A COMPLEX VARIABLE BOUNDARY ELEMENT METHOD (CVBEM) FOR SOLVING SEEPAGE PROBLEMS

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1998

APPROVAL SHEET

This project report attached hereto, entitled "**A Complex Variable Boundary Element Method for Solving Seepage Problems**" prepared and submitted by Lee Lin Jye in partial fulfillment of the requirement of Bachelor's degree with Honours in Civil Engineering is hereby accepted.

Date 16/10/98

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A COMPLEX VARIABLE BOUNDARY ELEMENT METHOD (CVBEM) FOR SOLVING SEEPAGE PROBLEMS

Tesis
(Ijazah Pertama)

Lee Lin Jye

Tesis Dikemukakan kepada
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To my lovely wife and all family members.

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ABSTRACT

In the present project, the complex variable boundary element method (CVBEM) is applied to solve some seepage problems involving earth dams. The method is based on the Cauchy integral formulae. It reduces the seepage problems to the task of solving a system of linear algebraic equations. The coefficients of the linear algebraic equations can be easily computed. Hence the CVBEM for the seepage problems can be easily implemented on the computer.

A computer program in FORTRAN based on the CVBEM is developed to calculate the free seepage line and seepage velocity in a homogeneous isotropic dam.

Possible extension of the present project is also indicated.

ABSTRAK

Pada projek ini, kaedah "complex variable boundary element method (CVBEM)" ataupun "kaedah unsur sempadan anu komplek" digunakan untuk menyelesaikan masalah-masalah resipan yang berkaitan dengan empangan tanah. Kaedah ini adalah berasaskan "rumus kamiran Cauchy". Ia menukarkan masalah-masalah resipan kepada masalah menyelesaikan suatu sistem persamaan linear aljabar. Pekali persamaan linear aljabar dalam sistem itu boleh dikira dengan mudah. Maka CVBEM untuk menyelesaikan masalah-masalah resipan boleh dilaksanakan dengan mengunakan komputer.

Satu program dalam FORTRAN yang berasaskan CVBEM telah dihasilkan untuk mengirakan garis resipan bebas dan halaju resipan dalam empangan homogen isotropi.

Kemungkinan projek ini boleh disambung juga ditunjukkan.

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CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND

The use of earth dams to store water for human needs and protection can be traced back to more than 4800 years ago. Ancient civilizations in the valleys of the Nile, Tigris and Euphrates and Indus which relied on irrigation from rivers for their production of crop were known to construct embankment dams to store water from seasonal rainfalls. One of the earliest embankment dams was constructed at Saad-El-Katara in Egypt about 4800 years ago. The height of the dam was 12 m. Prior to this century, there were very few earth dams which were more than 30 m high. The knowledge and technology to construct huge dams were then lacking.

Significant advancements were made from 1925 onwards after the establishment of the theory of soil mechanics by Karl Terzaghi (refer to e.g. Terzaghi, [9] and William et al. [10]). A proper understanding of flow in porous media is necessary for solving systematically some of the problems that arise in the construction of dams.

Modern embankment dams can be categorized into several types, e.g. earthfill dams and rockfill dams. For further details refer to Robin et al. [8].

Embankment dams may fail due to a variety of factors. Bharat et al. [4] listed some of the major causes responsible for the failures of embankment dams (refer to *Table 1.1*). From *Table 1.1*, it is clear that an understanding of the seepage effect in a dam is important in its construction. The seepage line should be well controlled within the downstream face of the dam section. If the dam section is homogeneous without a proper drainage, seepage will emerge on the downstream face of the dam. This will result in 'sloughing' or softening of the downstream face and may lead to local failure near the toe of the dam. The local failure may then gradually propagate upward. This problem can often be solved by providing a free draining zone on the downstream face or by intercepting the seepage inside the dam section using an internal drainage.

1.2 OBJECTIVE OF THE PROJECT

If the porous medium occupying the dam is isotropic and homogeneous the problem of determining the seepage flow in the dam can be formulated in terms of a mathematical problem which requires the solution of the Laplace's equation subject to certain conditions. For further details, refer to chapter two. The mathematical problem is difficult (if not impossible) to solve exactly. One has to resort to approximate or numerical techniques. There are a few numerical methods which are available for the solution of this type of problems, e.g. finite element method (FEM), finite difference method (FDM), and the boundary element method (BEM).

