Data-Driven SIRMs-Connected FIS for Prediction of External Tendon Stress

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Abstract. This paper presents a novel harmony search (HS)-based data-driven single input rule modules (SIRMs)-connected fuzzy inference system (FIS) for the prediction of stress in externally prestressed tendon. The proposed method attempts to extract causal relationship of a system from an input-output pairs of data even without knowing the complete physical knowledge of the system. The monotonicity property is then exploited as an additional qualitative information to obtain a meaningful SIRMs-connected FIS model. This method is then validated using results from test data from the literature. Several parameters, such as initial tendon depth to beam ratio; deviators spacing to the initial tendon depth ratio; and distance of a concentrated load from the nearest support to the effective beam span are considered. A computer simulation for estimating the bond reduction coefficient (Ω_u) is then reported. The contributions of this paper is two folds; (1) it contributes towards a new monotonicity-preserving data-driven FIS model in fuzzy modeling and (2) it provides a novel solution for estimating the Ω_u even without a complete physical knowledge of unbonded tendons.

Keywords: bond reduction coefficient; externally prestressed tendon stress; harmony search; monotonicity index; single input rule modules (SIRMs)-connected fuzzy inference system (FIS)

1. Introduction

Externally prestressed beam is a structural concrete member where the prestressing tendons are placed on the outside of the concrete section and are attached by anchors and deviators at discrete locations (Naaman and Alkhairi 1991a, Ng 2003). The idea of prestressing tendon placement (or sometime known as externally prestressing technique) has been growing rapidly in rehabilitating and strengthening existing structure due to progressive aging and corrosion of steel reinforcement (Ariyawardena and Ghali 2002, Naaman and Alkhairi 1991a, Ng 2003). Comparing to the conventional prestressing technique (i.e., bonded tendon), externally prestressing technique has some advantages, such as simpler to construct, easier to inspect and maintain (Naaman and Alkhairi 1991a, Ng 2003). Regardless of its popularity, the structural behaviour of externally

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prestressed beam is still not fully understood due to its complicity compared to the conventional prestressing technique (Tan and Tjandra, 2007). The study of externally prestressed beam is difficult because of the increase of stress in the external tendons which depends on the overall deformation of a beam member that varied over the entire length of the beam. This causes the eccentricity of the unbonded tendons to vary under loading i.e., second-order effects (Alkhairi and Naaman 1993, Ariyawardena and Ghali 2002, Mutsuyoshi *et al*. 1995, Naaman and Alkhairi 1991a, b, Ng and Tan 2006a) and it involved complicated numerical analysis (Alkhairi and Naaman, 1993, Mutsuyoshi *et al*. 1995, Rao and Mathew, 1996).

The common approach used for estimating the stress in externally prestressed tendon at ultimate, f_{ps}, for an externally prestressed beam is to determine the stress increase caused by an external loading(s), Δf_{ps}, beyond the effective prestress, f_{pe}, i.e., $f_{ps} = \Delta f_{ps} + f_{pe}$ (Alkhairi and Naaman 1993, Naaman and Alkhairi 1991a). Since the member analysis method involved complicated analysis, it can be further simplified by a "pseudo-section analysis" by considering the bond reduction coefficient, Ω_u (Alkhairi and Naaman 1993, Mutsuyoshi *et al.* 1995, Naaman and Alkhairi 1991a, Ng 2003). A search in the literature reveals that efforts to predict an accurate Ω_u (via analytical approaches and/or by experiment), either directly or indirectly, have been reported. A number of parameters have been identified to have contributed to Ω_u; e.g., concrete strength, $f_c^{'}$, area of prestressed reinforcement, A_{ps}, area of non-prestressed reinforcement, A_s, span to depth ratio, L/d_{ps0}, the effective prestressed, f_{pe}, ratio of initial tendon depth to beam depth, d_{ps0}/h, the ratio of deviators spacing to the initial tendon depth, S_d/d_{ps0}, ratio of the distance of a concentrated load from the nearest support to the effective beam span, L_s/L and so on (Harajli *et al*. 1999, Mutsuyoshi *et al*. 1995, Naaman and Alkhairi 1991b, Ng 2003). For details, refer to Figs. 1-2.

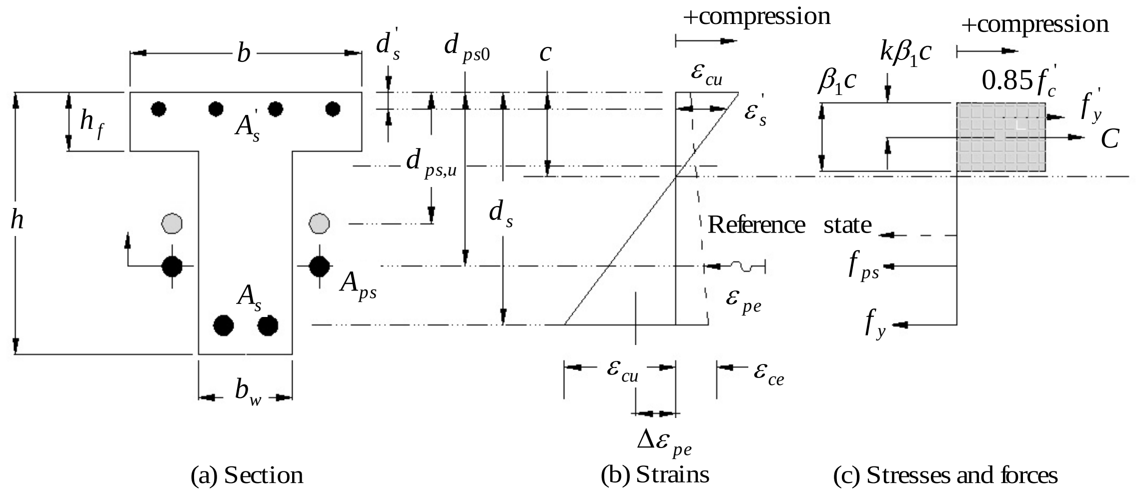


Fig. 1 Strain and stress distributions at critical section of an externally prestressed beam at ultimate flexural limit state. (Ng 2003)

