ANALYSIS OF SQUARE SLABS WITH OPENING USING YIELD LINE METHOD

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BY

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SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE BACHELOR OF ENGINEERING (HONS.) IN CIVIL ENGINEERING.

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APPROVAL SHEET

This Final Year Project Report entitled "ANALYSIS OF SQUARE SLABS WITH OPENING USING YIELD LINE METHOD" prepared and submitted by TIMOTHY AK. JULIUS EDWARD in partial fulfillment of the requirement for Bachelor of Engineering (Civil) is hereby accepted.

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ABSTRACT

Reinforced concrete slabs are among the most common structural elements, but despite the large number of slabs designed and built, the details of the elastic and plastic behavior of slabs are not always appreciated or properly taken into action. This occurs at least partially because of the mathematical complexity of equations related to elastic and plastic analysis. The method for the limit analysis and an extensive study on plastic analysis of reinforced concrete slabs is known as yield line theory. It was initiated by Ingerslev and was perfected by Johansen in 1843. This method is an upper bound approach. The ultimate load of the slab system is estimated by postulating a collapse mechanism for the slab.

This research project will cover an extensive study on plastic analysis in which it is associated to Yield Line Theory. The research project will involve the application of yield line theory on its analysis in order to determine the ultimate load that will be carried by the slab. In addition, this research will also involve a case study on the effects of an opening towards a slab. A yield line pattern will be postulated and an equation is derived to calculate the ultimate load applied on the slab. The results obtained from this analysis will be used to form a relationship between an opening and the load bearing capacity carried by a slab.

ABSTRAK

Papak konkrit merupakan salah satu daripada elemen yang sering terdapat dalam struktur bangunan. Di sebalik pelbagai jenis papak konkrit yang direka dan dibina, aspek keelastikan dan keplastikan tidak pernah di ambil kira. Ini berlaku disebabkan oleh kaedah pengiraan yang agak rumit serta berhubung kait dengan analisa keelastikan dan keplastikan. Kaedah analisa dan kajian yang lebih lanjut lagi terhadap sifat-sifat keplastikan papak konkrit dikenali sebagai Teori Garis Lentur. Teori in telah diperkenalkan oleh Ingersley dan telah diperkemaskan oleh Johansen pada tahun 1843.

Projek tahun akhir ini meliputi kajian yang lebih mendalam lagi dalam aspek analisis keplastikan di mana ia berkait rapat dengan teori garis lentur. Kajian ini melibatkan pengaplikasian teori garis lentur untuk menentukan beban maksimum yang terpaksa ditanggung oleh sebuah papak. Di samping itu, kajian ini juga melibatkan pengaruh bukaan yang dibentuk di atas papak. Satu corak garis lentur akan dibentuk dan satu bentuk rumus pengiraan akan dibuat berdasarkannya bagi menentukan bebanan maksimum yang dapat ditanggung oleh papak tersebut. Keputusan yang diperolehi daripada keseluruhan analisis digunakan untuk menghubung perkaitan antara sesebuah bukaan dengan bebanan maksimum yang ditanggung oleh papak.

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LIST OF NOTATIONS

A_s = Area of steel

$A_{req.}$ = Area of steel required

$A_{prov.}$ = Area of steel provided

A_{sx} = Area of steel reinforcement in x direction

A_{sy} = Area of steel reinforcement in y direction

b = Width of slab

b` = Distance of an opening from edge of the slab

d = Effective depth

d` = Effective depth for compression reinforcement

f_{cu} = Concrete characteristic strength

f_y = Steel reinforcement characteristic strength

f_s = Estimated design service stress

F_{cc} = Resulting force

F_{st} = Resulting tensile force

G_k = Dead load (normally self weight of the slab)