The main aim of this project is to investigate the possibility of applying a version of the BEM based on the Cauchy integral formulae, developed recently by Ang and Park [12], for determining the seepage flow in a dam. This Complex Variable Boundary Element Method (CVBEM) is used to solve two prototype problems.

Problem 1:

The problem is as sketched in *Figure 1.1*. A careful mathematical formulation of the problem is given in section 2.4 of chapter two. The curve AD is the seepage line and it is an unknown to be determined in the process of solving the problem. It is required to determine the potential and stream functions which describe the seepage flow in the dam.

Problem 2:

The problem is shown in *Figure 1.2*. This problem is similar to problem 1. The difference is that problem 2 has a tail water at the other side of the dam.

1.3 WHY CVBEM ?

As we shall see, one of the advantages of using the CVBEM for solving the problems described in section 1.2 is that it is not necessary to discretise the entire flow domain of the dam. Using the CVBEM, one only needs to discretise the boundary of the dam. If the FEM is used, the entire flow domain in the dam has to be discretised. A comparison between the discretisations needed in the CVBEM and the FEM is made in *Figure 1.3*.

With the CVBEM discretisation, the resulting system of linear algebraic equations to solve is smaller. Furthermore, with the CVBEM the coefficients of the linear algebraic equations are easy to compute. All this mean that the CVBEM is easier to implement on the computer. For further comparison between BEM and other numerical methods, refer to El-Zafrany [1].

TABLE 1.1

Failure Factors	Percentage
Overtopping	30
Seepage effects, piping and sloughing	25
Slope slides	15
Conduit leakage	13
Damage to slope paving	5
Others	12

FIGURE 1.1

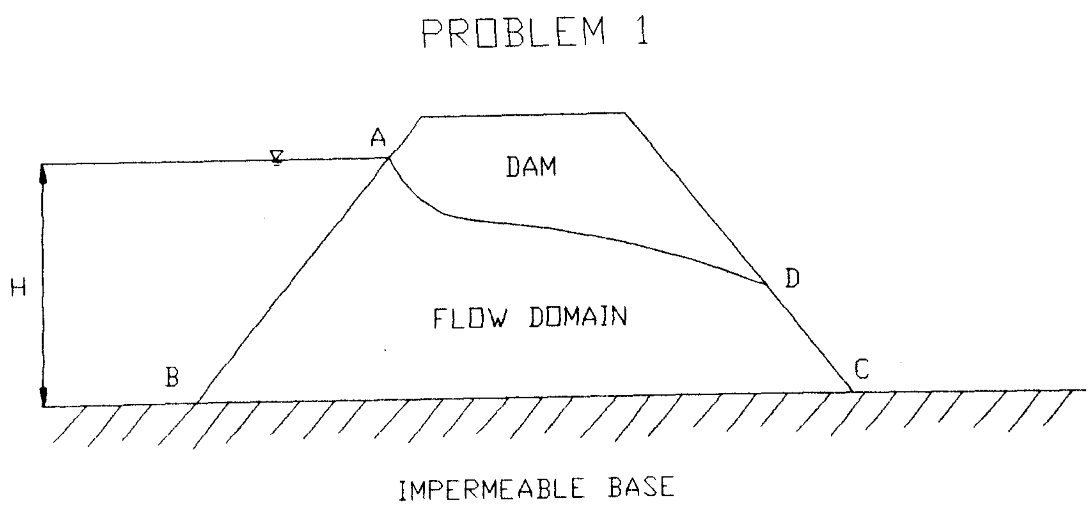
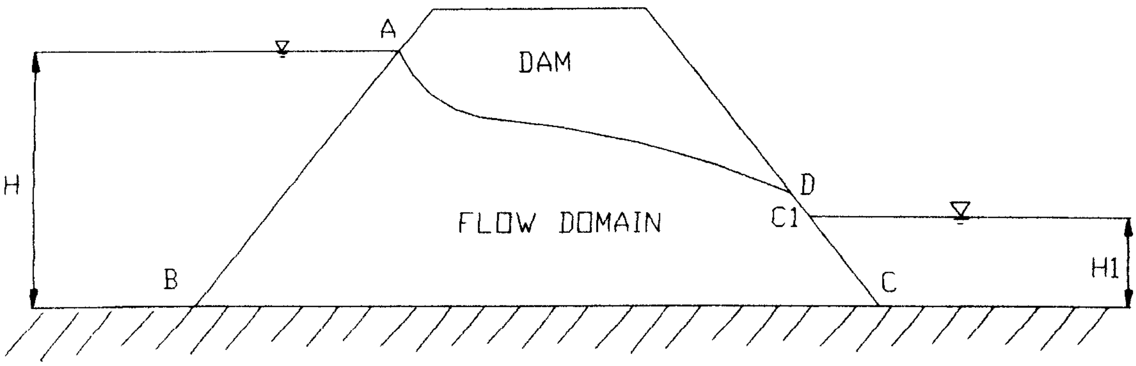


