary has a unique solution for the union of the domain which coincides with the exact solution in the subdomains. The partial differential equation can be solved iteratively by alternating between the subdomains with *and* $\cap$. Also, in each subdomain, the solution is obtained by using the Dirichlet boundary conditions. The steps repeated as the solution advances in alternating steps between the two subdomains. The method is inherently sequential as it requires the solution in the subdomain to be obtained first before another subdomain. By putting a bit of modification, parallel solution in each subdomain can be solved. This is accomplished just by replacing the boundary condition with the value that available from the previous iteration.

Therefore, the solution can proceed in parallel in each subdomain, potentially using separate processors. It is essential that the relevant message passing between the processors be achieved sufficiently fast, thus the inter-processor transfer does not slow down the iteration cycle. The interested reader should consult manuals of the library of routines for implementing parallel tasks in C++ and Fortran known as the Message Passing Interface (MPI), and related monographs (Pacheco, 1997, Gropp et al., 1999).

2.3.2 Steklov-Poincare method

When the subdomains are not overlapping, the performance of Schwarz method was compromised by inadequate accuracy of the interpolation process, especially when non-conforming grids are employed. In such cases, it is important to enforce continuity of both value of the potential and its normal derivative on the internal boundary, known as transmission condition. Associated framework is called the Stecklov-Poincare Framework and provides an alternative approach to parallel computation in the case of non-overlapping grids.

For an instance, the non-overlapping example of Steklov-Poincare method been used and assumed that the model was posed on the full domain. Also, the transmission condition between the subdomains requires both solution and the local flux be continuous on the internal boundary.

The problem in the two subdomains can be combined in a hybrid formulation that allows the parallel solution. To ensure the iteration converges, relaxation is used for both solution and the flux on the interior boundary. Hence, we can compute the interface quantity, which allows for the hybrid problem can be solved in parallel by fulfil the Neumann boundary conditions on the interior boundary in the first domain. However, the Dirichlet boundary condition had been fulfilled in the interior boundary condition at another subdomain.

Furthermore, the problem is solved in parallel at full iteration count by reversing the Dirichlet and Neumann conditions in the two subdomains. The relaxation parameters are determined and may provide different values mid and full iteration count. Thus, the alternating fulfils of Dirichlet and Neumann boundary conditions within each iteration, to make sure that the transmission conditions is enforced on the interior boundary. Hence, the convergence is confirmed, provided that the potential and flux on the interior boundary are computed by an accurate method especially in the case of non-conforming grids.

2.4 Domain Decomposition and Communication

Domain decomposition is one of the most important techniques commonly used in parallel computation. The basic idea is to divide the global domain into many subdomains and then the governing differential equations are solved in several or all subdomains simultaneously; problem like data communication and load balance arise naturally and become the bottleneck of parallel computation.

The weather prediction had been employs by the Radio Frequency System (RFS) model for both in a coarse grid and a fine grid. Number of grid points are $161 \times 91 \times 20$ for the coarse grid and $91 \times 91 \times 20$ for the grid. Each in directions of longitude, latitude and vertical, respectively. Given a solution space, one can decompose the given domain in one or more directions. The choice mainly depends on the amount of data communication burden each kind of decomposition would cause for parallel processing.

The next query is which direction should be chosen for the decomposition. Vertical direction was not considered since it has only 20 layers. In contrast, computation required high latitude area is much less than that for low latitude area which would cause unbalanced load distribution should be decomposed in the direction. Therefore, the decomposition in the longitudinal direction is used.

The sequential RFS model was coded in Fortran and the loop index in the code was arranged in the sequence of longitude, latitude and vertical. When partitioning in the longitudinal direction, the above arrangement of the loop index will cause discontinuity of memory access because Fortran uses column-wise memory allocation sequence. An index swapping routine is designed and added to switch the loop index of the code into the sequence of latitude, vertical and longitude. The result show that the new sequence having faster parallel execution time.

The RFS model employs an explicit solution scheme therefore no global information is needed at any grid point. Inter-processor communication takes place only when exchanging information on inner boundary of adjacent subdomains. These data can be updated by the neighbouring processors and then be transferred into the memory of current processor. The communication among processors were done using Message Passing Interface (MPI) communication library routines. MPI is suitable for Single Program Multiple Data (SPMD) parallel model. It provides subroutine calls to initialize or finalize a parallel session and to get the number of processors and processor identification. Other Capabilities of MPI include point to point message passing, collective communication, data broadcasting, gather and scatter, global reduction operations, etc. Information passing by MPI should be contiguous array data. Otherwise users must pack them into a buffer before sending out and unpack them from the received buffer into data items. With MPI virtual topology routines, user can partition along multiple dimensions using Cartesian topology. Current version of MPI does not provide parallel Input/Output, so all Input/output must be done in one processor and then broadcast or scatter to each processor.

Since some of the longitudinal index begins with 2 or 3, for simplicity the code is designed such that each subdomain contains at least 3 grid points which will be assigned to one processor for calculation. Since there are 91 grid points in the longitudinal direction for the fine grid, this makes the parallel code capable of running on up to 30 processors. In which case 29 processors would contain 3 grid points and the last processor 4 grid points. User of this parallel code needs only specify the number of processors in the execution command without any modification of the program.