K = Design constant

$K^`$ = Reference for design constant

l = Length of the yield line

l_x = Length of short span

l_y = Length of long span

L = Length on the edges of the slab

L_0 = Length of an opening

m = Moment of resistance

$m`$ = Negative moment

m_N = Moments on neutral axis

m_{sx} = Moments on short span

m_{sy} = Moments on long span

m_T = Moments on T axis

$M_{ult.}$ = Ultimate moment

W = Ultimate loading per unit area

Q_k. = Imposed load (live load applied to slab)

v = Nominal design shear stress

v_c = Design ultimate shear stress

V = Maximum shear at support

$W_{ult.}$ = Ultimate loading

z = Lever arm

α_{sx} = Moment coefficient in short span

α_{sy} = Moment coefficient in long span

β_b =Ratio of moments after and before redistribution

δ = Deflection on slabs

θ = Rotation in the yield line

γ_m = Partial safety factor

CHAPTER ONE

INTRODUCTION

1.1 GENERAL

A structure is an assembly of members each of which is subjected to bending or direct force (tensile and compressive force) or to a combination of bending and direct force. These primary influences will be absolutely accompanied by action of shearing and torsional forces. Reynolds and Steedman (1938) suggested that effects due to changes in temperature and to shrinkage and creeps of concrete, and also possibility of damages resulting from overloading, local damage, abrasion, vibration, frost, chemical attack and similar causes may also have to be considered.

Therefore, reinforced concrete design includes calculation of or other means of assessing and providing resistance against moment, forces and other effects on members. An efficiently designed structure is considered to have all members adequately arranged in order to carry loadings, self weight and forces and transmitted to the foundation of the structure. In this context, it involves more than providing suitable sizes for concrete members and the provision of calculated amount of reinforcement in an economical manner. It implies that steel bars (reinforcements) can be easily placed, thus reinforcements are provided to resist secondary forces and against all likely cause of damage to the structure.

When an assumed load is not likely to be exceeded and specified quality of concrete is fairly certain to be obtained, high design of strength or service stress can be employed. The more factors allowed for calculation of design, the higher the strength of service. If a magnitude of loading or other factor is unknown, it is essential to study the effects of the probable largest or smallest value of the factor and provide resistance for the most adverse case.

Kong and Evans (1975) stressed out that there are three important aims in reinforced concrete design. They have suggested that;

i) The structure must be safe, for society demands security in the structure it inhabits.

ii) The structure must fulfill its intended purpose during its intended life span.

iii) The structure must be economical with regard to the first cost and to maintenance costs; indeed, most decisions are implicitly or explicitly, economic decision.

One type of structural member to be discussed is a two-way spanning reinforced concrete slab. Reinforced concrete slab is slab used in floor, walls and roof used in buildings and also as decks of bridge. This form of structure can be taken in many forms such as *in situ* solid slabs, ribbed slabs or precast units. (Mosely et al., 1999).

1.2 TWO-WAY SPANNING RECTANGULAR SLAB

A two-way spanning rectangular slab is referred as a slab that carries and transmits loads in two directions. A two-way slab is a form of construction unique to reinforced concrete, among the major structural members. It is definitely considered as an efficient, economical and widely used structural system.

Referring to design of reinforced concrete slab (two-way spanning slab apply to all shapes, support and fixity) in BS8110, it is recommended that bending moments, torsional moments and shear forces are determined either by elastic analysis suggested by Nilson (1997) or analysis considering condition of failure proposed by Johansen (1843) yield line theory. If an elastic analysis is made, loads should be presumably considered as service loads and resulting moments and shear should be resisted accordingly. However, it is more satisfactory to factor loads and design the resulting section based on Ultimate Limit-State principles from BS8110 (1985).

BS8110 (1985) tabulates empirical bending moment coefficients for solid slabs that are loaded uniformly and spanning in two directions. Apart from the case of a freely supported slab without torsional restraint, these coefficients have been derived by means of yield-line analysis and they should be used for all uniformly distributed loadings of a rectangular slab. For non-rectangular and partially loaded slabs, an analysis at failure using yield-line analysis is probably preferable. Although analysis and design of a two-way spanning slab are for the ultimate limit-state, the serviceability limit-state of deflection would often be the ruling criterion and it is important to estimate accurately an appropriate effective depth before carrying through the entire design calculation in too much detail.

