

**COMPARISON OF FLEXURAL DESIGN
BETWEEN BS 8110 (1985) AND ACI 318 (1995)**

LOO SOON WAH



Universiti Malaysia Sarawak
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**COMPARISON OF FLEXURAL DESIGN
BETWEEN BS 8110 (1985) AND ACI 318 (1995)**

by
Loo Soon Wah

**A dissertation submitted
in partial fulfillment of the requirements for the
degree of Bachelor of Engineering (Hons.)
in Civil Engineering**

**Faculty of Engineering
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BORANG PENYERAHAN TESIS

Judul: COMPARISON OF FLEXURAL DESIGN BETWEEN
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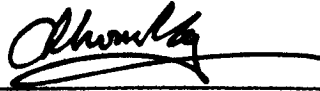
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This project report attached here to, entitled "Comparison of flexural design between BS 8110 (1985) and ACI 318 (1995)," prepared and submitted by Loo Soon Wah in partial fulfillment of the requirement for the degree of Bachelor of Engineering (Civil) is hereby accepted.

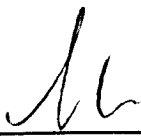


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ABSTRACT

This study compares the flexural design for arbitrary rectangular sections of reinforced concrete beams between BS 8110 (1985) and ACI 318 (1995). The main objectives of this study are to identify a more conservative design approach and to obtain a set of design aids such as charts and tables for ACI 318 (1995). This study includes the different analysis and design procedures of both codes and the interpretation of effects in flexural design.

In this study, singly reinforced beams and doubly reinforced beams were analyzed by using 4 different grades of concrete, associated either with mild steel or high-tensile steel in various percentages of longitudinal reinforcement ratios. Therefore, four Mathcad worksheets had been written to generate data for both codes. The depth of neutral axis and ultimate moment for every combination of concrete grades and steel ratios can be obtained easily through these worksheets. These were basically to eliminate the tedious iteration procedures and providing a more accurate solution for flexural design. From the obtained data, several graphs of ultimate moment ratio M/bd^2 versus percentage of longitudinal reinforcement ratio $100A_s/bd$ had been plotted. Comparison among the graphs between both codes had been carried out and the more conservative approach in flexural design had been identified.

ABSTRAK

Kajian ini dijalankan dengan membandingkan rekabentuk lenturan bagi rasuk konkrit bertetulang di antara BS 8110 (1985) dengan ACI 318 (1995). Objektif utamanya ialah untuk mengenalpasti rekabentuk yang lebih konservatif dan seterusnya menghasilkan carta serta jadual baru untuk memudahkan prosidur rekabentuk lenturan khususnya bagi ACI 318 (1995). Kajian ini adalah melibatkan perbezaan analisis dan rekabentuk prosidur di antara kedua-dua kod serta interpretasi tentang kesan-kesan yang mempengaruhi rekabentuk tersebut.

Dalam kajian ini, rasuk bertetulang tunggal dan bertetulangan kembar telah dianalisis dengan menggunakan 4 jenis gred konkrit, bergabung dengan keluli lembut atau keluli jenis tegangan tinggi mengikut pelbagai nisbah yang ditetapkan. Dengan ini, 4 kertas kerja Mathcad telah dihasilkan untuk mendapatkan data-data rekabentuk lenturan bagi kedua-dua kod tersebut. Paksi neutral dan momen muktamad bagi setiap kombinasi di antara konkrit dengan nisbah tetulang tertentu juga dapat diperolehi dengan mudah. Keadaan sedemikian telah berjaya menghapuskan kerja-kerja cuba-jaya yang membosankan. Melalui data-data yang diperolehi, beberapa graf bagi nisbah momen muktamad M/bd^2 melawan peratusan nisbah tetulang $100A_s/bd$ telah diplotkan. Akhirnya,

Perbandingan telah dijalankan dengan menginterpretasi analisis yang dilaksanakan dan seterusnya mengenalpasti rekabentuk lenturan yang lebih konservatif di antara kedua-dua kod tersebut.

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Table 3.1 Comparison of notations and parameters used for ACI 318 (1995) and BS 8110 (1985)

NOTATIONS

- A_s = Area of tension reinforcement
- A'_s = Area of compression reinforcement
- a = Effective depth of concrete compressive stress
- b = Width of the cross section
- $C(x)$ = Concrete compression force [BS 8110 (1985)]
- $C(c)$ = Concrete compression force [ACI 318 (1995)]
- c = Depth of neutral axis [ACI 318 (1995)]
- d = Effective depth of tension reinforcement
- d' = Effective depth of compression reinforcement
- E_s = Modulus of elasticity of reinforcement
- f_{cu} = Characteristic strength of concrete [BS 8110 (1985)]
- f_c = Characteristic strength of concrete [ACI 318 (1995)]
- f_y = Characteristic strength of reinforcement
- f_s = Stress in tension reinforcement
- f'_s = Stress in compression reinforcement
- h = Height of beam
- M = Ultimate moment of resistance
- M_n = Nominal moment of resistance
- $T(c)$ = Tensile force of tensile reinforcement [ACI 318 (1995)]
- $T(x)$ = Tensile force of tension reinforcement [BS 8110 (1985)]