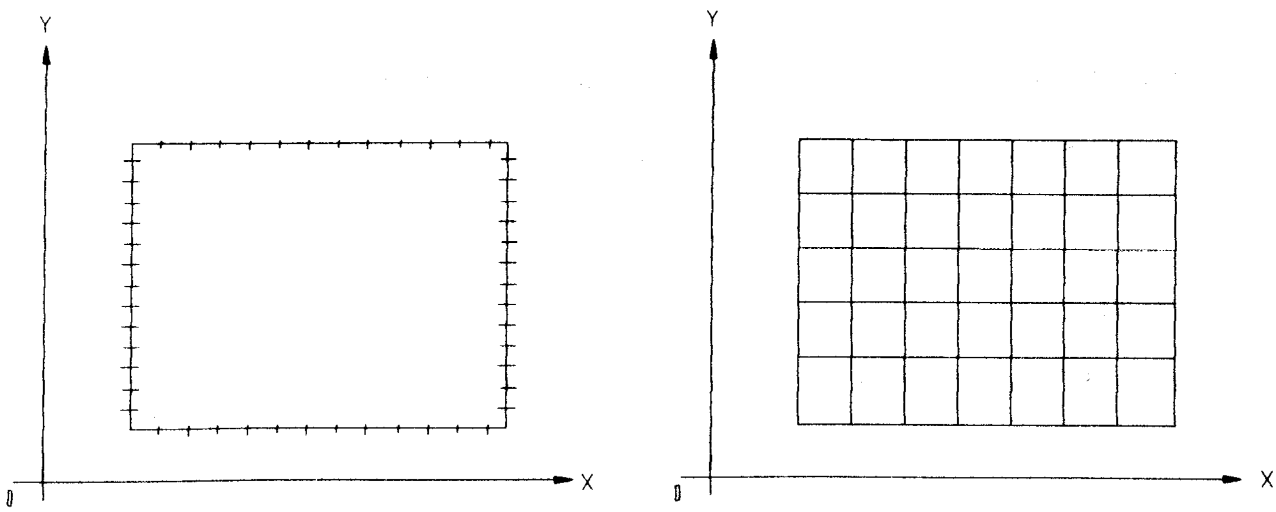
FIGURE 1.2

PROBLEM 2



IMPERMEABLE BASE

FIGURE 1.3



CVBEM FEM

CHAPTER TWO

FLOWS IN POROUS MEDIA

2.1 DARCY'S LAW

To solve the seepage problems stated in chapter one, it is necessary to understand the physics and mathematics of the flow of water through porous media.

In the 1950s, H. Darcy conducted a series of experiments to study one-dimensional flows in soils. A sketch of his experimental set up is given in *Figure 2.1*. From the experiments, it was discovered that the velocity at which water flowed through the soil was given by

$$u = -k(dh/dL) \tag{2.1}$$

where dh/dL denotes the rate of change of the hydrostatic head h per unit distance of the sand column tested and k is the permeability coefficient of the soil. For further details, refer to William and Robert [9].

