

# Monotone Fuzzy Rule Relabeling for the Zero-Order TSK Fuzzy Inference System

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**Abstract**—To maintain the monotonicity property of a fuzzy inference system, a monotonically-ordered and complete set of fuzzy rules is necessary. However, monotonically-ordered fuzzy rules are not always available, e.g. errors in human judgements lead to non-monotone fuzzy rules. The focus of this paper is on a new monotone fuzzy rule relabeling (MFRR) method that is able to relabel a set of non-monotone fuzzy rules to meet the monotonicity property with reduced computation. Unlike the brute-force approach, which is susceptible to the combinatorial explosion problem, the proposed MFRR method explores within a reduced search space to find the solutions; therefore decreasing the computational requirements. The usefulness of the proposed method in undertaking Failure Mode and Effect Analysis problems is demonstrated using publicly available information. The results indicate that the MFRR method can produce optimal solutions with reduced computational time.

**Index Terms**— TSK Fuzzy Inference system, monotonicity property, fuzzy rules relabeling, Failure Mode and Effect Analysis

## I. INTRODUCTION

FUZZY inference systems (FISs) constitute a popular computing framework that has been successfully applied to solving different problems [1], [2]. Two popular variants are the Mamdani-type [3] and Takagi-Sugeno-Kang (TSK)-type [4], [5] of FIS models. A number of methods to construct FIS models have been proposed in the literature. Among the commonly used methods include gathering fuzzy rules from humans [3], using (multi-objective) evolutionary computation optimization or tuning [6] techniques, neural learning techniques [2], [7], or the Wang-Mendel [8], [9] technique.

In regards to FIS modelling, the importance of the monotonicity property has been highlighted in a number of recent publications [10]-[16]. The key reasons that demand the monotonicity property are as follows: (i) many real-world systems and control problems obey the monotonicity property between the input and output [10], [12]; (ii) the validity of the FIS output needs to be ensured for undertaking comparison,

selection, and decision making problems [12], [13], [17]-[19]; (iii) in the case where the number of data samples is small or the fuzzy rule set is incomplete, it is important to fully exploit the available qualitative information or knowledge pertaining to the system [11], [18], [19]; (iv) additional qualitative information or knowledge of the system is useful for overcoming issues related to noise and inconsistencies in the data samples, as well as the over-fitting problem [11].

In this paper, we focus on analysing the monotonicity property of the zero-order TSK-FIS model constructed with fuzzy rules from humans. Note that the zero-order TSK-FIS model can also be considered as a special case of the Mamdani-FIS model [20]. The sufficient conditions for constructing a monotone TSK-FIS model were first derived in [10]. The sufficient conditions suggest that a monotone fuzzy rule base is important [14] (a must as stated in [15]) for constructing a monotone zero-order TSK-FIS model. In terms of applications, the sufficient conditions are useful in reliability engineering, e.g. in Failure Mode and Effect analysis (FMEA) [18], [19].

Despite the popularity of the TSK-FIS model, to the best of our knowledge, most of the studies in the existing literature assume that fuzzy rules collected from humans are free from judgement errors. This, however, may not be the case in practice. Indeed, when fuzzy rules are gathered from humans, it is possible for the resulting fuzzy rule base to violate the monotonicity property due to judgement errors. In this paper, we argue it is a challenge task, in practice, to always ensure that fuzzy rules gathered from humans (even domain experts) are free from judgement errors [21]. In this paper, such judgement errors are defined as noise that can lead to a set of non-monotone fuzzy rules.

In regards to the traditional TSK-FIS model, it is not clear how noisy fuzzy rules should be handled in order to produce a monotone FIS model [4], [5]. To handle the presence of noise in the fuzzy rule base, the monotone fuzzy rule relabeling (MFRR) method has been proposed [22], [23]. MFRR attempts to identify noisy fuzzy rules and relabel them to become a set of monotone fuzzy rules. The obtained new fuzzy rules should fulfil the following three criteria: (1) satisfying the monotonicity property as the first priority; (2) having the minimum number of relabelled rules as the second priority; (3) having the minimum loss measure as the third priority. In our previous research, the genetic algorithm (GA) was used to search for a set of monotone fuzzy rules [22], but the finding indicated that an optimal solution could hardly be obtained. A brute-force fuzzy rule relabelling (BFRR) method was developed in [23]. Even though BFRR could produce a set of

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