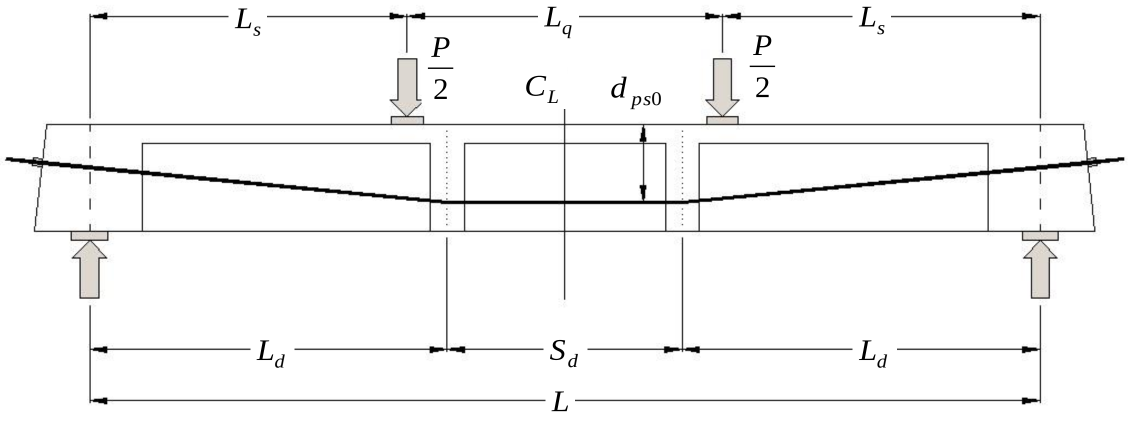


Fig. 2 Type of loading and configurations of external tendons and deviators (Ng 2003)

To predict f_{ps} at ultimate flexural failure, numerical technique (Pisani, 2009), nonlinear analysis (Dall'Asta *et al*. 2007, Zona *et al*. 2009), rational analysis (Ozkul *et al*. 2008), and finite element analysis (Lou and Xiang 2006, Sivaleepunth *et al.* 2006, 2007) have been adopted to estimate Ω_u analytically. However, it was argued that the aforementioned analytical solutions maybe tedious due to the second-order effects (Harajli *et al*. 1999, Mutsuyoshi *et al*. 1995) and maybe inconsistent with actual test values (He and Liu 2010, Nataraja *et al. 2006,* Sivaleepunth *et al*. 2006). Many tests had been carried out to estimate f_{ps}, that is, indirectly estimating, Ω_u (Alkhairi and Naaman 1993, Ariyawardena and Ghali 2002, Harajli *et al*. 1999, Lee *et al*. 1999, Mutsuyoshi *et al*. 1995, Naaman and Alkhairi 1991a, b, Ng and Tan 2006b, Ng 2003). Various statistical tools, e.g., linear regression (Naaman and Alkhairi 1991a) and correlation (Ng 2003), were used to approximate Ω_u. These lines of study results in various approximated mathematical models which best describe a set of experimental data. However, it is realized that most of these equations may not cover all the parameters that have significant effects on Δf_{ps} and tend to overestimate it.

Instead of using statistical tools in approximating Ω_u, soft computing approach is an alternative solution to solve this approximation problem. The soft computing model introduced herein is a data-driven harmony search (HS) zero-order single input rule modules (SIRMs)-connected fuzzy inference system (FIS) hereafter abbreviated as HS-SIRMs connected FIS. FIS model is used because of its interpretability capability to express the behaviour of the system in a human understandable way (Jin, 2000). It is worth mentioning that the use of fuzzy set related techniques in civil engineering is new. It is a popular research direction in the predictions of compressive strength (Subaşı *et al*. 2012) and shear strength (Nasrollahzadeh and Basiri, 2014) of concrete. SIRMs-connected FIS is chosen because of its capability to overcome the issue related to the curse of dimensionality when the number of input increases (Yubazaki *et al*. 1997). To improve the validity of the resulting SIRMs-connected FIS model, additional qualitative knowledge (i.e., monotonicity property) is imposed in the modelling process. Three non-dimensionless parameter (i.e., d_{ps0}/h, S_d/d_{ps0}, and L_s/L) are chosen. HS is then used to search for an SIRMs-connected FIS model which best fit the experimental data. To preserve the monotonicity property, monotonicity index (MI) from previous works (Lau, *et al.* 2013; Tay, *et al.* 2012) is used as the constraint of the HS search.

This study is significant because it contributes to a new data-driven SIRMs-connected FIS model with monotonicity preserving capability. A musical-inspired meta-heuristic optimizer (i.e., HS) is used to search for a set of parameters that best describe Ω_u. The proposed approach may further lead to time saving and cost reduction in externally prestressed beams analysis.

2.0 Background

In this section, existing prediction equations for externally prestressed tendon using simplified method (i.e., pseudo-section analysis) is described. This is followed by a review of a SIRMs-connected FIS, monotonicity index (MI), and HS.

2.1 Review of Prediction Equations

Several pseudo-section analysis based Ω_u equations have been developed to evaluate the f_{ps} and flexural strength of externally prestressed beam (Alkhairi and Naaman 1993, Mutsuyoshi *et al*. 1995, Naaman and Alkhairi 1991b, Ng and Tan 2006a, Ng 2003).

Naaman and Alkhairi (1991) proposed that

$$f_{ps} = f_{pe} + \Omega_u E_{ps} \varepsilon_{cu} \left(\frac{d_{ps0}}{c} - 1 \right) \leq f_{py} \tag{1}$$

where

$$\Omega_u = \begin{cases} \dfrac{2.6}{\left(\dfrac{L}{d_{ps0}} \right)} \text{(for 1 point load)} \\ \dfrac{5.4}{\left(\dfrac{L}{d_{ps0}} \right)} \text{(for third point loading or uniform)} \end{cases} \tag{2}$$

in which E_{ps} is modules of elasticity of tendon; ε_{cu} is concrete strain in top tendon at ultimate; d_{ps0} is initial depth of the external tendon; c is depth of neutral axis at critical section at ultimate; L is total span length; f_{pe} and f_{py} is effective prestress and yield strength of prestressing tendons respectively.