It uses a STARTEND subroutine to decide the start and end indices along the longitudinal direction to avoid the divisibility problem. The routine will divide the longitudinal grid points evenly among processors. The amount of computation and boundary data exchange is almost the same for all processors which resulted in a well-balanced parallel program. Initial data is read into one processor and then broadcast to all processors. Each processor will have a copy of the whole set of data but update only those grid points in its subdomain. Although it will waste some memory space in the distributed machine case, the advantage that Central Weather Bureu settles on is the convenience for maintenance and future expansion of the code.

2.5 Summary of proposed systems

	Work A	**Work B**	**Work C**
Title	Parallel Computing using Schwarz Domain Decomposition Methods for Aeroacoustic Problems	Finite-Difference Methods for Equilibrium Problems	Parallel Implementation of East Asia Weather Forecast System
Author and year	Romeo Resiga, H. M. Atassi, 1998	Nikolaos D. Katopodes, 2019	Shiung-Ming Deng, 1998
Purpose of study	To enhance the performances of the domain decomposition	To study the inheritance of the methods	To exchange large amount data among boundary of subdomains
Methods	Numerical Method and Differential Equation	Numerical Method and Differential Equation	Numerical Method and Differential Equation
Major findings	To solve the large systems of linear equation by using the parallel computing	To solve the problem iteratively by alternating between subdomains	To divide global domain into many subdomains

Table 1.0: Comparison of proposed system

Chapter 3: Methodology

3.1 Introduction

In this chapter, we will use the concept of mathematical modelling process to obtain the whole procedures. We will briefly explain the overview of the methodology that had been used in the study of domain decomposition to generate the equation from Laplace's equation to achieve our goals in solving the problem.

3.2 Modelling Process

3.2.1 Define Goal

The aims of this work are to determine and analyse how the decomposition of the Schwarz domain can solve the equation of the Laplace using parallel method. We can verify at the end of the analysis, any other problem that related can be solved using the problem of decomposition of the domain.

3.2.2 Characterize Model

To generate the equation that been used in solving the domain decomposition algorithm from the main type of equation that had been derived, its iterative solutions and understand the type of boundary condition that need to fulfil in generating the equations.

3.2.3 Formulate Model

To formulate the model, we need to identify and derive the equation regarding the domain decomposition problem. We formulate the domain decomposition algorithm by using the Poisson equation which turn into Laplace equation with the Dirichlet boundary condition and then solve the domain decomposition by using Schwarz domain method accordingly with the parallel computation.

3.2.4 Formulation Equation

Schwarz Model

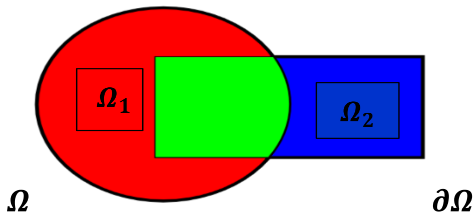

Figure 1: Overlapping between two subdomains of disk and rectangle

Figure 1.0 show the model of original domain used by Schwarz with the associated domain decomposition into two subdomains, which are geometrically simpler name disk and rectangle.

Schwarz invents method to proof that the solution exists for the general domain $\Omega := \Omega_1 \cup \Omega_2$. We solve the domain decomposition by using the derivative of Poisson equation,

$$\Delta \boldsymbol{u} = \boldsymbol{f} \tag{1}$$

which reduce later into Laplace equation,

$$\Delta \boldsymbol{u} = \mathbf{0} \tag{2}$$

with the Dirichlet boundary conditions,

$$\boldsymbol{u = g\ on\ \partial\Omega}$$

(3)

First, we will split the domain into 2: $\Omega \coloneqq \Omega_1 \cup \Omega_2$ and hence, we will get,

$$\Delta\left(\mathbf{u_1^{n+1}}\right) = \mathbf{0\ in\ \Omega_1}$$

$$\boldsymbol{u_1^{n+1} = g\ on\ \partial\Omega \cap \overline{\Omega_1}}$$

$$\boldsymbol{u_1^{n+1} = u_2^{n}\ on\ \Gamma_1}$$

(4)

$$\Delta\left(\mathbf{u_2^{n+1}}\right) = \mathbf{0\ in\ \Omega_2}$$

$$\boldsymbol{u_2^{n+1} = g\ on\ \partial\Omega \cap \overline{\Omega_2}}$$

$$\boldsymbol{u_2^{n+1} = u_1^{n+1}\ on\ \Gamma_2}$$

(5)

Let **n=0** in the **(4)** and in the **(5)**, we obtain

$$\Delta\left(\mathbf{u_1^{1}}\right) = \mathbf{0\ in\ \Omega_1}$$

$$\boldsymbol{u_1^{1} = g\ on\ \partial\Omega \cap \overline{\Omega_1}}$$

$$\boldsymbol{u_1^{1} = u_2^{0}\ on\ \Gamma_1}$$