For three-dimensional flows, if we refer to a $Ox_1x_2x_3$ Cartesian frame and if the velocity field is given by $\underline{q} = u_1\underline{i} + u_2\underline{j} + u_3\underline{k}$, (2.1) can be generalized to

$$u_i = -\sum_{j=1}^{3} k_{ij}\, \partial h/\partial x_j \tag{2.2}$$

where $k_{ij} = k_{ji}$ $(i,j = 1,2,3)$ are the permeability coefficients of the porous medium under consideration. For isotropic porous media, $k_{ij} = \delta_{ij}k$ (δ_{ij} is the Kronecker-delta) and (2.2) becomes

$$u_i = -k\partial h/\partial x_i. \qquad (2.3)$$

Today, it is widely accepted that (2.2) is valid for flows of water in any porous media, provided that the flows are non-turbulant. Equations (2.2) are often referred to as Darcy's law for flow of water in an anisotropic porous media.

2.2 GOVERNING EQUATIONS

If the soil and water are assumed to be incompressible, then, according to the law of conservation of mass, the velocity $\underline{q}$ must satisfy the equation

$$\underline{\nabla} \cdot \underline{q} = 0$$

or

$$\sum_{i=1}^{3} \partial u_i/\partial x_i = 0. \qquad (2.4)$$

Substituting (2.2) into (2.4), we find that

$$\sum_{i=1}^{3} \sum_{j=1}^{3} \partial(k_{ij}\, \partial h/\partial x_j)/\partial x_i = 0 \qquad (2.5)$$

Thus, the hydrostatic head $h(x_1, x_2, x_3)$ in a porous medium which is incompressible must satisfy the partial differential equation (2.5).

For isotropic porous media where $k_{ij} = \delta_{ij}\, k$, (2.5) can be reduced to

$$\partial[k(\partial h/\partial x_1)]/\partial x_1 + \partial[k(\partial h/\partial x_2)]/\partial x_2 + \partial[k(\partial h/\partial x_3)]/\partial x_3 = 0. \qquad (2.6)$$

In general, k can be a function of x_1, x_2 and x_3. But if k is a constant i.e. if the soil is homogeneous, then (2.6) is further reduced to

$$\partial^2 h/\partial x_1^2 + \partial^2 h/\partial x_2^2 + \partial^2 h/\partial x_3^2 = 0, \tag{2.7}$$

i.e. the well-known Laplace's equation.

If the flow is two-dimensional, i.e. if h depends on only two Cartesian coordinates x_1 and x_2 (say), (2.7) gives

$$\partial^2 h/\partial x_1^2 + \partial^2 h/\partial x_2^2 = 0. \tag{2.8}$$

In the project, it will be assumed that the flow of water in the dam (see section 1.2 in chapter one) is two-dimensional and that the porous medium occupying the dam is in-compressible, homogeneous and isotropic. The flow is therefore governed by the two-dimensional Laplace's equation in (2.8).

For convenience, we shall now use x and y to denote x_1 and x_2 respectively. The hydrostatic head is therefore h(x,y) and (2.8) can be rewritten as

$$\partial^2 h/\partial x^2 + \partial^2 h/\partial y^2 = 0. \tag{2.9}$$

2.3 BOUNDARY VALUE PROBLEMS (BVP)

If the boundary of the flow domain is given by the (fixed) simple closed curve C (refer to *Figure 2.2*) and if the flow is governed by (2.9) then the task of analyzing the flow can be formulated as a BVP which requires the solution of (2.9) in R (R is the region enclosed by C) subject to certain conditions on C. Typically, at each and every point on C, either h or $\underline{n} \cdot \underline{\nabla} h$ (not both) is known ($\underline{n}$ is the normal outward vector to C).

BVP: solve (2.9) in R subject to:

$$h = f(x,y) \quad \text{on} \qquad C_1 \tag{2.10}$$

$$\underline{n} \,.\, \underline{\nabla}h = g(x,y) \quad \text{on} \qquad C_2 \tag{2.11}$$

where f and g are suitably prescribed functions and C_1 and C_2 are non-intersecting curves such that $C = C_1 \cup C_2$.

In chapter three, a Complex Variable Boundary Element Method (CVBEM) for solving the above stated BVP is presented.

2.4 SEEPAGE ANALYSIS IN DAMS

For problems 1 and 2 which require us to analyze the seepage flow in the dam, as stated in section 1.2 of chapter one, the boundary defined by the curve AD (*Figure 1.1* and *1.2*) is given by y = h(x,y). Since h is an unknown, the curve AD is an unknown. The BVPs involved are therefore more complicated to solve than the one stated in section 2.3 where the boundary C is fixed and known.