Mutsuyoshi *et al*. (1995) then modified Ω_u in Naaman's Equation (Naaman and Alkhairi 1991a) based on a numerical analysis and introduced a depth reduction factor, R_d, to estimate the tendon depth at ultimate. The tendon stress is given as:

$$f_{ps} = f_{pe} + \Omega_u E_{ps} \varepsilon_{cu} \left(\frac{R_d d_{ps0}}{c} - 1 \right) \le f_{py} \tag{3}$$

with the depth reduction coefficient given by

$$R_d = 1.0 - 0.022 \left(\frac{L}{d_{ps0}} - 5 \right) \left(\frac{S_d}{L} - 0.2 \right) + 0.0186 \left(\frac{L}{d_{ps0}} \right) \left(\frac{A_s f_y}{b d_s f_c^{'}} \right) \tag{4}$$

and bond reduction coefficient given by

$$\Omega_u = \begin{cases} \dfrac{4.36}{\dfrac{L}{d_{ps0}}} - 0.084 \left(\dfrac{S_d}{L} \right) \\ \dfrac{1.47 + 10.3 \left(\dfrac{L_d}{L} \right)}{\left(\dfrac{L}{d_{ps0}} \right)} - 0.29 \left(\dfrac{L_d}{L} \right) \left(\dfrac{S_d}{L} \right) \end{cases} \tag{5}$$

where S_d is deviator spacing; b is beam width of compression zone; L_d is distance between loading points; A_s, d_s, f_y is area, depth and yield strength of tension reinforcement respectively; and $f_c^{'}$ are cylinder compressive strength of the concrete.

Aravinthan *et al*. (1997) then improved the equation proposed by Mutsuyoshi *et al*. (1995) based on the investigation on simply-supported externally prestressed beams. The proposed equation considered several factors that influenced the second-order effect such as: distance between deviators-to-span ratio, S_d / L, span-to-effective depth ratio, L / d_{ps0}, bonded-to-total tendon area ratio $A_{ps,\text{int}} / A_{ps,tot}$.
The Ω_u is proposed as:

$$\Omega_u = \begin{cases} \dfrac{0.21}{\dfrac{L}{d_{ps0}}} + 0.04 \left(\dfrac{A_{ps,\text{int}}}{A_{ps,tot}} \right) + 0.04 \text{ for one point load} \\ \dfrac{2.31}{\dfrac{L}{d_{ps0}}} + 0.21 \left(\dfrac{A_{ps,\text{int}}}{A_{ps,tot}} \right) + 0.06 \text{ for third point loading} \end{cases} \tag{6}$$

with depth reduction factor given as;

$$R_d = \begin{cases} 1.14 - 0.005\left(\dfrac{L}{d_{ps0}}\right) - 0.19\left(\dfrac{S_d}{L}\right) \leq 1.0 \text{ for one point load} \\ 1.25 - 0.010\left(\dfrac{L}{d_{ps0}}\right) - 0.38\left(\dfrac{S_d}{L}\right) \leq 1.0 \text{ for third point loading} \end{cases} \tag{7}$$

From a series of theoretical and experimental investigations, Ng (2003) showed that the span to depth ratio, L/d_{ps0}, has insignificant effect on Δf_{ps}. A new dimensionless parameter, S_d/d_{ps0}, is introduced to cater for the second-order effects for longer span beam. Ng (2003) proposed a modified equation for Ω_u using the correlation of average strains in the external tendons obtained through the rational analysis based on strain compatibility and force equilibrium on externally prestressed beam:

$$\Omega_u = \left[0.895 - 1.365\left(\frac{L_s}{L}\right)\right]\left(\frac{d_{ps0}}{h}\right) - k_s \tag{8}$$

with coefficient accounting for second-order effect given as;

$$k_s = \begin{cases} 0.0096\left(\dfrac{S_d}{d_{ps0}}\right) \text{for } \dfrac{S_d}{d_{ps0}} \leq 15 \\ 0.144 \text{ for } \dfrac{S_d}{d_{ps0}} > 15 \end{cases} \tag{9}$$

where h is beam height and L_s is shear span.

The preceding description have identified several significant non-dimensionless parameters (i.e., d_{ps0}/h, S_d/d_{ps0}, and L_s/L) which then served as the basis for prediction of external tendon stress, f_{ps}, for externally prestressed beams using HS-SIRMs connected FIS model, which indirectly approximates the bond reduction coefficient, Ω_u.

2.2 A general formulation of the Zero-order SIRMs-Connected FIS

A relatively new fuzzy inference model, SIRMs-connected FIS model is proposed for multi-input fuzzy system with n-input (Yubazaki *et al*. 1997). Consider a zero-order SIRMs-connected FIS model with n-input (i.e., $y = f(\bar{x};\theta)$), where $\bar{x} = (x_1, x_2, ..., x_n)$ and $\theta = (w_1, w_2, ..., w_n; A_j^1, A_j^2, ..., A_j^n; c_1, c_2, ..., c_n)$. It consists of n fuzzy rule modules as in Fig. 3. Note that $SIRM - i$ represents the i-th rule module, where x_i is the sole variable in the antecedent, where $i = 1, 2, ..., n$. $R_i^{j_i}$ is the j-th rule in $SIRM - i$, where $j = 1, 2, ..., m_i$, while $c_i^{j_i}$ is a variable output value in the consequent part. A fuzzy rule $R_i^{j_i}$ can be viewed as a mapping from $A_i^{j_i}$ to $c_i^{j_i}$.

$$SIRM-1:\left\{R_1^{j_1}:\text{if } x_1=A_1^{j_1} \text{ then} y_1^{j_1}=c_1^{j_1}\right\}_{j=1}^{m_1}$$
$$\vdots$$
$$SIRM-i:\left\{R_i^{j_i}:\text{if } x_i=A_i^{j_i} \text{ then} y_i^{j_i}=c_i^{j_i}\right\}_{j=1}^{m_i}$$
$$\vdots$$
$$SIRM-n:\left\{R_n^{j_n}:\text{if } x_n=A_n^{j_n} \text{ then} y_n^{j_n}=c_n^{j_n}\right\}_{j=1}^{m_n}$$

Fig. 3 Fuzzy rules for a zero-order SIRMs-connected FIS model

The output of $SIRM-i$, i.e., $y_i(x_i)$ is obtained using Eq. (10). The membership function (MF) for $A_i^{j_i}$ is denoted as $\mu_i^{j_i}(x_i)$. The final inference result of SIRMs-connected FIS is obtained by a weighted sum of rule modules, as in Eq. (11). In which w_i reflects the relative importance of the $SIRM-i$ which is defined according to the contribution of the input item to the system performance.

$$y_i(x_i)=\frac{\sum_{j_i}^{m_i}\left[\mu_i^{j_i}(x_i).\,y_i^{j_i}\right]}{\sum_{j_i}^{m_i}\left[\mu_i^{j_i}(x_i)\right]} \tag{10}$$

$$y=\sum_{i=1}^{n} w_i \,.\, y_i(x_i) \tag{11}$$

2.3 A Monotonicity Index (MI) for Zero-order SIRMs-Connected FIS

Let $f(\bar{x})$ denote as n -input zero-order SIRMs-connected FIS model, where $\bar{x}=(x_1,x_2,...,x_n)\in X_1\times X_2\times...\times X_n$. The i-th input in $\bar{x}$ is represented by x_i is excluded from $\bar{s}$, i.e., $\bar{s}\subset\bar{x}$; $x_i\notin\bar{s}$. The definition for monotonicity of $f(\bar{x})$ can be formally written as:

Definition 1 *An SIRMs-connected FIS model is said to fulfill the monotonicity increasing or decreasing property between its output,* y*, and its input,* x_i*, when* y *monotonically increases or decreases respectively, as* x_i *increases, i.e.,* $f(\bar{s},x_i)\geq f(\bar{s},x_i^{'})$ *or* $f(\bar{s},x_i)\leq f(\bar{s},x_i^{'})$*, respectively, where* $x_i>x_i^{'}\in X_i$.

