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A boundary integral equation method for the two-dimensional diffusion equation subject to a non-local condition

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Abstract

A boundary integral equation method is proposed for the numerical solution of the two-dimensional diffusion equation subject to a non-local condition. The non-local condition is in the form of a double integral giving the specification of mass in a region which is a subset of the solution domain. A specific test problem is solved using the method. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

In non-dimensionalized form, the partial differential equation that governs two-dimensional linear and isotropic diffusion processes is given by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial t}. \tag{1}$$

A class of problems of practical interest is to solve Eq. (1) for the unknown function $u(x, y, t)$ for time $t \geq 0$ in a two-dimensional region R (on the $0xy$ plane) subject to the initial and boundary conditions

$$u(x, y, 0) = f(x, y) \qquad \text{for } (x, y) \in R, \tag{2}$$

$$u(x, y, t) = g(x, y, t) \qquad \text{for } (x, y) \in C_1 \text{ and } t \geq 0, \tag{3}$$

$$u(x, y, t) = h(x, y)z(t) \qquad \text{for } (x, y) \in C_2 \text{ and } t \geq 0, \tag{4}$$

$$\frac{\partial}{\partial n}[u(x, y, t)] = k(x, y, t) \qquad \text{for } (x, y) \in C_3 \text{ and } t \geq 0, \tag{5}$$

and the non-local (integral) condition

$$\int\int_S u(x, y, t)\,\mathrm{d}x\,\mathrm{d}y = m(t) \qquad \text{for } t \geq 0. \tag{6}$$

where f, g, h, k and m are known and suitably prescribed functions, z is an unknown function to be determined, the region R is bounded by a simple closed curve C, the curves C_1, C_2 and C_3 are non-intersecting and such that $C_1 \cup C_2 \cup C_3 = C$, S is a given subregion of R that is independent of time t and is bounded by a simple closed curve D given by $D = C_2 \cup C_4$, the open curve C_4 lies completely in the interior of R, and $\partial u/\partial n = \mathbf{n}\cdot\nabla u$, $\mathbf{n}$ is the unit normal vector on C pointing away from R. From a physical standpoint, Eq. (6) specifies the total amount of mass of the diffusing quantity u (or the total amount of heat energy, in the case of heat diffusion) which the region S can possess at any time t. Condition (4) with $z(t)$ being unknown implies that the concentration of the diffusing quantity (or the temperature) on some part of the boundary must be controlled in a certain way in order that the region S carries the specified amount of mass (or heat energy). For a sketch of the geometry of the problem, refer to Fig. 1.

The class of problems defined by Eqs. (1)–(6) arises in many practical applications in heat transfer, control theory, thermoelasticity and medical sciences. A specific application which involves the use of the absorption of light to measure the concentration of a diffusing chemical is described by Noye and Dehghan [6]. Special cases of the problem, such as R being a rectangular region, have been solved directly by many researchers, e.g. Gumel et al. [5], Noye and Dehghan [6,7], Noye et al. [8], Cannon et al. [3], and Wang and Lin [11] using the finite-difference methods. With the exception of Ref. [6], the case $S = R$, i.e. $C_4 = C_1 \cup C_3$, was studied in all the references just cited.

The present paper makes use of a boundary integral equation method (BIEM) for the numerical solution of Eqs. (1)–(6) in the Laplace transform (LT) space. The physical solution is recovered by using the Stehfest

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