On the unknown curve AD, it is known that $\underline{n} \,.\, \underline{\nabla}h = 0$, i.e. the flow does not have a velocity normal to AD. On the impermeable base BC, one can also impose the condition $\underline{n} \,.\, \underline{\nabla}h = 0$. On AB, h(x,y) = H (where H is the height of the water). For CD, different conditions apply for the problem 1 and 2. For problem 1, h(x,y) = y on CD. For problem 2, h(x,y) = y on C_1D but on CC_1, $h(x,y) = H_1$ (where H_1 is the tail water).

In chapter four, we shall see how the CVBEM described in chapter three can be applied iteratively to solve (2.9) for problems 1 and 2.

FIGURE 2.1

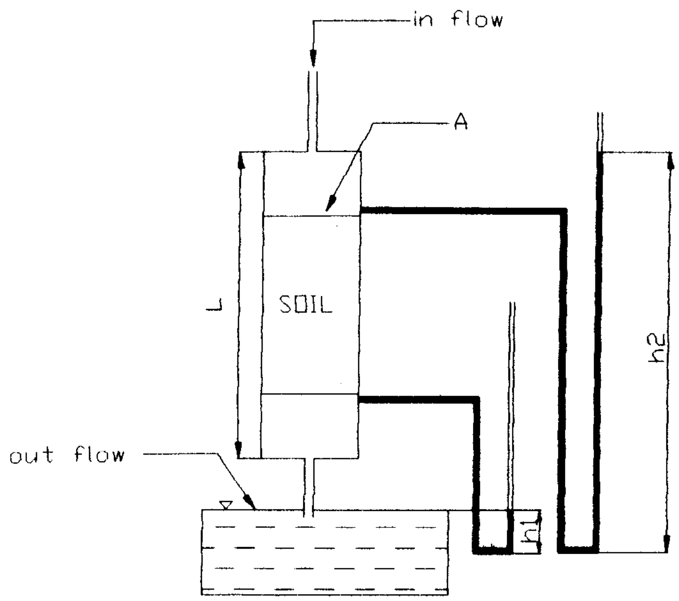
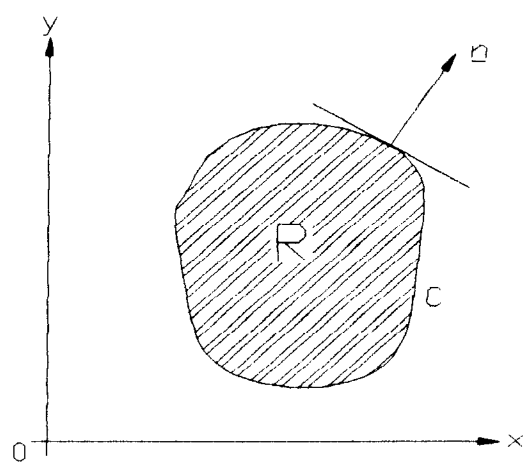


FIGURE 2.2



CHAPTER THREE

COMPLEX VARIABLE BOUNDARY ELEMENT METHOD (CVBEM)

3.1 INTRODUCTION

Ang and Park [12] introduced an alternative boundary element method based on the Cauchy's intergral formulae and the theory of complex hypersingular integrals. This method known as the complex variable boundary element method (CVBEM) is for solving the BVP described in section 2.3. It approximates the BVP as a system of linear algebraic equations and can be easily implemented on the computer.

In the present chapter, the CVBEM is described and used to solve a specific BVP which has exact solution. The numerical results obtained are compared with the exact solution.

3.2 CVBEM

In this section, the CVBEM proposed by Ang and Park [12] for solving the BVP defined in section 2.3 is described.

It is a well established fact that the real and imaginary parts of the complex function f(z) ($z = x_1 + ix_2$) which is holomorphic or analytic in the region R satisfy the Laplace's equation in R (Saff and Snider [5]). Thus for the solution of the BVP, we can write h(x,y) = Re{f(z)} where f(z) is a holomorphic function in R. The function f(z) must also be chosen to satisfy equations (2.10) - (2.11).