The proposed procedure for MI is summarized as follows:

(i) Determine the upper and lower limits of the universe of discourse for x_i, and denote as $\bar{x}_i$ and $\underline{x}_i$ respectively.

(ii) Divide x_i domain to n_i divisions. Determine the grid size of x_i, $s_i = \left(\bar{x}_i - \underline{x}_i\right)/n_i$.

(iii) Compare each pair of $y_i\left(x_i + s_i \times n_{s_i}\right)$ and $y_i\left(x_i + s_i \times \left(n_{s_i} + 1\right)\right)$ with a function denote as $monotone\left(y_i\left(x_i + s_i \times n_{s_i}\right)\right)$. Eq. (12) or Eq. (13) is adopted for a monotonic increasing or decreasing relationship respectively.

$$monotone\left(y_i\left(x_i + s_i \times n_{s_i}\right)\right) = \begin{cases} 1 \text{ if } y_i\left(x_i + s_i \times n_{s_i}\right) \le y_i\left(x_i + s_i \times \left(n_{s_i} + 1\right)\right) \\ 0 \text{ if } y_i\left(x_i + s_i \times n_{s_i}\right) > y_i\left(x_i + s_i \times \left(n_{s_i} + 1\right)\right) \end{cases} \tag{12}$$

$$monotone\left(y_i\left(x_i + s_i \times n_{s_i}\right)\right) = \begin{cases} 1 \text{ if } y_i\left(x_i + s_i \times n_{s_i}\right) \ge y_i\left(x_i + s_i \times \left(n_{s_i} + 1\right)\right) \\ 0 \text{ if } y_i\left(x_i + s_i \times n_{s_i}\right) < y_i\left(x_i + s_i \times \left(n_{s_i} + 1\right)\right) \end{cases} \tag{13}$$

(iv) Obtain the MI between y_i and x_i for an SIRMs-connected FIS model using Eq. (14)

$$MI_i = \frac{\sum_{n_{s_i}=1}^{n_{s_i}=n_i-1} monotone\left(y_i\left(x_i + s_i \times n_{s_i}\right)\right)}{\sum_{n_{s_i}=1}^{n_{s_i}=n_i-1} (1)} \tag{14}$$

2.4 Harmony Search (HS) Algorithm

The SIRMs-connected FIS model is then optimized using a music-inspired meta-heuristic optimizer (i.e., HS). The HS is conceptualized using the musical process of searching for a perfect state of harmony. HS is chosen because it does not require initial values for the decision variables. Besides, it uses a stochastic random search based on the memory considering rate (HMCR) and the pitch adjusting rate (PAR) so that derivative information is unnecessary (Geem *et al*. 2008, Geem *et al*. 2001). Consider an optimization problem with m variables (i.e., $\bar{z} = \left(z_1, z_2, ..., z_m\right)$). The aim is to search for a set of $\bar{z}$ in such that $g\left(\bar{z}\right)$ is optimized. Fig. 4 summarizes the optimization procedure for HS.

```
Begin
    Define objective function g(z̄), z̄ = (z_1, z_2,...,z_m)^T and
    Define harmony memory size (HMS), harmony memory considering rate (HMCR),
    pitch adjusting rate (PAR), and termination criterion (maximum number of search)
    Generate Harmony Memory (HM) with random harmonies
    while (t < max number of iterations)
        while (i ≤ number of variables)
            if (rand < PAR), choose a value from HM for the variable i
                if (rand < PAR), adjust the value by adding certain amount
                end if
            else Choose a random value
            end if
        end while
        Accept the new harmony (solution) if better
    end while
    Find the current best solution
end
```

Fig. 4 Pseudo code for HS algorithm (Geem *et al*. 2001)

3.0 Proposed Framework

In this section, the HS-SIRMs connected FIS model is expressed as a constraint optimization problem. In this study, the non-dimensional parameters considered are: d_{ps0}/h, S_d/d_{ps0}, and L_s/L.

3.1 The Zero-order SIRMs-Connected FIS for Estimating Ω_u

A zero-order SIRMs-connected FIS model with three inputs (i.e., $\Omega_u = f(\bar{x};\theta)$, where $\bar{x} = (d_{ps0}/h, S_d/d_{ps0}, L_s/L)$ and θ is the parameters describing the model is considered. It consists of three fuzzy rule modules as in Fig. 5, i.e., $i = 1, 2, 3$. $SIRM - d_{ps0}/h$ represents the d_{ps0}/h rule module, where $A_1^{j_1}$ is the sole variable in the antecedent. $R_1^{j_i}$ is the j_i-th rule in $SIRM - d_{ps0}/h$, where $j_i = 1, 2, \ldots, m_i$ and $c_1^{j_i}$ is a numerical output in the consequent or fuzzy singleton. Thus, a fuzzy rule $R_1^{j_i}$ can also be viewed as a mapping from $A_1^{j_1}$ to $c_1^{j_i}$, i.e., $R_1^{j_i} : A_1^{j_i} \rightarrow c_1^{j_i}$. The same applies to modules $SIRM - S_d/d_{ps0}$ and $SIRM - L_s/L$.

$$SIRM - d_{ps0}/h : \left\{ R_1^{j_1} : \text{if } d_{ps0}/h = A_1^{j_1} \text{ then} \Omega_{u_1}^{j_1} = c_1^{j_1} \right\}_{j=1}^{m_1}$$

$$SIRM - S_d/d_{ps0} : \left\{ R_2^{j_2} : \text{if } S_d/d_{ps0} = A_2^{j_2} \text{ then} \Omega_{u_2}^{j_2} = c_2^{j_2} \right\}_{j=1}^{m_2}$$

$$SIRM - L_s/L : \left\{ R_3^{j_3} : \text{if } L_s/L = A_3^{j_3} \text{ then} \Omega_{u_3}^{j_3} = c_3^{j_3} \right\}_{j=1}^{m_3}$$