- x** = Depth of neutral axis [BS 8110 (1985)]
- z** = Lever arm
- ϵ_{cu}** = Ultimate strain of concrete [BS 8110 (1985)]
- ϵ'_c** = Ultimate strain of concrete [ACI 318 (1995)]
- ϵ_s** = Strain of tension reinforcement
- ϵ'_s** = Strain of compression reinforcement
- γ_m** = Partial safety factor for steel reinforcement
- ϕ** = Reduction factor
- β_1** = Ratio of the average compressive stress
- ρ** = Longitudinal reinforcement ratio
- P** = Percentage of longitudinal reinforcement ratio

CHAPTER 1

INTRODUCTION

1.1 Introduction

Many codes and standards have been developed throughout the world for the design and control of the quality of the reinforced concrete. Among those codes, BS 8110 (1985) and ACI 318 (1995) are widely used. Both of the design codes are generally limit state design and based on the similar design concept, but the symbols and interpretation of formulas are different.

In Malaysia, BS 8110 (1985) is the only code that has been widely practiced so far. This code has just been updated in 1997 after 12 years from its predecessor in 1985. Conversely, the ACI Code is always been updated to reflect the latest technological changes in materials and design philosophy. Therefore, it can serve as an alternative of the design code that will be practiced in Malaysia later.

Comparison of the flexural design between BS 8110 (1985) and ACI 318 (1995) was carried out in this parametric study especially for the rectangular flexural members. In addition, the different analysis, parametric considerations and design procedures for both codes were also identified. Eventually, several design charts and tables were produced to

facilitate the flexural design especially in eliminating iteration and providing more exact solutions for an arbitrary cross section.

1.2 Objective

The objectives of this study are to compare the manners of flexural design between the BS 8110 (1985) and ACI 318 (1995) codes; to identify a more conservative design approach from both of the codes; to generate new design aids such as charts and tables for ACI 318 (1995); and the results from this study will serve as a guide for quick check of the concrete beam design.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

When a beam is subjected to bending moments (also termed as flexure), bending strains are produced. As normally defined, under positive moment, compressive strain is produced in the top of the beam and tensile strain is produced in the bottom. Therefore bending members must be able to resist tensile and compressive stresses.

For a concrete flexural member (beam, wall and slab) to have any significant load carrying capacity, its basic inability to resist tensile stresses must be overcome by embedding reinforcement in tension zones. Thus, reinforced concrete has been constructed to perform very adequately under flexure.

2.2 General Theory of Ultimate Flexural Design

In general, the existing design methods in American and British Codes are based on the similar theory with the following assumptions. These assumptions are not exactly correct but are justifiable for practical purposes. (Kong and Evans 1987):

- a) The strains in the reinforcing steel and the concrete are directly proportional to the distances from the neutral axis where the strains is zero.
- b) The ultimate limit state of collapse is reached when the concrete strain of the extreme compression fibre reaches an ultimate value, ϵ_{cu} .
- c) The distribution of concrete compression stress is defined by an idealized stress-strain curve.
- d) The tensile strength of the concrete is ignored.
- e) The stress in the reinforcement is derived from the appropriate stress-strain curve, with the assumption that plane sections remain plane.

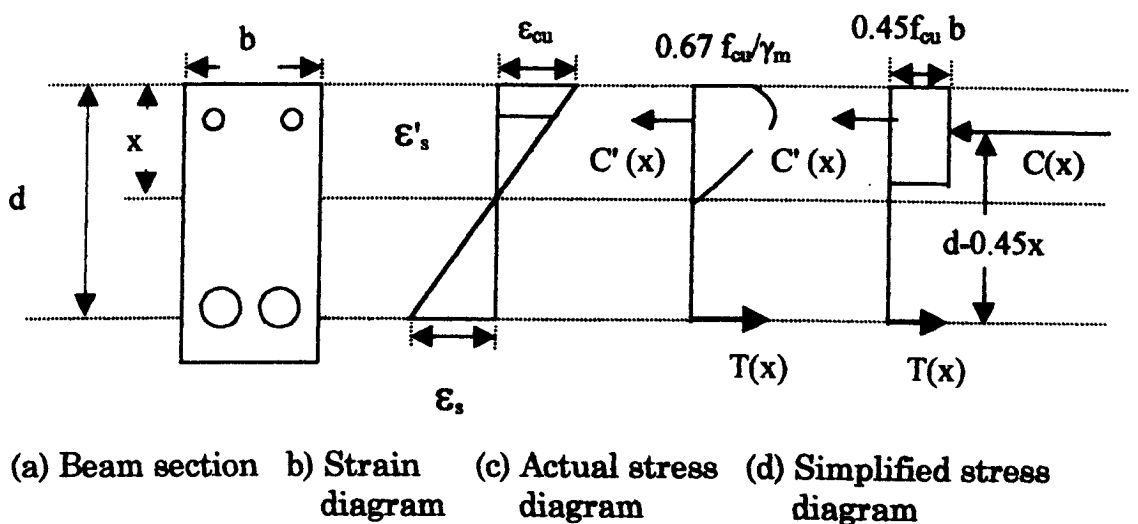


Fig. 2.1 Derivation of formula from stress-strain diagram in BS 8110 (1985)