1.2.3 Limit-State Design

Based on limit-state design philosophy, a structure has to be designed to sustain safely loads and deformations that may occur during construction or in use and should have adequate durability during the life of the structure. A structure is considered unfit for use when it reaches the limit state. The limit-state is defined as a particular state in which it cease to fulfill the function or to satisfy certain condition for which it was designed. Limit-state design is divided into two groups ;

i) **Ultimate Limit-State** is reached when the structure collapse. Collapse may occur from rupture of one or more critical sections, from the transformation of the structure into a mechanism from elastic to inelastic instability, or from loss of equilibrium as a rigid body and so on.

ii) **Serviceability Limit-State** is described as those of excessive deflection, cracking, vibration and so on.

Referring to clause 2.2.2 of BS8110, Part 1 (1985), ultimate limit-state is stressing more on structural stability in which a structure should be so designed that adequate means exist to transmit design ultimate dead, wind and imposed loads safely from the highest supported level to the foundation of the structure. The layout of a structure and interaction between structural members should be such as to ensure a robust and stable design. In clause 2.2.2.2 of BS8110: Part 1 (1985), robustness is referring to a structure planned and designed so that they are not unreasonably susceptible to the effect of accident.

Serviceability limit-states according to clause 2.2.3 of BS8110: part1 (1985) deals with design properties of materials and design loads. Such criteria associated to these limit-states are deflection due to vertical loading, response to wind loads, cracking and vibration. Section 3 of BS8110: Part 2 (1985) has further details incorporating with serviceability limit-states. In order to ensure acceptable compliances with these limit states, various partial factors of safety are employed in these limit-states design in which particular values selected for these factors depend on accuracy known for load or strength to which the factors are applied.

1.3 OBJECTIVES

The main objective of this project is to study the effects of openings in a rectangular slab. The analysis to determine the effects of opening for these slabs would require an adequate knowledge in plastic theory and yield line analysis.

Analysis will be done using different various sizes of opening. The results obtained through the application of yield line analysis will be used to plot a graph. Based on this graph we will generate a conclusion on the effect of opening towards a slab and further recommendation to enhance the analysis of yield line could be proposed.

CHAPTER TWO

LITERATURE REVIEW

2.1 GENERAL

Various methods, based on elastic and collapse consideration are used to design slabs spanning in two directions. BS8110 is a code practice used as guidance for two-way spanning slab design. Under overload condition in a slab failing in flexure, steel reinforcement will yield first in a region of high moment. When this occurs, this portion of slab acts as a plastic hinge, able to resist the hinging moment but cannot exceeds more.(MacGregor, 1988). The bands in which yielding has occurred are referred to as ***yield lines*** and dividing slabs in several elastic plates.

Yield line analysis was introduced by Johansen (1843) and it is considered to be the most powerful method in slab design. K.W. Johansen published this theory in his doctoral thesis in 1843 at the Danish Technical University. This analysis is based on an extensive study on plastic analysis and yielding of an under-reinforced concrete slab section. Due to an increment of loading, slab tends to crack at a point in which high moments are acting and at the same time, deflections occur. Due to the increasing value of deflection, cracks (yield lines) will be formed until slab is broken (MacGinley & Choo, 1978).

2.2 ANALYSIS OF SECTION

2.2.1 Stress-Strain Relationship

A possible analysis of section should be kept simple and it should be based on the behavior of the reinforced concrete member, (Mosely et al., 1999). They produced 3 most important principles;

i) Stresses and strains are related by the material properties including the stress-strain curves for concrete and steel.

ii) Distribution of strains must be compatible with the distorted shape of the cross section.

iii) Resultant force developed by the section must balance the applied loads of static equilibrium.

Short-term stress-strain curves are presented in Fig.2.1 and 2.2 of Part 1 of BS8110 (1985). The behavior of structure concrete is represented by a parabolic stress-strain relationship. The ultimate design stress is given by;

$$\frac{0.67\ f_{cu}}{\gamma_m} = \frac{0.67\ f_{cu}}{1.5}$$

$$= 0.45\ f_{cu}$$

where, f_{cu} is the concrete characteristic strength; γ_m is the partial safety factor. The partial safety factor is base on Table 2.2 of Part 1 of BS8110 (1985). A safety factor of 1.5 is selected for concrete in axial or flexural load.

2.2.2 Bending and the Equivalent Rectangular Stress Block

The design of most reinforced concrete structures will usually commence from the design for the condition at the ultimate limit state in which it is then followed by checking to ensure that the structure is adequate for serviceability limit states; deflection or cracking. For the design of ultimate limit state, the stress block is applied. Consider a singly reinforced section as shown below.