Fig. 5 Fuzzy rules for a zero-order SIRMs-connected FIS model

The zero-order SIRMs-connected FIS model is written as Eq. (15).

$$\begin{aligned}\Omega_u &= f\left(d_{ps0}/h,\ S_d/d_{ps0}, \text{and}\, L_s/L\right) \\ &= w_1 \times \Omega_{u_1} + w_2 \times \Omega_{u_2} + w_3 \times \Omega_{u_3} \\ &= w_1 \frac{\sum_{j_i=1}^{m_1}\left[\mu_1^{j_1}\left(d_{ps0}/h\right).c_1^{j_1}\right]}{\sum_{j_i=1}^{m_1}\left[\mu_1^{j_1}\left(d_{ps0}/h\right)\right]} + w_2 \frac{\sum_{j_i=1}^{m_2}\left[\mu_2^{j_2}\left(S_d/d_{ps0}\right).c_2^{j_2}\right]}{\sum_{j_i=1}^{m_2}\left[\mu_2^{j_2}\left(S_d/d_{ps0}\right)\right]} + w_3 \frac{\sum_{j_i=1}^{m_3}\left[\mu_3^{j_3}\left(L_s/L\right).c_3^{j_3}\right]}{\sum_{j_i=1}^{m_3}\left[\mu_3^{j_3}\left(L_s/L\right)\right]}\end{aligned} \tag{15}$$

3.2 Monotonicity Index (MI)

The monotonicity relationship between the inputs and output of the zero-order SIRMs-connected FIS model can be observed from experiments. It is generally agreed that when d_{ps0}/h increases, Ω_u increases. Besides, when S_d/d_{ps0} and L_s/L increase, Ω_u decrease (Ng, 2003). An input $x_i \in \left(d_{ps0}/h, S_d/d_{ps0}, L_s/L\right)$ is considered, the proposed procedure is summarized as follows:

(i) Determine the upper and lower limits of the universe of discourse for x_i, and denote as $\bar{x}_i$ and $\underline{x}_i$ respectively.

(ii) Divide x_i domain to n_i divisions. Determine the grid size of x_i, $s_i = \left(\bar{x}_i - \underline{x}_i\right)/n_i$.

(iii) Compare each pair of $\Omega_{u_i}\left(x_i + s_i \times n_{s_i}\right)$ and $\Omega_{u_i}\left(x_i + s_i \times \left(n_{s_i} + 1\right)\right)$ with a function denote as $monotone\left(\Omega_{u_i}\left(x_i + s_i \times n_{s_i}\right)\right)$. Eq. (16) or Eq. (17) is adopted for a monotonic increasing or decreasing relationship respectively.

$$monotone\left(\Omega_{u_i}\left(x_i + s_i \times n_{s_i}\right)\right) = \begin{cases} 1 \text{ if } \Omega_{u_i}\left(x_i + s_i \times n_{s_i}\right) \le \Omega_{u_i}\left(x_i + s_i \times \left(n_{s_i} + 1\right)\right) \\ 0 \text{ if } \Omega_{u_i}\left(x_i + s_i \times n_{s_i}\right) > \Omega_{u_i}\left(x_i + s_i \times \left(n_{s_i} + 1\right)\right) \end{cases} \tag{16}$$

$$monotone\left(\Omega_{u_i}\left(x_i + s_i \times n_{s_i}\right)\right) = \begin{cases} 1 \text{ if } \Omega_{u_i}\left(x_i + s_i \times n_{s_i}\right) \ge \Omega_{u_i}\left(x_i + s_i \times \left(n_{s_i} + 1\right)\right) \\ 0 \text{ if } \Omega_{u_i}\left(x_i + s_i \times n_{s_i}\right) < \Omega_{u_i}\left(x_i + s_i \times \left(n_{s_i} + 1\right)\right) \end{cases} \tag{17}$$

(iv) Obtain the MI between Ω_{u_i} and x_i for an SIRMs connected FIS model using Eq. (18)

$$MI_i = \frac{\sum_{n_{s_i}=1}^{n_{s_i}=n_i-1} monotone\left(\Omega_{u_i}\left(x_i + s_i \times n_{s_i}\right)\right)}{\sum_{n_{s_i}=1}^{n_{s_i}=n_i-1}(1)} \tag{18}$$

In this paper, MI_i is preprocessed with Eq. (19)

$$MI_i^{'} = \begin{cases} 0, MI_r = 1 \\ 1, MI_r \neq 1 \end{cases} \tag{19}$$

3.2 A Monotonicity Preserving HS-SIRMs Connected FIS Model for Estimating Ω_u

A HS-SIRMs connected FIS model, i.e., $\Omega_u = f(\bar{x};\theta)$, is considered. A system identification problem attempts to determine a set of $\theta^{'}$, in such a way that $f(\bar{x};\theta^{'})$ best represents a system when it is observed by j desired input-output pairs of data, i.e., $(\bar{x}_k, \Omega_{u_k})$, where $\bar{x}_k = \left((d_{ps0}/h)_k, (S_d/d_{ps0})_k, (L_s/L)_k\right)$, $k = 1, 2, \ldots, j$. Fig. 6 shows the schematic diagram of parameter identification for an HS-SIRMs connected FIS.