Fig. 2.1 shows a cross section of a reinforced concrete beam with the strain and stress distributions. The symbols used were based on BS 8110 (1985) which are defined as follows: b = width of beam; A_s , d = Area and effective depth of longitudinal tension reinforcement respectively; A'_s , d' = area and effective depth of longitudinal compression reinforcement respectively; f_{cu} = Characteristic cube strength of concrete; x = neutral axis depth; $T(x)$ = tensile force of the tension reinforcement; $C(x)$ = compressive force of the concrete; $C'(x)$ = compressive force of the compression reinforcement; ϵ_s = tensile strain of the tension reinforcement; ϵ'_s = compressive strain of the compression reinforcement; ϵ'_c and ϵ_{cu} = maximum compression strain [= 0.003 according to ACI 318 (1995) and = 0.0035 according to BS 8110 (1985)]

From assumption (b) as mentioned earlier, the maximum concrete compressive strain has a specified value ϵ_{cu} . Therefore, the concrete strains at distance d' and d can be obtained directly from compatibility. Perfect bonding is also assumed between the concrete and the steel. Thus, the strain ϵ_s and ϵ'_s can be derived as follows [Fig. 2.1 (b)]:

$$\frac{\epsilon_s}{\epsilon_{cu}} = \frac{d - x}{x}$$

$$\epsilon_s = \left(\frac{d - x}{x} \right) \epsilon_{cu} \quad (2.1)$$

$$\frac{\epsilon'_s}{\epsilon_{cu}} = \frac{x - d}{x}$$

$$\epsilon'_s = \left(\frac{x - d}{x} \right) \epsilon_{cu} \quad (2.2)$$

From the simplified stress block [Fig. 2.1 (d)]: the forces on the beam section can be expressed in terms of these characteristics:

$$C(x) = 0.45 f_{cu} b x \quad (2.3)$$

$$C'(x) = 0.87 A'_s f_s \quad (2.4)$$

$$T(x) = 0.87 A_s f_s \quad (2.5)$$

Tensile stress of steel, f_s , and compressive stress of steel, f'_s , are closely related to the strain ϵ_s and ϵ'_s , by the respective stress-strain curves for the reinforcement.

From equilibrium condition,

$$T(x) = C(x) + C'(x) \quad (2.6)$$

Substituting Eqs. (2.3) through (2.5) into Eq. (2.6) gives

$$0.87 f_s A_s = 0.405 f_{cu} b x + 0.87 f'_s A'_s \quad (2.7)$$

From equations (2.1), (2.2) and (2.7), the value of neutral axis x , is the only unknown, which is related to ϵ_s and ϵ'_s ; and hence f_s and f'_s .

Therefore, to satisfy the equilibrium condition, x can be solved by trial and error method.

In normal practice, a value of x is assumed and the strains and hence the stress are determined. If Eq.(2.7) is not satisfied, an adjustment is made to x by inspection. These procedures will be repeated until Eq.(2.7) is sufficiently satisfied. To overcome this tedious and time consuming process, the Mathcad Worksheets had been written to solve the iterations and to provide more accurate solutions for x value.

The ultimate flexural strength, (often denoted as ultimate moment of resistance), M , of the beam is then obtained by taking moment about the level of the tension reinforcement or other suitable axis such as the centroid of the concrete stress block. The equation of the flexural strength, M , can be obtained using the following equation:

$$M = 0.405 f_{cu} b x (d - 0.45x) + 0.87f_s'A_s (d - d') \quad (2.8)$$

This general theory is applicable for all the arbitrary cross section of rectangular concrete beams. The detail analyses for BS 8110 (1985) and ACI 318 (1995) are shown in the chapter 3, section 3.6.

2.3. Mode of the Flexural Failure

Ultimate strength design in civil engineering practice took place partially in recognition of the importance of structural safety. Therefore, the study of the three modes in which a beam may fail in flexure under overloads is the prime concern of the design engineer. The particular modes of failures are actually depending on the amount of tensile reinforcement.

2.3.1 Balance Failure

Balance failure occurs when the concrete fails and the steel yields simultaneously at ultimate load. The concrete strain is 0.0035 or 0.003 for BS 8110 (1985) and ACI 318 (1995) respectively and the steel strain is 0.002. The amount of steel to give this situation can be determined by equating the internal forces, $C(x)$ and $T(x)$ in the concrete. This is the theoretical balance design case. (Choo and Macginley, 1990)

2.3.2 Tension Failure

Tension failure occurs when the steel yield followed by concrete compression failure. If the steel is stressed to its yield point before the concrete crushed in compression, the section will not fail. A small