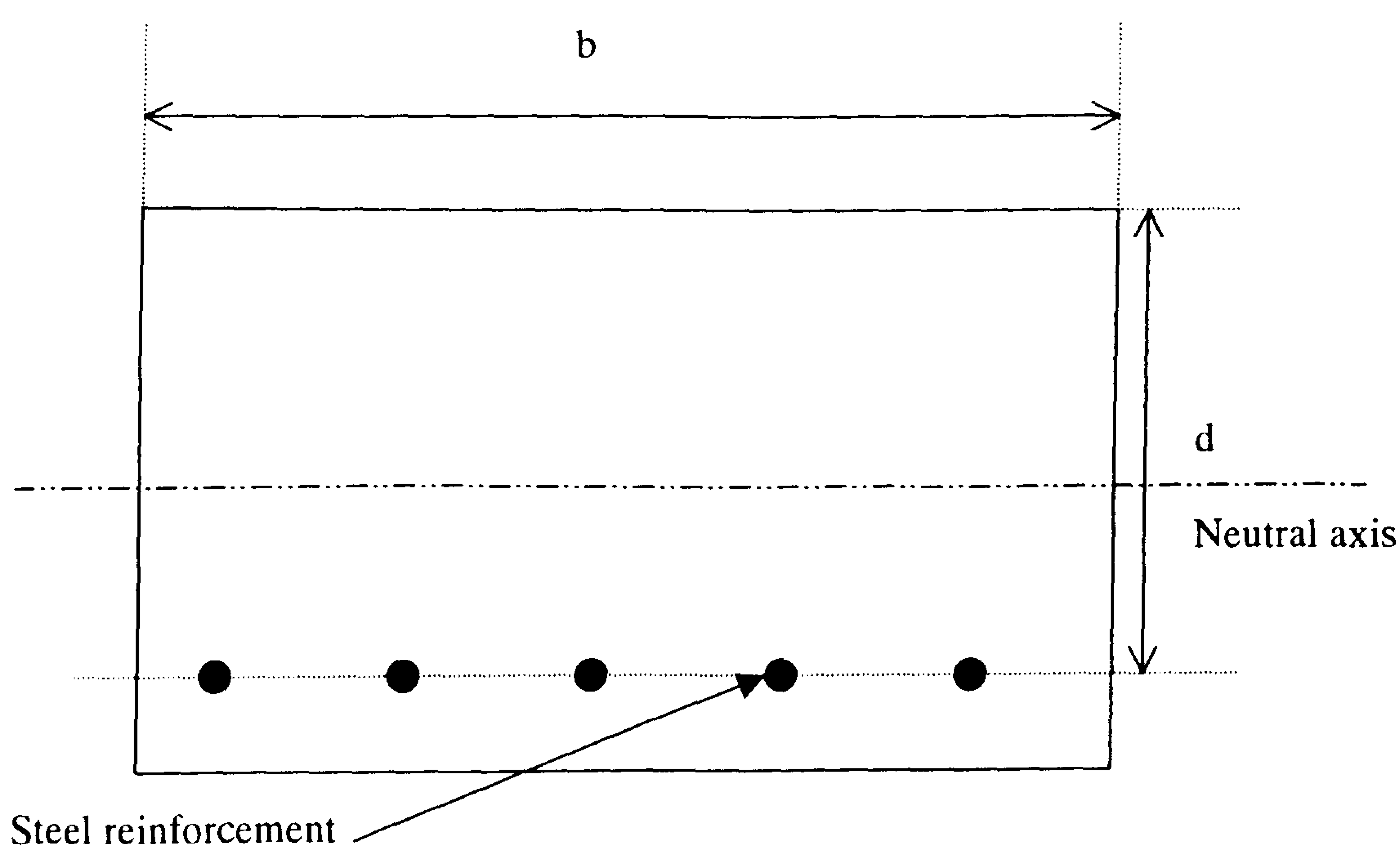

Fig. 2.1 A cross sectional area of a slab

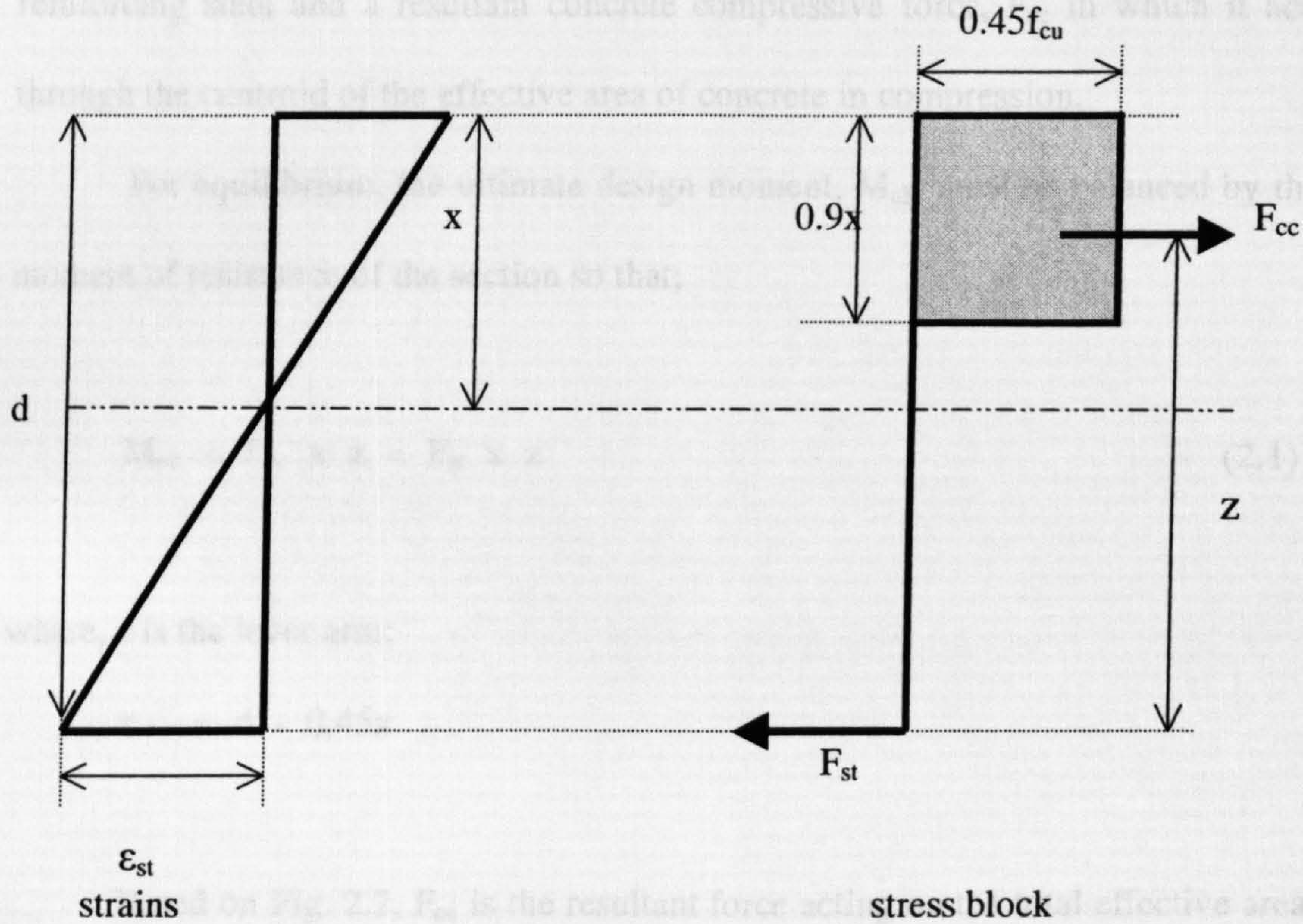

Fig. 2.2 Rectangular stress diagram

Fig. 2.1 is a cross section of a slab with a width of b and an effective depth of d in which is singly reinforced throughout the slab area. The stresses and strains associated to the slab is expressed in Fig. 2.2. It can be seen from the figure that the stress does not extend to the neutral axis of the section but has a depth of 0.9x. This will result in the centroid of the stress block being 0.45 from the top edge of the section.