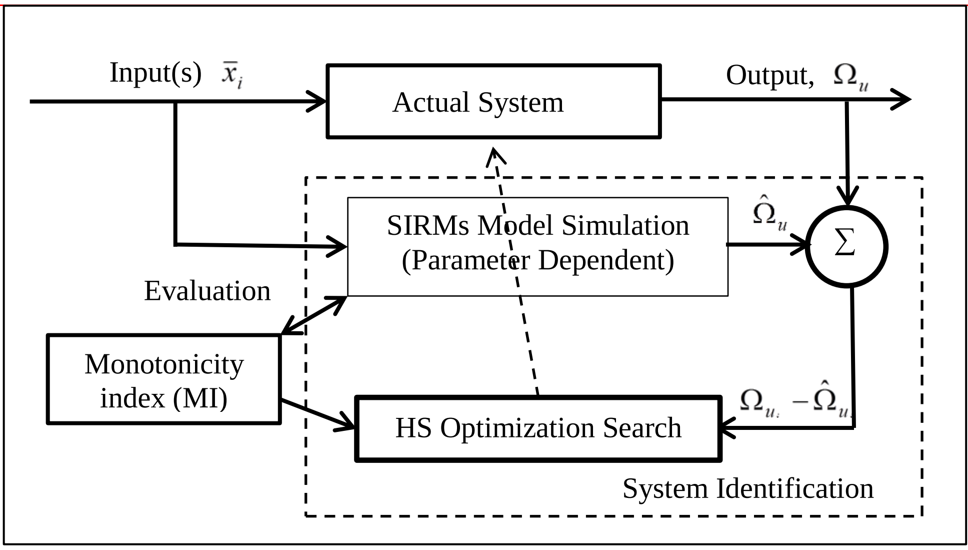


Fig. 6 Proposed model

A data set composed of j desired input-output pairs $\left(\bar{x}_k, \Omega_{u_k}\right)$, where $k = 1,2,3,\ldots, j$, is used to construct an HS-SIRMs connected FIS model. The inputs (i.e., d_{ps0}/h, S_d/d_{ps0}, and L_s/L) is applied to both the system and the HS-SIRMs connected FIS model, while the square of the difference between the target system output (i.e., Ω_u) and the model output (i.e., $\hat{\Omega}_u$), is $\left(\Omega_u - \hat{\Omega}_u\right)^2$. The total of $\left(\Omega_u - \hat{\Omega}_u\right)^2$ for the j set of data is used to give an indication of how near the FIS model with the target system. The constrained optimization problem is formulated as Eq. (20):

$$\text{Minimize } Error(\theta) = \sum_{k=1}^{k=j}\left(\Omega_{u_k} - f\left(\bar{x}_k;\theta\right)\right)^2 \tag{20}$$

Subjected to $MI_i^{'} = 1$, in which $i \in [1, 2, 3]$

Thereafter, the objective function to be minimized by HS is as shown in Eq. (21).

$$\textit{Objective function} = \mathrm{Error}(\theta) + w \times \sum_{i=1}^{n_3} MI_i^{'} \tag{21}$$

where w is a weightage constant.

4.0 Model Development

4.1 Data Collection

A total of 27 beams $(j = 27)$ from experimental investigations were used to examine the applicability of the proposed model in estimating Δf_{ps} and f_{ps} of the externally prestressed beams and hence Ω_u. The flexural strength of the beams were analysed based on strain compatibility and force equilibrium on beams reported in Table 1 and Figs. 1-2. These beams were: (i) Series T, ST, and SR beams with straight tendons, tested under third-point loading in (Ng 2003); (ii) Series M beams with an effective span of 5200 mm and draped tendons, tested under two symmetrical loads spaced at 900 mm apart by (Mutsuyoshi *et al*. 1995) and (iii) Series Y with an effective span of 4000 mm and straight tendons, tested under two symmetrical loads spaced at 600 mm apart (Yaginuma 1995). Data used in this paper is obtained from (Mutsuyoshi *et al*. 1995, Ng 2003), as summarized in Table 1.

Table 1. Parameter for predicting bond reduction equation [18-21]

Beam No	S_d/d_{ps0}	d_{ps0}/h	L_s/L	Δf_{ps}	f_{ps}
M-1	7.2000	0.7692	0.4135	357.4000	1347.6000
M-2	12.0000	0.7692	0.4135	341.5000	1331.7000

NA-1	3.5211	0.7100	0.4250	179.2000	670.3000
OA-1	14.2349	0.7025	0.4250	160.9000	651.8000
SA-1	2.1127	0.7100	0.4250	187.8000	676.7000
SR1	0.0000	1.1000	0.3333	815.5000	1784.5000
SR2	0.0000	1.1000	0.3333	602.6000	1647.6000
SR3	0.0000	1.1000	0.3333	422.4000	1809.4000
SR4	0.0000	1.1000	0.3333	426.6000	1737.6000
SR5	0.0000	0.7000	0.3333	621.3000	1704.3000
SR6	0.0000	0.7000	0.3333	360.7000	1462.7000
ST-1	0.0000	0.6667	0.3333	443.2000	1207.6000
ST-2	0.0000	0.6667	0.3333	380.9000	1152.1000
ST-2C	0.0000	0.6667	0.3333	330.2000	1099.3000
ST-2P	0.0000	0.6667	0.5000	259.2000	1017.7000
ST-3	0.0000	0.6667	0.3333	409.2000	1159.6000
ST-4	0.0000	0.6667	0.3333	366.2000	1122.7000
ST-5	0.0000	0.6667	0.3333	269.6000	1029.9000
ST-5A	10.0000	0.6667	0.3333	376.0000	1137.7000
ST-5B	7.5000	0.6667	0.3333	412.4000	1154.2000
T-0	15.0000	0.6667	0.3333	410.8000	1707.5000
T-0A	22.5000	0.6667	0.3333	260.7000	1005.1000
T-0B	30.0000	0.6667	0.3333	224.0000	965.9000
T-1	0.0000	0.6667	0.3333	589.5000	1786.0000
T-1A	0.0000	0.8333	0.3333	810.9000	1137.6000
T-1D	0.0000	0.8333	0.3333	954.9000	1242.8000
T-2	5.0000	0.6667	0.3333	527.5000	1709.4000

4.2 Simulation

In the simulation, Gaussian membership function (MF) is used. It is further assumed that there are five Gaussian MFs for each of the non-dimensional parameter. The parameters setting for HS-SIRMs connected FIS model is depicted in Table 2.

Table 2 Parameter setting used for the simulation

Parameter	Setting
Harmony memory size (HMS)	30
Harmony consideration rate (HMCR)	0.90
Pitch adjusting rate (PAR)	0.20
Number of iterations	10,000

Number of inputs, N	3
Number of MF for each input	5
Grid size for MI, n_i	1000
Weight, w	0; 1,000,000

5.0 Results and Discussions

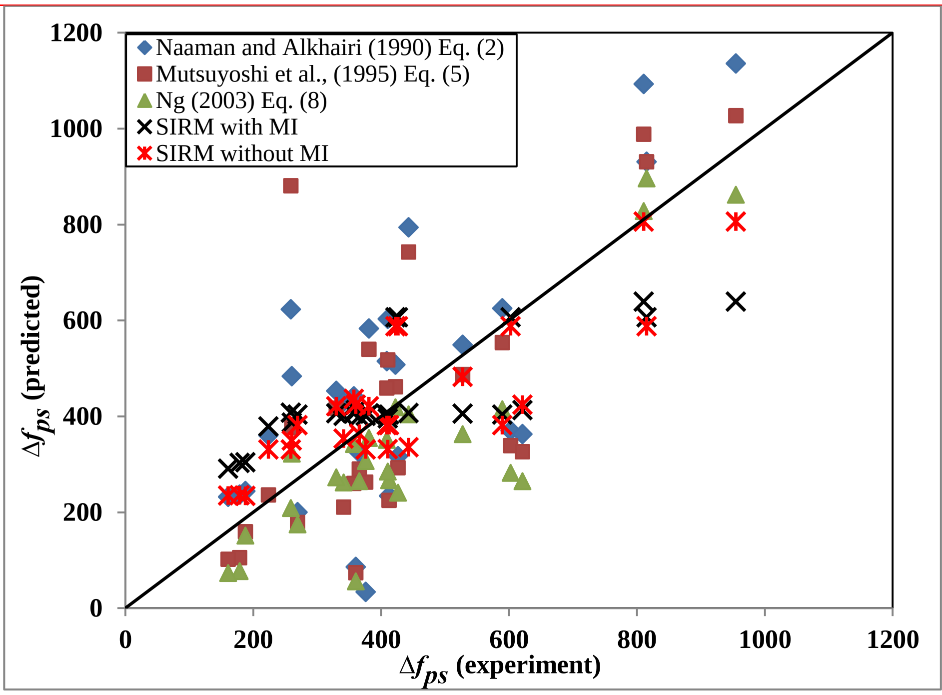
5.1 Comparison with Previous Equations

Fig. 7 show the plot for $\Delta f_{ps}(\text{predicted})$ versus $\Delta f_{ps}(\text{experimental})$ and $f_{ps}(\text{predicted})$ versus $f_{ps}(\text{experimental})$ in externally prestressed tendons between the existing prediction equations (Mutsuyoshi *et al*. 1995, Naaman and Alkhairi 1991a, Ng 2003) and the proposed model. It is observed that most of the existing prediction equations can reasonably predict f_{ps}, but they tend to overestimate Δf_{ps}, except for Ng (2003) which underestimates Δf_{ps}, and showed the scattering phenomenon far from the perfect line. Besides, it is observed that the existing prediction equations (Mutsuyoshi *et al*. 1995, Naaman and Alkhairi 1991b, Ng 2003) tend to give inconsistent and unconservative results in predicting Δf_{ps} and f_{ps}.

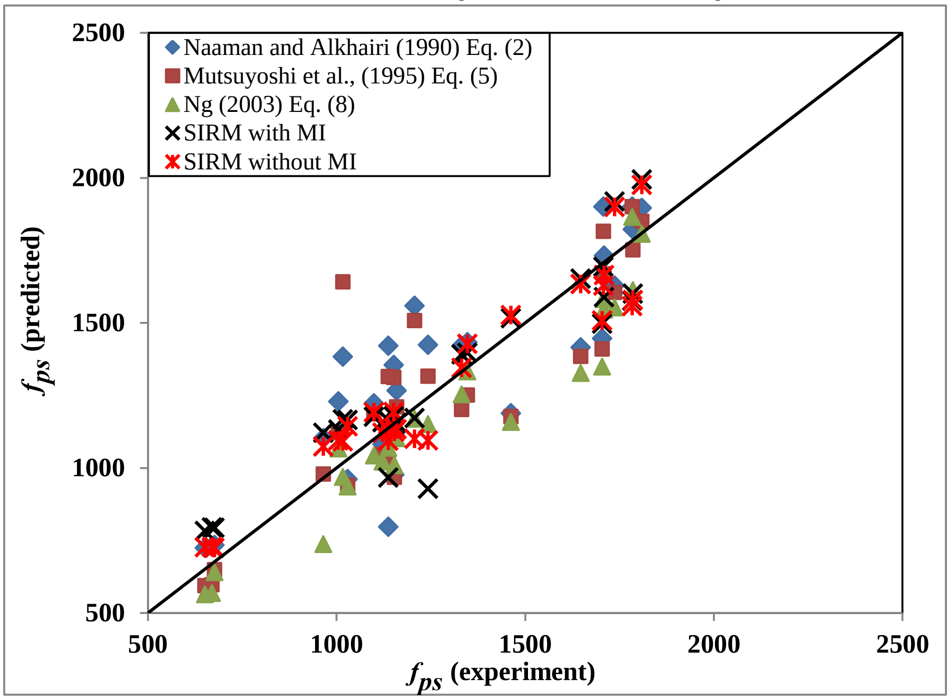
Table 3 further shows the results of the correlation between the experimental results (Alkhairi and Naaman 1993, Mutsuyoshi *et al*. 1995, Ng 2003) and the proposed model with and without considering monotonicity property for beams listed in Table 1 and the predicted values. The HS-SIRMs connected FIS model is first tested with the data from Table 1 without considering MI as a constraint. The study shows a relatively good coefficient correlation of 0.8468 and 0.9538 respectively for Δf_{ps}, and f_{ps} in the external tendons at ultimate, with relatively less variability compared to the other proposed equations (Naaman and Alkhairi, 1991b; Mutsuyoshi *et al*. 1995; Ng 2003).

Table 3 Correlation of test results and theoretical predictions for Δf_{ps} and f_{ps}

Methodology	Stress increase, Δf_{ps}		Ultimate stress, f_{ps}	
	Correlation Coefficient	Variability within 95% confidence	Correlation Coefficient	Variability within 95% confidence
Proposed Without MI	0.8468	0.0874	0.9538	0.0321
Proposed With MI	0.7815	0.1097	0.9262	0.0404
Naaman (Naaman and Alkhairi, 1991b)	0.7356	0.1572	0.8661	0.0563
Mutsuyoshi (Mutsuyoshi et al. 1995)	0.7228	0.1610	0.8613	0.0569
Ng (Ng 2003)	0.8573	0.0971	0.9532	0.0308



(a) Experimental Δf_{ps} versus predicted Δf_{ps}



(b) Experimental f_{ps} versus predicted f_{ps}

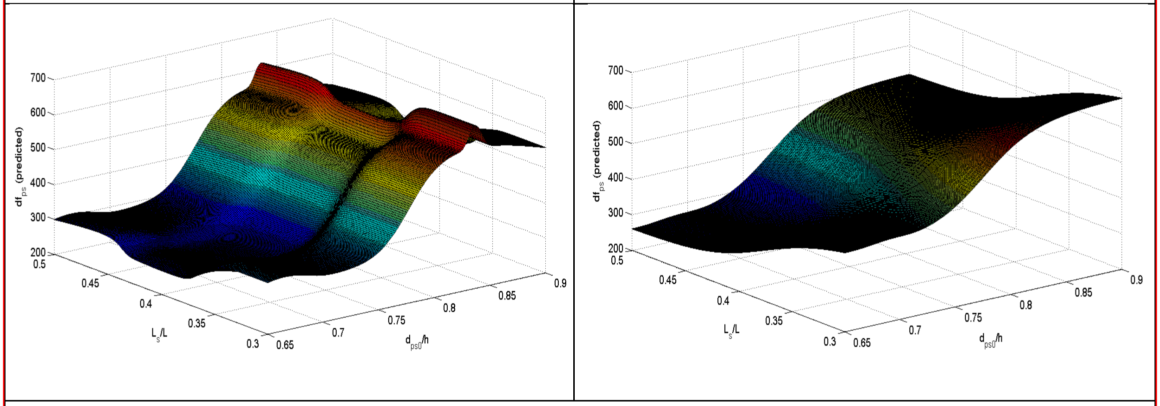
Fig. 7 Comparison of experimental results with predicted values

5.2 Comparison between Model with and without MI

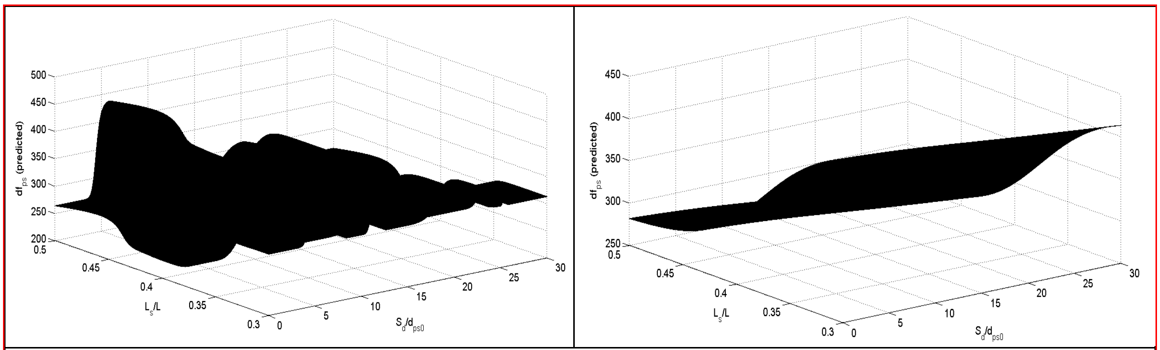
An evaluation of HS-SIRMs connected FIS model with and without MI as a constraint was also carried out in this study. It is observed that although HS-SIRMs connected FIS model without MI yields better correlations and less variability compared to other researchers as in Table 3, the monotonicity property of the model is not preserved as shown in Fig. 8. To improve the validity of the model, MI is considered as a constraint to the HS-SIRMs connected FIS model. It is identified that the model yields slight increase in correlation coefficient value and variability but the monotonicity property of the model is fulfilled as depicted in Fig. 8.

Fig. 8 shows in detail the surface plot for the HS-SIRMs connected FIS model with and without MI as a constraint. By considering S_d/d_{ps0} to be constant (i.e., $S_d/d_{ps0} = 12$), it is observed that when the ratio of d_{ps0}/h increases, the observed output increases monotonically; when the ratio of L_s/L increases, the observed output decreases monotonically. Furthermore, when the ratio of d_{ps0}/h is kept to be constant (i.e., $d_{ps0}/h = 0.7$), the increased in the ratio of S_d/d_{ps0} and L_s/L caused the observed output to decrease monotonically. Eventually, when the ratio of L_s/L is kept constant (i.e., $L_s/L = 0.4$), the increase in the ratio of S_d/d_{ps0} causes the observed output to decrease, while the increase in the ratio of d_{ps0}/h causes the observed output to increase monotonically.

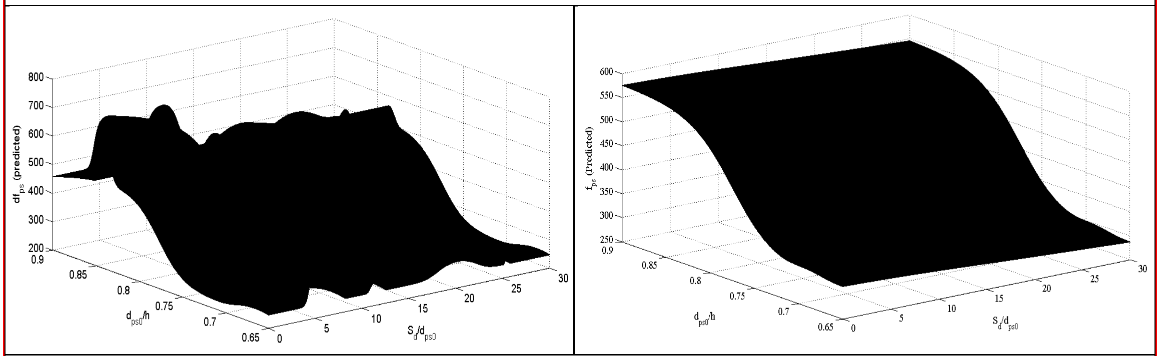
Without MI as a constraint	With MI as constraint



(a) Surface plot for $S_d/d_{ps0} = 12$



(b) Surface plot for $d_{ps0}/h = 0.7$



(c) Surface plot for $L_s/L = 0.4$

Fig. 8 Surface plots for the effects of MI to HS-SIRMs connected FIS model

6.0 Conclusions

In this paper, HS-SIRMs connected FIS model is proposed in a data-driven FIS model. The *objective function* is formulated as a constrained optimization problem. A HS optimization procedure is then used to search for a set of variables that obey the monotonicity property as the sufficient conditions. Experiments are conducted with data obtained from the Ω_u developed by other researchers to study Δf_{ps} and f_{ps} in externally prestressed beams. The results show that the proposed approach is useful to generate a HS-SIRMs connected FIS model with HS optimization procedure for the predicting Ω_u and hence Δf_{ps} and f_{ps} with acceptable computation complexity and error.

The parametric study was carried out for determining Δf_{ps} and f_{ps} in externally prestressed beams. The influential non-dimension parameters include (i) d_{ps0}/h; (ii), S_d/d_{ps0}; and (iii) L_s/L were examined by HS-SIRMs connected FIS model analysis. The proposed model overall provides a better correlation for predicting Δf_{ps} and f_{ps} compared with other existing prediction equations while preserving the monotonicity property of the model.

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